1. The points A, B and C have position vectors $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ respectively. M is the midpoint of BC.

(a) Find the position vector of the point D such that $\overrightarrow{BC} = \overrightarrow{AD}$. [3]

(b) Find the magnitude of $\overrightarrow{AM}$. [3]

2. The point A has position vector **i** – 2**j**. The point B is such that $|\overrightarrow{OB}| = |\overrightarrow{OA}|$ and $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OA}$.

(a) (i) Find $|\overrightarrow{OB}|$. [2]

(ii) Find the two possible directions of $\overrightarrow{OB}$, giving your answers correct to the nearest degree. [2]

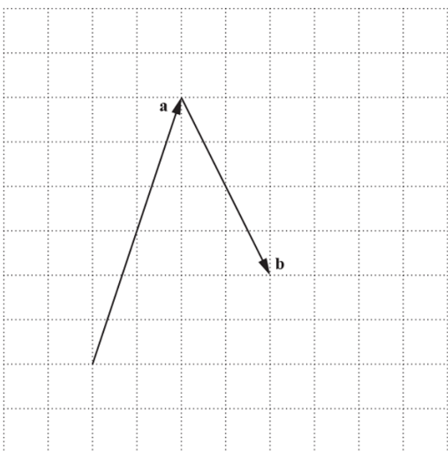
The point C is such that $|\overrightarrow{AC}| = 2$.

(b) Find the maximum and minimum values of $|\overrightarrow{OC}|$. [4]

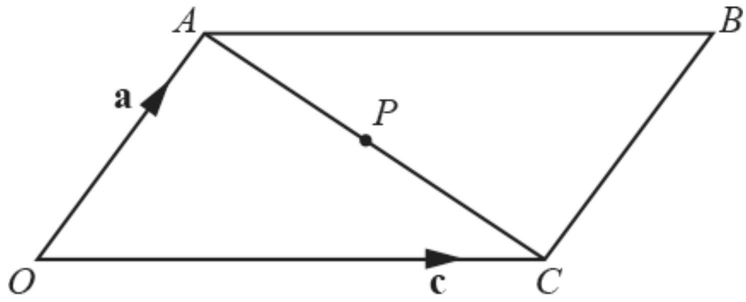
3. Vectors **a** and **b** are defined as follows: **a** = 2**i** + 6**j** and **b** = 2**i** – 4**j**.

(a) Given that p**a** + q**b** = 6**i** – 7**j**, find the values of the constants p and q. [3]

(b) It is now given instead that |**a** + k**b**| = 5. Use the diagram below to find the two possible values of the constant k. [4]



4. $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is the midpoint of AC.



(a) Find the following in terms of **a** and **c**, simplifying your answers.

(i) $\overrightarrow{AC}$ [1]

(ii) $\overrightarrow{OP}$ [2]

(b) Hence prove that the diagonals of a parallelogram bisect one another. [4]

5. Vector $\mathbf{v} = a\mathbf{i} + 0.6\mathbf{j}$, where a is a constant.

(a) Given that the direction of **v** is 45°, state the value of a. [1]

(b) Given instead that **v** is parallel to $8\mathbf{i} + 3\mathbf{j}$, find the value of a. [2]

(c) Given instead that **v** is a unit vector, find the possible values of a. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		a	$\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \mathbf{d} - \mathbf{a} = \overrightarrow{AD}$ $\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$	B1(AO1.1) M1(AO3.1a) A1(AO1.1) [3]	soi
		b	$\overrightarrow{OM} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $\overrightarrow{AM} = \overrightarrow{OM} - \overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$ $\lvert\overrightarrow{AM}\rvert = \sqrt{6^2 + 3^2} = 3\sqrt{5}$	B1(AO1.1) M1(AO1.1) A1(AO2.2a) [3]	soi Accept 6.71
			Total	6	
2		a	i) $\lvert\overrightarrow{OB}\rvert = \sqrt{1^2 + 2^2}$ Mag = $\sqrt{5}$ or 2.24 (3 sf)	M1(AO1.2) A1(AO1.1) [2]	
		a	ii) Direction (= $\tan^{-1}(0.5)$) = 27° & (180° + 27° or $\tan^{-1}(-0.5)$) = 207°	M1(AO1.1a) A1f(AO1.1) [2]	ft their 27°
		b	For max & min *OC*, *C* lies on *OA* *OC* = *OA* ± 2 Max *OC* = $\sqrt{5} + 2$ or 4.24 (3 sf) Min *OC* = $\sqrt{5} - 2$ or 0.236 (3 sf)	M1(AO2.1) M1(AO3.1a) A1(AO2.2a) A1(AO1.1) [4]	May be implied, eg by diagram Their *OA* (from (a)) ± 2
			Total	8	
3		a	$2p + 2q = 6$ $6p - 4q = -7$ eg $4p + 4q = 12$	B1(AO3.1a)	

