1. a. Given that $|t| = 3$, find the possible values of $|2t - 1|$.

[3]

b. Solve the inequality $|x - \sqrt{2}| > |x + 3\sqrt{2}|$.

[4]

2. i. Give full details of a sequence of two transformations needed to transform the graph of $y = |x|$ to the graph of $y = |2(x + 3)|$.

[3]

ii. Solve the inequality $|x| > |2(x + 3)|$, showing all your working.

[5]

3. It is given that $|x + 3a| = 5a$, where a is a positive constant. Find, in terms of a, the possible values of

$$|x + 7a| - |x - 7a|.$$

[6]

4. (a) If $|x| = 3$, find the possible values of $|2x - 1|$. [3]

(b) Find the set of values of x for which $|2x - 1| > x + 1$. Give your answer in set notation. [4]

5. (a) Given that $|n| = 5$, find the greatest value of $|2n - 3|$, justifying your answer. [3]

(b) Solve the equation $|3x - 6| = |x - 6|$. [3]

6. Solve the equation $|2x - 1| = |x + 3|$. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			Either Attempt solution of linear equation or inequality with signs of x different	M1		
			Obtain critical value $-\sqrt{2}$	A1	or equiv (exact or decimal approximation)	
			Or 1 Attempt to square both sides	M1	obtaining at least 3 terms on each side	
			Obtain $x^2-2\sqrt{2}x+2>x^2+6\sqrt{2}x+18$	A1	or equiv; or equation; condone > here	
			Or 2 Attempt sketches of $y=\left\|x-\sqrt{2}\right\|$, $y=\left\|x+3\sqrt{2}\right\|$	M1		
			Obtain $x=-\sqrt{2}$ at point of intersection	A1	or equiv	
			Conclude with inequality of one of the following types: $x<k\sqrt{2},\quad x>k\sqrt{2},\quad x<\frac{k}{\sqrt{2}},\quad x>\frac{k}{\sqrt{2}}$	M1	any integer k	
			Obtain $x<-\sqrt{2}$ or $-\sqrt{2}>x$ as final answer	A1	final answer $x<-\frac{2}{\sqrt{2}}$ (or similar unsimplified version) is A0 <u>Examiner's Comments</u> It is disappointing to record the fact that only 44% of candidates earned all four marks on this inequality. The more popular approach involved squaring both sides of the inequality. There were some errors, usually involving the square of $3\sqrt{2}$, but most did square both sides accurately. There were then errors involving signs and the manipulation of the surds. Other candidates dealt with either an equation or inequality (or occasionally four such) where each side was linear in x. Often the critical value $x=-\sqrt{2}$ was reached but it was then a rather haphazard process to reach a conclusion. A neat approach involves careful sketches of $y=\left\|x-\sqrt{2}\right\|$ and	

					$y=\left\lvert x+3\sqrt{2}\right\rvert$ the critical value and the answer are immediately apparent. Such an approach was not seen very often.	
			Total	4		
2		i	Refer to translation and stretch	M1	in either order; ignore details here; allow any equiv wording (such as move or shift for translation) to describe geometrical transformation but not statements such as add 3 to *x*	
		i	Either State translation in negative *x*-direction by 3	A1	or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; accept horizontal to indicate direction; term 'translate' or 'translation' needed for award of A1	
		i	State stretch by factor 2 in *y*-direction	A1	or parallel to *y*-axis or vertically; term 'stretch' needed for award of A1; these two transformations can be given in either order SC: if M0 but details of one transformation correct, award B1 for 1/3 (in Either, Or 1, Or 2 cases)	
		i	Or 1 State stretch by factor $\frac{1}{2}$n *x*-direction	A1	or parallel to *x*-axis; term 'stretch' needed for award of A1 or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0	
		i	State translation in negative *x*-direction by 3	A1 [3]	**Examiner's Comments** The vast majority of candidates recognised that a translation and stretch were the transformations involved. Generally the details of the two transformations were correct if sometimes the use of language was not as precise as it might have been. As in previous series, use of terms such as 'move' and 'shift' instead of 'translate' meant that the accuracy mark was not	

