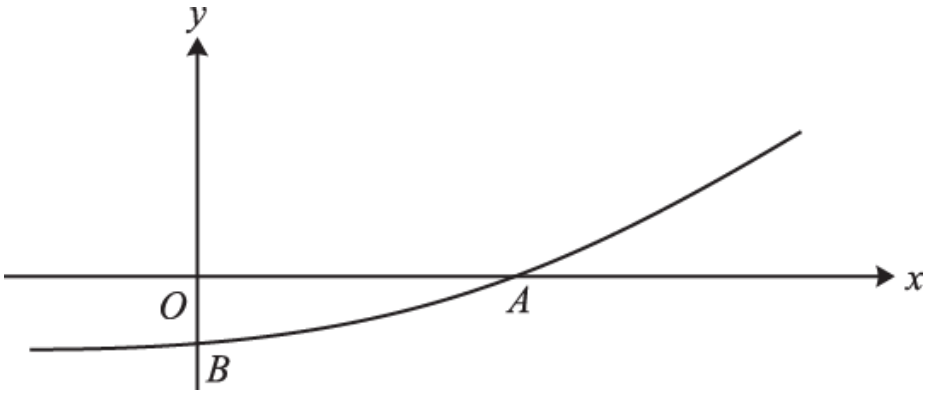
1.



The diagram shows the curve with equation

$$x = (y + 4)\ln(2y + 3).$$

The curve crosses the x-axis at A and the y-axis at B.

i. Find an expression for $\frac{dx}{dy}$ in terms of y.

[3]

ii. Find the gradient of the curve at each of the points A and B, giving each answer correct to 2 decimal places.

[5]

2. For each of the following curves, find the gradient at the point with x-coordinate 2.

i. $y = \frac{3x}{2x + 1}$

[3]

ii. $y = \sqrt{4x^2 + 9}$

[3]

3. Find the exact value of the gradient of the curve

$$y = \sqrt{4x - 7} + \frac{4x}{2x + 1}$$

at the point for which $x = 4$.

[6]

4. The functions f and g are defined for all real values of x by

$$f(x) = 2x^3 + 4 \quad \text{and} \quad g(x) = \sqrt[3]{x-10}.$$

i. Evaluate $f^{-1}(-50)$.

[2]

ii. Show that $fg(x) = 2x - 16$.

[2]

iii. Differentiate $gf(x)$ with respect to x.

[3]

5. Given that $y = 4x^2 \ln x$, find the value of $\frac{d^2y}{dx^2}$ when $x = e^2$.

[5]

6. Find the equation of the tangent to the curve $y = \frac{5x+4}{3x-8}$ at the point , (2, –7).

[5]

7. The curves C_1 and C_2 have equations

$$y = \ln(4x - 7) + 18 \text{ and } y\, a(x^2 + b)^{\frac{1}{2}}$$

respectively, where a and b are positive constants. The point P lies on both curves and has x-coordinate 2. It is given that the gradient of C_1 at P is equal to the gradient of C_2 at P. Find the values of a and b.

[8]

8. Find the equation of the tangent to the curve

$$y = 3x^2(x + 2)^6$$

at the point (–1, 3), giving your answer in the form $y = mx + c$.

[5]

9. The equation of a curve is $y = e^{2x} \cos x$. Find $\frac{dy}{dx}$ and hence find the coordinates of any stationary points for which $-\pi \leqslant x \leqslant \pi$. Give your answers correct to 3 significant figures.

[6]

10. A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant. Given that the curve has a minimum point
when $x = -2$

- find the value of k,
- show that the curve has a point of inflection which is not a stationary point.

