1.

i. Show that the equation $2\sin x = \dfrac{4\cos x - 1}{\tan x}$ can be expressed in the form

$6\cos^2 x - \cos x\, 2 = 0$.

[3]

ii. Hence solve the equation $2\sin x = \dfrac{4\cos x - 1}{\tan x}$, giving all values of x between 0° and 360°.

[4]

2. Solve each of the following equations, for $0° \leqslant x \leqslant 360°$.

i. $\sin\frac{1}{2}x = 0.8$

[3]

ii. $\sin x = 3\cos x$

[3]

3. i. Show that the equation

$$\sin x - \cos x = \frac{6\cos x}{\tan x}$$

can be expressed in the form

$$\tan^2 x - \tan x - 6 = 0.$$

[2]

ii. Hence solve the equation $\sin x - \cos x = \dfrac{6\cos x}{\tan x}$ for $0° \leqslant x \leqslant 360°$.

[4]

4. **In this question you must show detailed reasoning.**

Solve the equation $2\cos^2 x = 2 - \sin x$ for $0° \leq x \leq 180°$. [5]

5. **In this question you must show detailed reasoning.**

Solve the equation $3 \sin^2\theta - 2 \cos \theta - 2 = 0$ for $0° \leq \theta \leq 360°$. [5]

6. **In this question you must show detailed reasoning.**

Given that $5\sin 2x = 3\cos x$, where $0° < x < 90°$, find the exact value of $\sin x$. [4]

7. **In this question you must show detailed reasoning.**

Solve the equation $\tan 2x = -\sqrt{3}$ for $0° \leqslant x < 360°$. [5]

8. The cubic polynomial $f(x)$ is defined by $f(x) = 4x^3 + 9x - 5$.

(a) Show that $(2x - 1)$ is a factor of $f(x)$ and hence express $f(x)$ as the product of a linear factor and a quadratic factor. [4]

(b) (a) Show that the equation

$$4\sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13\tan 2\theta$$

can be expressed in the form

$$4 \sin^3 2\theta + 9 \sin 2\theta - 5 = 0.$$ [4]

(b) Hence solve the equation

$$4\sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = 13\tan 2\theta$$

for $0 \leq \theta \leq 2p$. Give each answer in an exact form. [4]

9. (a) Show that the equation

$$2 \sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ [3]

(b) Hence solve the equation

$$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of θ between 0° and 180°, correct to 1 decimal place. [5]

10. (a) Solve the equation $\sin^2\theta = 0.25$ for $0° \le \theta < 360°$. [3]

(b) **In this question you must show detailed reasoning.**

Solve the equation $\tan 3\phi = \sqrt{3}$ for $0° \le \emptyset < 90°$. [3]

11. (a) Given that $\sqrt{2\sin^2\theta + \cos\theta} = 2\cos\theta$, show that $6\cos^2\theta - \cos\theta - 2 = 0$. [2]

(b) In this question you must show detailed reasoning. [3]

Solve the equation

$$6\cos^2\theta - \cos\theta - 2 = 0,$$

giving all values of i between 0° and 360° correct to 1 decimal place. [4]

(c) Explain why not all the solutions from part **(b)** are solutions of the equation

[1]

$$\sqrt{2\sin^2\theta + \cos\theta} = 2\cos\theta.$$

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$2\sin x\ {}^{\sin x}/_{\cos x} = 4\cos x - 1$ $2\sin^2 x = 4\cos^2 x - \cos x$ $2 - 2\cos^2 x = 4\cos^2 x - \cos x$ $6\cos^2 x - \cos x - 2 = 0$ AG	M1	Use $\tan x = {}^{\sin x}/_{\cos x}$ and rearrange to a form not involving fractions	Must be used and not just stated Must multiply all terms by $\cos x$ so $4\cos^2 x - 1$ is M0, but allow M1 for $\cos x(4\cos x - 1)$ even if subsequent errors
		i		M1	Use $\sin^2 x = 1 - \cos^2 x$	Must be used and not just stated Must be used correctly, so M0 for $1 - 2\cos^2 x$ Not dependent on previous M mark, so M0 M1 possible Must be attempting quadratic in $\cos x$ so M0 for $\cos^2 x = 1 - \sin^2 x$
		i		A1	Obtain $6\cos^2 x - \cos x - 2 = 0$ with no errors seen	Must be equation ie = 0 Allow poor notation (eg cos not $\cos x$, or $\tan x = {}^{\sin}/_{\cos}(x)$) as long as final answer is correct __Examiner's Comments__ Most candidates could quote both of the required identities and then attempt to use them. Whilst $\sin^2 x = 1 - \cos x$ was usually used correctly, the use of $\tan x$ caused more problems as candidates were expected to also deal with the fraction to gain the method mark and a number struggled to do so. Some candidates used poor notation, such as omitting the x from their trigonometric ratios, and others spoiled an otherwise