			$10p = 5$ $p = 0.5,\ q = 2.5$	M1(AO 1.1) A1(AO 1.1) [3]	Both Correct method to solve and achieve any correct equation in either p or q Both
		b	Vectors $3\mathbf{i} + 4\mathbf{j}$ and $5\mathbf{i}$ shown on diagram, each starting at start point of vector **a** $k = 0.5$ or 1.5	(AO1.2) B1B1(AO1.1) B1(AO2.2a) B1(AO1.1) [4]	or just end points of these vectors shown
			Total	7	
4		`	Allow without arrows or squiggles throughout		<u>Examiner's Comments</u> In all three parts of this question, many candidates did not use correct vector notation.
		a	(i) c – a oe	B1 (AO1.2) [1]	<u>Examiner's Comments</u> Almost all candidates answered this question correctly.
		a	(ii) $\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$ or $\mathbf{c} + \frac{1}{2}(\mathbf{a} - \mathbf{c})$ $= \frac{1}{2}(\mathbf{a} + \mathbf{c})$ or $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$	M1 (AO3.1a) A1 (AO1.1b) [2]	$\mathbf{a} + \frac{1}{2}$ their (i) or $\mathbf{c} - \frac{1}{2}$ their (i) Correct ans without wking: M1A1

					Examiner's Comments Most answered this question correctly. A few made a sign error, for example $\mathbf{c} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$.
		b	$\overrightarrow{OB} = (\mathbf{a} + \mathbf{c})$ $\Rightarrow \overrightarrow{OP} = \frac{1}{2}\overrightarrow{OB}$ Must see previous line ⇒ P is midpt of OB or OPB is a straight line and OP = PB Hence diagonals of /m bisect one another	M1 (AO3.1a) A1* (AO1.1) dep*A1 (AO2.1) E1 (AO2.2a) [4]	$\overrightarrow{PB} = \mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$ or $\mathbf{a} + \frac{1}{2}$ their(a)(i) or $\mathbf{c} + \frac{1}{2}(\mathbf{a} - \mathbf{c})$ (= $\frac{1}{2}(\mathbf{a}+\mathbf{c})$ oe) ft their (a)(i) or $\overrightarrow{PB} = \mathbf{c} - \frac{1}{2}$ their(a)(i) NB $\overrightarrow{PB} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$ without justification: M0A0A0E0 $\Rightarrow \overrightarrow{PB} = \overrightarrow{OP}$ or similar with $\overrightarrow{BP}$ or $\overrightarrow{BO}$ dep M1A1A1 **Examiner's Comments** This question proved challenging for a significant majority of candidates. Many assumed the result by starting with, for example, $PB = \frac{1}{2}(\mathbf{c} + \mathbf{a})$, instead of deriving this result. Some candidates considered the modulus of some vectors. Some candidates seemed unaware of the meaning of the word "bisect", in

					some cases confusing it with "perpendicular". Thus many wrote that a + c is perpendicular to a – c, and that this somehow proves that the diagonals bisect one another. Perhaps the majority of candidates did not know how to start answering this question at all. An example of a candidate's solution that suggested they had no understanding of proof by vectors was as follows: "BO = AC. As they are the same length it means they would both meet in the centre, hence meaning they bisect one another."
			Total	7	
5		a	$a = 0.6$	B1 (AO 1.2) [1]	State correct value for *a*
		b	$3k = 0.6$, so $k = 0.2$ $a = 8 \times 0.2 = 1.6$	M1 (AO 1.1a) A1 (AO 1.1) [2]	Attempt to find scale factor Obtain $a = 1.6$ **OR** $0.6k = 3$, so $k = 5$
		c	$\sqrt{a^2 + 0.6^2} = 1$ $a^2 = 0.64$ $a = \pm 0.8$	B1 (AO 1.2) M1 (AO 1.1a) A1 (AO 1.1) [3]	Correct definition for unit vector seen or implied Attempt to find at least one value for *a* Both correct values for *a* Allow BOD for $a^2 + 0.6^2 = 1$, with no square root seen
			Total	6	