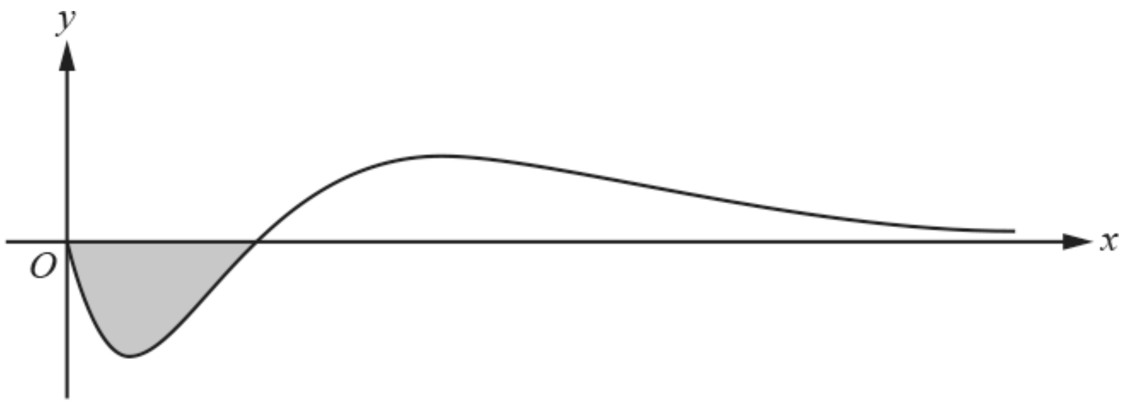
[7]

11. The equation of a curve has the form $y = e^{x^2}(ax^2 + b)$, where a and b are non-zero constants. It is given that $\frac{d^2y}{dx^2}$ can be expressed in the form $e^{x^2}(cx^4 + d)$, where c and d are non-zero constants. Prove

[5]

that $5a + 2b = 0$.

12. **In this question you must show detailed reasoning.**



The function f is defined for the domain $x \geq 0$ by

$$f(x) = (2x^2 - 3x)e^{-x}.$$

The diagram shows the curve $y = f(x)$.

(a) Find the range of f. [6]

(b) The function g is defined for the domain $x \geq k$ by

$$g(x) = (2x^2 - 3x)e^{-x}.$$

Given that g is a one-one function, state the least possible value of k. [1]

(c) Find the exact area of the shaded region enclosed by the curve and the x-axis. [7]

13. **In this question you must show detailed reasoning.**

A curve has equation $y = \frac{\ln x}{x}$.

(a) Find the x-coordinate of the point where the curve crosses the x axis. [2]

(b) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis. [2]

(c) Find the coordinates of the turning point on the curve. [4]

(d) Determine whether this turning point is a maximum or a minimum. [5]

14.

A function f is defined for $x > 0$ by $f(x) = \frac{6}{x^2 + a}$, where a is a positive constant.

(a) Show that f is a decreasing function. [4]

(b) Find, in terms of a, the coordinates of the point of inflection on the curve $y = f(x)$. [5]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Attempt use of product rule	M1	to produce expression of form (something non-zero) ln(2 y + 3) + $\dfrac{\text{linear in } y}{\text{linear in } y}$; ignore what they call their derivative	
		i	Obtain ln(2y + 3) …	A1	with brackets included with brackets included as necessary	
		i	Obtain … + $\dfrac{2(y+4)}{2y+3}$	A1	<u>Examinner's Comments</u> A few candidates did not recognise the need to use the product rule here but most did and, indeed, 58% of candidates duly earned all three marks. A common error was the differentiation of ln(2y + 3) to produce $\dfrac{1}{2y+3}$ and, in many cases, one of the terms was ln(2y + 3). Full credit was not given when necessary brackets were absent; ln 2y + 3 appeared in many answers. As soon as a correct expression for $\dfrac{dx}{dy}$ was produced, the marks were awarded. This was fortunate for many candidates as some subsequent horrendous 'simplification' was perpetrated, including the correct $\dfrac{dx}{dy} = \ln(2y+3) + \dfrac{2y+8}{2y+3}$ becoming $\ln(2y+3) + \dfrac{8}{3}$.	

		ii	Substitute $y = 0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1		
		ii	Obtain 0.27 for gradient at A	A1	or greater accuracy 0.26558…; beware of 'correct' answer coming from incorrect version $\ln(2y+3)+\frac{8}{3}$ of answer in part (i)	
		ii	Attempt to find value of y for which $x = 0$	M1	allowing process leading only to $y = -4$	
		ii	Substitute $y = -1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1		
		ii	Obtain 0.17 or $\frac{1}{6}$ for gradient at B	A1	or greater accuracy 0.16666…; value following from correct working <u>Examiner's Comments</u> This question assessed the specification item 'understand and use the relation $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \div \frac{\mathrm{d}x}{\mathrm{d}y}$'. It was not answered well in general and only 13% of candidates were able to earn all five marks. There were several major problems as far as candidates were concerned. Many candidates thought that the gradients could be found by substitution into the expression from part (i); many therefore claimed the gradient at A as 3.77 and rather fewer decided on 6 as the gradient at B. Other candidates, with a little awareness of a difference between $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}x}{\mathrm{d}y}$, decided that all would be well if x and y were interchanged throughout and relevant x-values substituted into their adjusted expression from part (i). Another problem arose for those candidates trying to produce an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. There may already have been some 'simplification' as mentioned above but, for those still with	