						correct solution by failing to give an equation as their final answer.
		ii	$(3\cos x - 2)(2\cos x + 1) = 0$ $\cos x = {}^2/_3$, $\cos x = {}^{-1}/_2$ $x = 48.2°, 312°, 120°, 240°$	M1	Attempt to solve quadratic in $\cos x$	This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods) Condone any substitution used, including $x = \cos x$
		ii		M1	Attempt to find x from root(s) of quadratic	Attempt $\cos^{-1}$ of at least one of their roots Allow for just stating $\cos^{-1}$(their root) inc if $\lvert\cos x\rvert > 1$ Not dependent so M0 M1 possible If going straight from $\cos x = k$ to $x =$ …then award M1 only if their angle is consistent with their k
		ii		A1	Obtain at least 2 correct angles	Allow 3sf or better Must come from correct solution of quadratic – ie correct factorisation or correct substitution into formula so A0 if two correct roots from eg $(3\cos x + 2)(2\cos x + 1) = 0$ Allow radian equivs – 0.841, 5.44, ${}^{2\pi}/_3$ or 2.09, ${}^{4\pi}/_3$ or 4.19
		ii		A1	Obtain all 4 correct angles, with no extra in given range	Must now be in degrees **SR** If no working shown then allow **B1** for 2 correct angles (poss in rads) or **B2** for 4 correct angles, no extras Examiner's Comments This part of the question was done very well by the majority of the candidates, who were able to identify the fact that

						the given equation was a quadratic in cosx and attempt an appropriate method to solve it. The four required roots then usually followed, though some candidates struggled to find the secondary angles with 318.2° being a common wrong answer. Others lost marks by discarding the negative root to the quadratic, failing to realise that this would also lead to valid solutions.
			Total	7		
2		i	$\frac{1}{2}x = 53.1°, 126.9°$	B1	Obtain 106°, or better	Allow answers in the range [106.2, 106.3] Ignore any other solutions for this mark Must be in degrees, so 1.85 rad is B0
		i	$x = 106°, 254°$	M1	Attempt correct solution method to find second angle	Could be 2(180° – their 53.1°) or (360° – their 106°) Allow valid method in radians, but M0 for eg (360 – 1.85)
		i		A1	Obtain 254°, or better **Examiner's Comments** Most candidates were able to correctly find the first angle though a few halved rather than doubled the result of $\sin^{-1} 0.8$. Finding the second angle proved more challenging with the most common error being to simply subtract their first answer from 180°. The fact that this resulted in a second angle that was smaller than the first did not seem to deter them. Whilst some candidates were able to use the symmetry of the $\sin\frac{1}{2}x$ graph to find the second angle, the more successful method was to find the possible solutions for $\frac{1}{2}x$ from the sin x graph and then double all the solutions.	Allow answers in the range [253.7°, 254°] A0 if in radians (4.43) A0 if extra incorrect solutions in range **SR** If no working shown then allow B1 for 106° and B2 for 254° (max B2 if additional incorrect angles)