					earned. A minority of candidates used a stretch with scale factor $\frac{1}{2}$ parallel to the *x*-axis; this needs care with the translation and with the order of the two transformations and this care was often absent in such cases. Occasionally a third transformation was included, reflection in the *x*-axis, obviously the result of candidates recalling some aspect connected with the graphs of modulus functions.	
		i	Or 2 State translation in negative *x*-direction by 6	A1	or state translation by $\begin{pmatrix}-6\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1	
		i	State stretch by factor $\frac{1}{2}$ in *x*-direction	A1 [3]	or parallel to *x*-axis; term 'stretch' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0	
		ii	Either Solve linear eqn / ineq to obtain critical value −6	B1		
		ii	Attempt solution of linear eqn / ineq where signs of *x* and 2 *x* are different	M1		
		ii	Obtain critical value −2	A1		
		ii	Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a < x < b$ or of form $x < a$, $x > b$(where $a < b$); allow ≤ here	
		ii	Obtain $-6 < x < -2$	A1	as final answer; must be < not ≤; allow "$x > -6$ and $x < -2$" **Examiner's Comments** There were some algebraic slips but, generally, candidates were able to find the two critical values of −6 and −2. Squaring both sides of the given inequality was the more usual method. There was more difficulty in deciding the correct set of values of *x*. Some candidates were able to write down the answer immediately but others had no method for determining the answer and conclusions often seemed somewhat haphazard. Candidates concluding with $-6 \le x \le -2$ did not earn the final mark	

					and nor did candidates offering $x < -2$, $x > -6$.	
		ii	Or Square both sides to obtain $x^2 > 4(x^2 + 6x + 9)$	B1	or equiv	
		ii	Attempt solution of 3-term quadratic eqn / ineq	M1	with same guidelines as in Q2(ii) for factorising and formula	
		ii	Obtain critical values −6 and −2	A1		
		ii	Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a < x < b$ or of form $x < a$, $x > b$(where $a < b$); allow ≤ here	
		ii	Obtain $-6 < x < -2$	A1 [5]	as final answer; must be < not ≤; allow '$x > -6$ and $x < -2$'	
			Total	**8**		
3			Obtain $2a$ as one value of x	B1		Allow solution leading to $a = \frac{1}{2}x$ (B1) and $a = -\frac{1}{8}x$ (M1A1)
			Attempt to find second value of x	M1	By solving equation with signs of x and $5a$ different, or by squaring both sides and attempting solution of quadratic equation with three terms	If using quadratic formula to solve equation, substitution must be accurate
			Obtain $-8a$	A1	And no other values of x	
			Substitute each of at most two values of x (involving a) leading to one final answer in each case and showing correct application of modulus signs in at least one case	M1		
			Obtain $4a$ as final answer	A1	Obtained correctly from $x = 2a$	
			Obtain $-14a$ as final answer	A1	Obtained correctly from $x = -8a$ **Examiner's Comments** This question proved to be one of the more demanding requests in the paper and only 38% of the candidates recorded full marks. The slightly unfamiliar nature of the request and the presence of a were presumably factors causing the difficulties. It was also plain	