					a correct expression for $\frac{dx}{dy}$, there was the problem of finding its reciprocal. A few did this correctly after expressing $\frac{dx}{dy}$ with a common denominator but, for far too many, $\frac{dx}{dy} = \ln(2y+3) + \frac{2y+8}{2y+3}$ became $\frac{dy}{dx} = \frac{1}{\ln(2y+3)} + \frac{2y+3}{2y+8}$. A final problem concerned the values of y to be substituted. Most realised that $y = 0$ was appropriate for the point *A* but finding the *y*-value for *B* required more thought which, commendably, some candidates did display with a comment about $y = -4$ being impossible because it led to the logarithm of a negative number. Successful candidates usually avoided all the algebraic problems by substituting each of the two *y*-values into the expression for $\frac{dx}{dy}$ and then finding the reciprocal of each numerical value. This uncomplicated approach was not seen very often.
			Total	8	
2		i	Either Attempt use of quotient rule	M1	allow numerator wrong way round but needs minus sign in numerator and both terms in numerator involving *x*; for M1 condone minor errors such as absence of square in denominator, absence of brackets, …
		i	Obtain $\frac{3(2x+1)-6x}{(2x+1)^2}$ or equiv	A1	give A0 if necessary brackets absent unless subsequent calculation indicates their 'presence'

		i	Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv but A0 for final $\frac{3}{5^2}$	
		i	Or Attempt use of product rule for $3x(2x+1)^{-1}$	M1	allow sign error; condone no use of chain rule	
		i	Obtain $3(2x+1)^{-1} - 6x(2x+1)^{-2}$ or equiv	A1		
		i	Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv Examiner's Comments This question was answered well with 75% of candidates earning all three marks. Use of the quotient rule was the usual approach; a few candidates had the terms in the numerator the wrong way round but a more common error, and an avoidable one, was the simplification of $3(2x+1) - 6x$ in the numerator to give 1. Some candidates opted for the product rule and were not always successful, failure to apply the chain rule being the principal cause of error.	
		ii	Differentiate to obtain form $kx(4x^2+9)^n$	M1	any non-zero constants k and n (including 1 or $\frac{1}{2}$ for n)	
		ii	Obtain $4x(4x^2+9)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv	
		ii	Substitute 2 to obtain $\frac{8}{5}$ or 1.6	A1	or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$ Examiner's Comments This was also answered well, again with 75% of candidates earning three marks. Failure to include a factor x in the derivative was the most common error. In this part, and in part (i), a number of candidates omitted to substitute 2 to find the gradient as requested.	
			Total	6		
3			Differentiate first term to obtain form $k(4x-7)^{-\frac{1}{2}}$	*M1	any non-zero constant k; M0 if this differentiation is carried out in the midst of some incorrect involved expression	

			Obtain $2(4x-7)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Attempt use of quotient rule or, after adjustment, product rule	*M1	for QR, allow numerator wrong way round but needs – sign in numerator; condone a single error such as absence of square in denominator, absence of brackets, …; for PR, condone no use of chain rule M0 if this differentiation is carried out in the midst of some incorrect involved expression
			Obtain $\dfrac{4(2x+1)-8x}{(2x+1)^2}$ or $4(2x+1)^{-1} - 8x(2x+1)^{-2}$	A1	or (unsimplified) equivs; give A0 if brackets absent unless subsequent calculation indicates their 'presence'
			Substitute 4 into expression for first derivative so that (initially at least) exactness is retained	M1	dep *M *M
			Obtain $\dfrac{58}{81}$	A1	answer must be exact Note: using $y=\sqrt{4x-7}+\dfrac{4}{2x+1}$: do not apply MR **Examiner's Comments** This question was answered very well and 63% of candidates recorded all 6 marks. The first term was usually differentiated correctly but there were a few more problems with the second term. Careless simplification often led to an expression $\dfrac{1}{(2x+1)^2}$ for those candidates using the quotient rule. Some candidates rewrote the expression as $4x(2x\ 1)^{-1}$; a few did not use the product rule and, for some others, there were errors as the chain rule was not used. The vast majority of candidates recognised the need to give an exact answer and there were few instances where candidates resorted to decimal approximations.
			Total	6	
4		i	Either: State $2x^3 + 4 = -50$	B1	
		i	State −3 and no other	B1	**Examiner's Comments**