		ii	$\tan x = 3$	B1	State $\tan x = 3$	Allow B1 for correct equation even if no, or an incorrect, attempt to solve Give BOD on notation eg $^{\sin}/_{\cos}(x)$ as long as correct equation is seen or implied at some stage
		ii	$x = 71.6°$, $252°$	M1	Attempt to solve $\tan x = k$	Not dep on B1, so could gain M1 for solving eg $\tan x = ⅓$ Could be implied by a correct solution
		ii		A1	Obtain 71.6° and 252°, or better **Examiner's Comments** This part of the question was better attempted, and most candidates scored full marks with ease. A few struggled to find the second angle, or lost the final mark through a lack of precision when rounding. Some candidates made life difficult for themselves by attempting to square both sides and use $\sin^2 x + \cos^2 x \equiv 1$, but this was very rarely done correctly. Potential pitfalls included forgetting to square the coefficient of 3, omitting to use both the positive and negative square roots and finally realising that only two of the four solutions were valid. At least this was a valid method, which could not be said for those candidates who started with $\sin x + \cos x = 1$.	A0 if extra incorrect solutions in range **Alt method:** **B1** Obtain $10\sin^2 x = 9$ or $10\cos^2 x = 1$ **M1** Attempt to solve $\sin^2 x = k$ or $\cos^2 x = k$ (allow M1 if just the positive square root used) **A1** Obtain 71.6° and 252°, with no extra incorrect solutions in range **SR** If no working shown at all then allow B1 for each correct angle (max B1 if additional incorrect angles), but allow full credit if $\tan x = 3$ seen first
			Total	6		
3		i	$\tan x(\sin x - \cos x) = 6 \cos x$ $\tan x(^{\sin x}/_{\cos x} - 1) = 6$ $\tan x(\tan x - 1) = 6$	M1	Use $\tan x = \frac{\sin x}{\cos x}$ correctly once	Must be used clearly at least once – either explicitly or by writing eg 'divide by $\cos x$' at side of solution Allow M1 for any equiv eg $\sin x = \cos x \tan x$ Allow poor notation eg writing just tan rather than $\tan x$
		i	$\tan^2 x - \tan x = 6$ $\tan^2 x - \tan x - 6 = 0$ **AG**	A1	Obtain $\tan^2 x - \tan x - 6 = 0$	Correct equation in given form, including = 0

					Examiner's Comments A variety of methods were seen for this proof, some more efficient than others. Most candidates did get there in the end, but full credit was only given if the correct notation had been used throughout. Candidates must also ensure that each step is clearly and convincingly detailed when a proof has been requested.	Correct notation throughout so A0 if eg tan rather than tan$x$ seen in solution
		ii	$(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3$, $\tan x = -2$	M1	Attempt to solve quadratic in tan x	This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods) Condone any substitution used, inc $x = \tan x$
		ii	$x = \tan^{-1}(3)$, $x = \tan^{-1}(-2)$	M1	Attempt to solve tan $x = k$ at least once	Attempt $\tan^{-1}k$ at least once Not dependent on previous mark so M0M1 possible If going straight from tan $x = k$ to $x = \ldots$, then award M1 only if their angle is consistent with their k
		ii	$x = 71.6°, 252°, 117°, 297°$	A1	Obtain two correct solutions	Allow 3sf or better Must come from a correct method to solve the quadratic (as far as correct factorisation or substitution into formula) Allow radian equivs ie 1.25 / 4.39 / 2.03 / 5.18
		ii		A1	Obtain all 4 correct solutions, and no others in range **Examiner's Comments** This question was generally very well done, and many candidates gained full marks on this question. The most common error was to completely discount the solution resulting from $\tan^{-1}(-2)$ as it resulted in a negative angle rather than appreciating it would still generate other angles within the given range. It was also disappointing to see candidates with a correct method failing to gain full marks due to rounding errors. As in previous questions involving	Must now all be in degrees Allow 3sf or better A0 if other incorrect solutions in range 0° – 360° (but ignore any outside this range) **SR** If no working shown then allow **B1** for each correct solution (max of **B3** if in radians, or if extra solns in range).

					trigonometry, some candidates did not ensure their calculator was in the correct mode before proceeding. Angles given in radians could gain some credit, but candidates did not actually consider which measure they were using so the typical error was $\tan^{-1}(3) = 1.25$ and hence 189.25.	
			Total	6		
4			DR $2(1 - \sin^2 x) = 2 - \sin x$ $2\sin^2 x - \sin x = 0$ $\sin x(2\sin x - 1) = 0$ $\sin x = \frac{1}{2}$ so $x = 30$ or $x = 150$ $\sin x = 0$ so $x = 0$ or $x = 180$	M1(AO3.1a) A1(AO1.1) M1(AO1.1a) A1(AO1.1) A1(AO1.1) [5]	Use $\cos^2 x = 1 - \sin^2 x$ and simplify Obtain $2\sin^2 x - 1\sin x = 0$ Attempt to solve a 2 term quadratic in sin x and use correct order of operations to obtain x Both values are required Both values are required	One step of simplification must be seen Use any valid method Must be seen
			Total	5		
5			DR $3(1 - \cos^2\theta) - 2\cos\theta - 2 = 0$ $3\cos^2\theta + 2\cos\theta - 1 = 0$	M1(AO3.1a) A1(AO1.1) M1(AO1.1a)	Attempt to use $\sin^2\theta = 1 - \cos^2\theta$	