					that many candidates did not understand the meaning of the modulus function. Successful candidates were able to complete the answer in just a few lines, obtaining the two possible values of x from two simple linear equations and then substituting each value into $\|x + 7a\| - \|x - 7a\|$ without fuss. For example, $\|-8a + 7a\| - \|-8a - 7a\| = \|-a\| - \|-15a\| = a - 15a = -14a$ as part of the solution left no doubt that the candidate really understood what was needed. Not all candidates realised that a sensible strategy was to start by finding possible values of x. They started by trying to manipulate $\|x + 7a\| - \|x - 7a\|$, often squaring the two terms and simplifying to obtain $28ax$. Sketch graphs were employed by some candidates but were seldom of any help. A majority of the candidates did solve $\|x + 3a\| = 5a$ correctly to obtain $-8a$ and $2a$, although those adopting a method involving squaring were prone to errors, forgetting to square the right-hand side or being unable to deal with a quadratic equation including both x and a. A few candidates, apparently thinking everything had to be positive in a question involving modulus, rejected $-8a$ as a possibility or changed it to $8a$. For many of the candidates who had found the two possible values of x, the latter part of the question revealed their uncertainties with this topic. As well as the presence of modulus signs tending to prompt a spurious process of squaring, their presence also sometimes prompted the haphazard deployment of ± signs. Many candidates proceeded to provide a long list of possible values by evaluating $\|x + 7a\| - \|x - 7a\|$, using $x = 2a$, $x = -2a$, $x = 8a$, $x = -8a$ in turn and even, in some cases using different values of x in the same evaluation, evaluating for example $\|2a + 7a\| - \|-8a - 7a\|$. Sometimes the modulus signs were treated just as if they were brackets, and final answers presented as $\|x\| = 4a$ and $x = \|4a\|$ were not convincing either.	
			Total	6		

4		a	5 Substituting $x = -3$ into $\|2x - 1\|$ 7	B1(AO1.1) M1(AO1.1a) A1(AO1.1) [3]		
		b	$2x - 1 > x + 1$ therefore $x > 2$ $-(2x - 1) > x + 1$ (Allow ± in bracket) $x < 0$ $\{x : x < 0\} \cup \{x : x > 2\}$	B1(AO 1.1) M1(AO3.1a) A1(AO1.1) A1(AO2.5) [4]	OR **B1** for a sketch of $y = \|2x - 1\|$ and $y = x + 1$ on the same axes **M1** attempt to find the points of intersection **A1** obtain $x > 2$ and $x < 0$ **A1** $\{x : x < 0\} \cup \{x : x > 2\}$ OR **B1** $(2x - 1)^2 > (x + 1)^2$ seen **M1** attempt to multiply out and simplify, then solve quadratic **A1** obtain $x > 2$ and $x < 0$ **A1** $\{x : x < 0\} \cup \{x : x > 2\}$	
			Total	7		
5		a	$n = \pm 5$ $\|2(-5) - 3\| = 13$ $\|2(5) - 3\| = 7$ 13, as $13 > 7$	B1(AO1.2) M1(AO2.1) A1(AO2.4) [3]	Identify that $n = \pm 5$ Substitute both $n = \pm 5$ into expression Conclude with 13	

					<table><tr><td>and explanation</td><td></td></tr></table>	
		b	$3x - 6 = x - 6$, so $x = 0$ $3x - 6 = -x + 6$ $4x = 12$ $x = 3$	B1(AO1.1) M1(AO1.1a) A1(AO1.1) [3]	<table><tr><td>Obtain $x = 0$ Attempt to solve equation, with 3x and x of opposite signs Obtain $x = 3$</td><td>**OR M1** square both sides **A1** Obtain $x = 0$ **A1** Obtain $x = 0$</td></tr></table>	
			Total	6		
6			Attempt process for finding both values $3x^2 - 10x - 8 (= 0)$ Obtain 4 and $-\frac{2}{3}$	M1(AO 1.1a)E A1(AO 1.1)E A1(AO 1.1)E [3]	<table><tr><td>e.g. squaring both sides to obtain 3 terms on both sides ($4x^2 - 4x + 1 = x^2 + 6x + 9$) **BC**</td><td>Or consider two linear equations $(2x - 1) = \pm (x + 3)$ 1 correct solution for A1 **SC** one correct solution from one linear equation B1</td></tr></table> __Examiner's Comments__ This question was answered extremely well with nearly all candidates correctly solving this equation involving the modulus function. Of the two main methods for solving this type of equation the first, which involved re-writing as $(2x - 1)^2 = (x + 3)^2$ was far more successful	

					than those candidates who decided to re–write as two linear equation as many made sign errors even though most started from the correct two equations $(2x - 1) = \pm (x + 3)$.	
			Total	3		