					There were few problems with part (i) and 83% of candidates earned both marks. Few seemed to realise that the answer can be obtained by solving f (x) = −50 and the common approach was to find the inverse function. Many were guilty of careless notation, writing the inverse in a way to suggest $\frac{\sqrt[3]{x-4}}{2}$ when they clearly meant $\sqrt[3]{\frac{x-4}{2}}$; provided they proceeded to carry out the correct calculation, both marks were earned. Further carelessness was evident on some scripts where 50 was substituted.	
		i	Or: Obtain $\sqrt[3]{\frac{1}{2}(x-4)}$ for inverse of f	B1	or equiv; using any letter	
		i	State −3 and no other	B1		
		ii	Show composition of functions the right way round	M1	AG; necessary detail needed	
		ii	Obtain 2x – 16	A1	**Examiner's Comments** Part (ii) was answered extremely well with almost all candidates showing sufficient detail and recording two marks.	first step 2(x – 10) + 4 acceptable but then two more steps needed
		iii	Obtain $\sqrt[3]{2x^3-6}$ or $(2x^3-6)^{\frac{1}{3}}$ for gf (x)	B1	or unsimplified equiv	
		iii	Apply chain rule to function which is cube root of a non-linear expression	M1	condone incorrect constant; otherwise use of chain rule for their function must be correct	may use u = 2x^3 −6; M1 earned for expression involving u
		iii	Obtain $x = \sqrt[4]{9+8x-x^2}$	A1	or similarly simplified equiv; do not accept final answer with $\frac{6}{3}$ unsimplified **Examiner's Comments**	… in terms of x

					There were more problems with part (iii). Almost all candidates had the correct expression for gf (x) but some chose not to carry out the obvious simplification, or simplified it to $\sqrt[3]{-20x^3-40}$ or to $(2x^3+4)^{\frac{1}{3}}-10^{\frac{1}{3}}$. Common errors with the differentiation included a factor $6x$ instead of $6x^2$, an expression involving $(2x^3-6)^{-\frac{1}{3}}$ and a final answer not suitably simplified.	
			Total	7		
5			Attempt use of product rule to find first derivative	M1	producing form … ± … where one term involves ln x and the other does not	
			Obtain $8x\ln x + 4x$	A1	or unsimplified equiv	
			Attempt use of correct product rule to find second derivative	M1	with one term involving ln x	
			Obtain 8ln x + 12	A1	or unsimplified equiv **Examiner's Comments** This question was a suitable introduction to the paper for the majority of candidates, and 77% of them duly earned all five marks. Some provided very concise solutions taking only a few lines of working; the two applications of the product rule were handled without fuss. For many other candidates, solutions were more protracted with each attempt at the product rule needing some work at the side as functions u and v were defined and differentiated. Assembling the parts to form each derivative was prone to error and one that occurred frequently was a failure to include the derivative of $4x$ in the expression for the second derivative. Sound advice for candidates setting out solutions is to carry out obvious simplifications as the solution progresses. It was surprising that a significant number of candidates did not do this in this question. Having found the first derivative as $8x\ln x+\frac{4x^2}{x}$, they continued by correctly applying the product rule to the first term and then using the quotient	

			Obtain 28	A1	rule to deal with the second term. A few candidates did not see the need to use the product rule at all and the first step in a few cases was the substitution of e^2.	
			Total	5		
6			Attempt use of quotient rule or, after adjustment, product rule Obtain $\frac{5(3x-8)-3(5x+4)}{(3x-8)^2}$ or equiv Substitute 2 to obtain −13 or equiv Attempt to find equation of tangent Obtain $y = -13x + 19$ or $13x + y - 19 = 0$	*M1 A1 A1 M1 A1	For M1 allow one slip in numerator but must be minus sign in numerator and square of $3x - 8$ in denominator; allow M1 for numerator the wrong way round Allow if missing brackets implied by subsequent simplification or calculation Dep *M; equation of tangent not normal Or similarly simplified equiv with 3 non-zero terms **Examiner's Comments** This opening question was answered very well in general with 74% of the candidates recording full marks. The majority applied the quotient rule accurately although lack of care with brackets in the numerator did lead to some sign errors. Some candidates opted for use of the product rule and this was not handled quite so convincingly. There were some cases where candidates stopped as soon as they had found the gradient but, in general, candidates proceeded without difficulty to produce the equation of the tangent and to present it in an acceptable form.	For product rule attempt, *M1 for $k_1(3x - 8)^{-1} + k_2(5x + 4)(3x - 8)^{-2}$ form and A1 for correct constants 5 and −3;
			Total	5		
7			State, at some stage, $a(4 + b)^{\frac{1}{2}} = 18$	B1		