			$(3\cos\theta - 1)(\cos\theta + 1) = 0$ $\cos\theta = {}^1/_3$ $\cos\theta = -1$ $\theta = 70.5^\circ, 289^\circ, 180^\circ$	A1(AO2.2a) A1(AO1.2) [5]	Obtain correct equation Attempt to solve quadratic Obtain at least two correct angles Obtain all 3 angles, and no others	Factorise or **BC**
			Total	5		
6			DR $5\sin 2x = 3\cos x \Rightarrow 10\sin x\cos x = 3\cos x$ $\cos x(10\sin x - 3) = 0$ $\cos x \neq 0$ for $0° < x < 90°$ so $\sin x = \frac{3}{10}$	B1(AO 1.1) M1(AO1.1a) E1(AO2.1) A1(AO1.1) [4]	Use $\sin 2x = 2\sin x\cos x$ to obtain correct identity Attempt to factorise	**SC2** For use of identity followed by cancelling $\cos x$, leading to $\sin x = \frac{3}{10}$.
			Total	4		
7			**DR** $2x = -60$	M1(AO1.1) M1(AO2.1)		

			2x = 180 – 60 or 360 – 60 x = 60 or 150 2x = 120 + 360 or 300 + 360 x = 60 or 150 or 240 or 330 | OR 2x = 120 OR 2x = 300 2x = 120 or 300 x = 60 or 150 x = 60 + 180 or 150 + 180 x = 60 or 150 or 240 or 330	A1(AO1.1) M1(AO2.1) A1(AO1.1) [5]	2x = –60 or x = –30° x = –30 + 90 or –30 + 2×90 x = 60 or 150 x = –30 + 3×90 or –30 + 4×90 | one value enough for M1 both both both all four	
			Total	5		
8		i	$f(^1/_2) = {}^1/_2 + {}^9/_2 - 5 = 0$ $f(x) = (2x - 1)(2x^2 + x + 5)$	B1 M1	Confirm f(½) = 0, with detail shown | 4(½)^3 + 9(½) – 5 = 0 is sufficient B0 for just f(½) = 0 No conclusion needed If using division to justify then must draw attention to the zero remainder Attempt complete division or equiv | Must be dividing by (2x – 1) Must be complete method - ie all 3 terms attempted Long division - must subtract lower line (allow one slip) Inspection - expansion must give at least three correct terms of the cubic Coefficient matching - must be valid attempt at all	

				Marks		Guidance	
				A1 A1 [4]	Obtain correct quotient	coeffs of quadratic, considering all relevant terms each time Synthetic division - must be using 0.5 (not – 0.5) and adding within each column (allow one slip); expect to see $\begin{array}{c\|cccc} 0.5 & 4 & 0 & 9 & -5 \\ & & 2 & 1 & \\ \hline & 4 & 2 & 10 & \end{array}$ Allow $4x^2 + 2x + 10$ from dividing by $x - ½$ Must be written as a product Allow $(x - ½)$ $(4x^2 + 2x + 10)$ ISW any attempt to write as 3 linear factors, or to find roots	

					Obtain $(2x-1)(2x^2+x+5)$ **Examiner's Comments** This part of the question was very well answered, with the majority of the candidates able to produce a correct product. Algebraic long division was the most common approach and candidates coped well with the lack of a quadratic term. Coefficient matching was less common, but was equally successful. Most candidates made explicit use of the factor theorem to show that $(2x-1)$ was a factor, and this invariably had sufficient detail to be convincing. If using algebraic long division it was not sufficient to simply have a 0 on the bottom line; attention had to be drawn to the lack of remainder for the mark to be awarded.	
		ii	(a) $4\sin 2\theta\cos 2\theta + \frac{5}{\cos 2\theta} = \frac{13\sin 2\theta}{\cos 2\theta}$ $4\sin 2\theta\cos^2 2\theta + 5 = 13\sin 2\theta$ $4\sin 2\theta(1 - \sin^2 2\theta) + 5 = 13\sin 2\theta$ $4\sin 2\theta - 4\sin^3 2\theta + 5 = 13\sin 2\theta$	B1 B1 B1	Use $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ or $\tan 2\theta\cos 2\theta = \sin 2\theta$ — Must be explicit, and correct notation when used. Allow even if errors elsewhere in equation Correct method to remove fraction(s) — Any correct equation seen no longer containing fractions (allow recovery from a slip in notation) Use $\cos^2 2\theta = 1 - \sin^2 2\theta$ — Must be explicit, and correct notation when used Allow even if errors elsewhere in equation	

				Mark	Guidance		
			$4\sin^3 2\theta + 9\sin 2\theta - 5 = 0$	B1 [4]	Obtain correct equation, from correct working	Must be correct notation throughout Dependent on B1 B1 B1 awarded **NB - must annotate answer space**	
					Examiner's Comments Candidates were clearly familiar with the relevant identities but many lacked the mathematical precision required for the marks to be awarded. The most common errors were to have the indices incorrectly placed and for the coefficient of 2 to disappear. Even if these errors were later corrected, the identity had to be fully correct at the point of use for the mark to be awarded. Candidates should also appreciate that each step in a proof should be clearly detailed; in some cases a number of steps were run together resulting in a lack of clarity of argument.		
		iii	(b) $(2\sin 2\theta - 1)(2\sin^2 2\theta + \sin 2\theta + 5) = 0$ $\sin 2\theta = {}^1/_2$	B1	State that $\sin 2\theta = ½$ oe	Could just be stated, or implied by later method	
			$2\theta = \frac{1}{6}\pi,\ \frac{5}{6}\pi, \frac{13}{6}\pi, \frac{17}{6}\pi$	M1	Attempt to solve $\sin 2\theta = \pm ½$ to find at least one root	Correct order of operations ie ½ ($\sin^{-1}$ ½) Allow M1 if angle(s) found in degrees (15°, 75° etc)	
			$\theta = \frac{1}{12}\pi,\ \frac{5}{12}\pi, \frac{13}{12}\pi, \frac{17}{12}\pi$	A1	Obtain at least 2 correct roots	Must be in radians, and given in an exact form Allow recurring	

				A1 [4]	<table><tr><td></td><td>decimals, or mixed numbers</td></tr><tr><td>Obtain 4 correct roots</td><td>Must be in radians, and given in an exact form Allow recurring decimals, or mixed numbers ISW any angles that come from an incorrect quadratic quotient, or an incorrect attempt to find the roots of the quadratic quotient A0 if any extras in range [0, 2p] that are not clearly from their quadratic roots</td></tr></table> **Examiner's Comments** Most candidates recognised the link with the previous part of the question and appreciated that they had to solve the cubic in sin2θ. However a significant minority did not realise that this was related to the cubic that they had already factorised in part (i) and made a fresh attempt to solve, sometimes even attempting the use of the quadratic formula on the cubic. Many did recognise the link and attempted to use the root of ½, but this was sometimes equated to sinθ rather than sin2θ. Candidates who correctly stated that sin2θ = ½ were usually able to solve the equation to find at least two roots, with only the most astute candidates giving all 4 roots as their answer. Candidates seem to be comfortable working with angles in radians in an exact form, and there were only a few instances of angles being given in degrees instead.	
			Total	12		

9	a	$$2\sin x\left(\frac{\sin x}{\cos x}\right)=\cos x+5$$ $2\sin^2 x = \cos^2 x + 5\cos x$ $2(1-\cos^2 x) = \cos^2 x + 5\cos x$ $2 - 2\cos^2 x = \cos^2 x + 5\cos x$ $3\cos^2 x + 5\cos x - 2 = 0$	M1 (AO 3.1a) M1 (AO 3.1a) A1 (AO 2.1) [3]	Uses $\tan x = \sin x / \cos x$ Uses $\sin^2 x = 1 - \cos^2 x$ **AG** – correct working throughout | Must show sufficient working to justify the given answer <u>Examiner's Comments</u> This question was done well by many. Because the answer aimed for was given, examiners paid close attention to the written detail and expected fully accurate working for the answer mark. Removing the denominator sometimes led to error, with $\cos^2 x + 5$ appearing on the RHS, and less regularly $2\sin x\cos x\sin x$ on the LHS.	
	b	$(3\cos 2\theta - 1)(\cos 2\theta + 2) = 0$ $$\cos 2\theta=\frac{1}{3}\ \left(\text{and }\cos 2\theta=-2\right)$$ $$\theta=\frac{1}{2}\arccos\left(\frac{1}{3}\right)$$	M1 (AO 1.1a) A1 (AO 1.1) M1 (AO 1.1)	Attempt to solve 3-term quadratic Condone $\cos x=\frac{1}{3}$ Correct order of operation to find one value of θ (or both | $(2\theta=)70.52877\ldots,$	

			$\theta = 35.3°$	A1 (AO 1.1)	values of 2θ correct) One correct value to the nearest integer or better	289.471…	
			$\theta = 144.7°$	A1 (AO 1.1) [5]	Cao (35.3 and 144.7)	Any additional values in the range loses final A mark if earned	
					<u>Examiner's Comments</u> This part starts with the word 'Hence' and what we expect from this is clear in the Specification Document. Most did start by solving a quadratic, not always producing two angles in the end. To gain full credit the two angles had to be given correct to 1 decimal place as requested. 144.8° was not uncommon. Some did not grasp the significance of 2θ and left the answer as 70.5°.		
			Total	8			
10		a	$\sin\theta = 0.5$ and -0.5 or $\sin\theta = \pm\sqrt{0.25}$ both	B1 (AO1.1a)	"–0.5" may be implied by **all 4 answers** Ignore other answers for this B1	$\sin\theta = 0.5$, $\theta = 30$ and 210 B0B0B0	
			$\theta = 30°$ and $150°$	B1 (AO1.1)			
			$\theta = 210°$ and $330°$	B1 (AO1.1)	NB Correct ans with no wking: B1B1B1	$\sin\theta = \pm0.5$, $\theta = 30$ and 210 B1B0B0	

				[3]	**Examiner's Comments** Many candidates omitted sin θ = –0.5, usually obtaining 30° but not always 150°. Some of those who included sinθ = –0.5 only gave one of the two other answers. Some candidates found $\sin^{-1}(0.25) = 14.5°$. Some of these then found 14.5^2 or $\sqrt{14.5}$.	
		b	DR 60° and 240° seen or implied 20° seen ø = 20°, 80° With no other sol'ns	B1 (AO1.1a) B1 (AO1.1) B1 (AO1.1) [3]	Both needed, but ignore other values SC: correct ans with no wking: B0B1B0 **Examiner's Comments** Most candidates obtained 3ϕ = 60° but not all included 3ϕ = 240o. A few started with $\tan\phi = \frac{\sqrt{3}}{3}$ and hence ϕ = 30. Some thought that $\tan^{-1}(\sqrt{3}) = 30°$. Some gave extraneous answers obtained by, for example, (180° – 60°) ÷ 3. Others gave answers outside the given	

					domain. Some gave answers in radians.	
			Total	6		
11		a	$2(1 - \cos^2\theta) + \cos\theta = 4\cos^2\theta$ $2 - 2\cos^2\theta + \cos\theta = 4\cos^2\theta$ $6\cos^2\theta - \cos\theta - 2 = 0$	M1 (AO3.1a) A1 (AO2.2a) [2]	Correctly removing square root and use of $\sin^2\theta = 1 - \cos^2\theta$ to obtain an equation in cos only **AG** – sufficient working must be shown to establish given result	
		b	DR $(2\cos\theta + 1)(3\cos\theta - 2) = 0$ $\cos\theta = -\frac{1}{2}$ and $\cos\theta = \frac{2}{3}$ $\cos\theta = \frac{2}{3} \Rightarrow \theta = 48.2,\ 311.8$ $\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120,\ 240$	M1 (AO1.1) A1 (AO1.1) A1 (AO1.1) M1 (AO2.2a) [4]	Correct method for solving quadratic — May use formula or completing the square Any two correct values — 48.189…, 311.810… All four correct values — And no others	
		c	E.g. since $\cos\theta \neq -\frac{1}{2}$n the RHS of the equation $\sqrt{2\sin^2\theta + \cos\theta} = 2\cos\theta$	E1 (AO 2.3) [1]		

			Total	7	