			Obtain derivative $\frac{4}{4x-7}$ for C_1	B1		
			Obtain derivative $kx(x^2+b)^{-\frac{1}{2}}$ for C_2	M1	Any non-zero constant k	
			Obtain correct $ax(x^2+b)^{-\frac{1}{2}}$	A1		
			Equate derivatives with $x = 2$	M1		
			Attempt values of a and b from two equations involving a and $(4+b)^{\frac{1}{2}}$	M1	Using correct process Correct equations are $a(4+b)^{\frac{1}{2}} = 18$ and $2a(4+b)^{-\frac{1}{2}} = 4$	
			Obtain $a = 6$	A1		
			Obtain $b = 5$	A1	**Examiner's Comments** Responses to this question varied considerably. Some candidates proceeded logically and efficiently and were able to produce a correct solution in half a page. For other candidates, there seemed to be no plan and solutions meandered on for several pages of complicated algebra. Careful reading of the question was essential and a major problem on many scripts was the failure to form the second equation $18 = a(4+b)^{\frac{1}{2}}$ resulting from the fact that the two curves meet where $x = 2$. Differentiation was often not correct, with the derivative of C_1 appearing as $\frac{1}{4x-7}$ a common error. Candidates who had differentiated correctly and also established the second equation were faced with the equations $18 = a(4+b)^{\frac{1}{2}}$ and $2a(4+b)^{-\frac{1}{2}} = 4$. Some candidates saw that it was then a straightforward process to eliminate one of a and b; they proceeded to the correct values without fuss. Many other candidates dealt with these equations by squaring both sides of both equations, leading them into quadratic equations with large coefficients and increasing the chances of careless slips. Algebraic errors such as $(4+b)^{\frac{1}{2}} = 2 +$	

					$b\frac{1}{2}$ occurred far too often for an examination at this level. Some candidates did not help themselves by delaying the substitution of $x = 2$ until a late stage of their solutions. Full marks were earned by 45% of the candidates but the solutions of other candidates didreveal uncertainties with the necessary algebra and calculus techniques.	
			Total	8		
8			Differentiate to produce form			
			$k_1x(x+2)^m + k_2x^2(x+2)^n$	*M1	For positive integers k_1, k_2, m, n ; allow M1 if slip to, for example, $(x + 3)$ in both brackets	
			Obtain $6x(x+2)^6 + 18x^2(x+2)^5$	A1	Or unsimplified equiv	
			Substitute $x = -1$ to obtain value 12	A1	From correct work only	
			Attempt equation of tangent (not normal) through point (–1, 3)	M1	Dep *M; using non-zero numerical value of gradient; condone slip in use of coordinates	
			Obtain $y = 12x + 15$	A1	Answer required in $y = mx + c$ form **Examiner's Comments** Most candidates found this a straightforward opening question and 80% of the candidates dulyrecorded full marks. The product rule was used efficiently by all but a few candidates. There were some careless errors when substituting –1 in the derivative and a few more as candidates manipulated the equation to the requested form. There were very few instances of candidates using a gradient of $-\frac{1}{12}$ and, in effect, finding the equation of the normal.	
			Total	5		
9			$\frac{dy}{dx} = \pm k e^{2x} \cos x \pm e^{2x} \sin x$	M1*	k is any constant	Product Rule

			$\frac{dy}{dx} = 2e^{2x}\cos x - e^{2x}\sin x$ oe	A1		
			their $\frac{dy}{dx} = 0$	M1dep*		
			$\tan x = 2$ or $\cos x = (\pm)\frac{1}{\sqrt{5}}$ or $\sin x = (\pm)\frac{2}{\sqrt{5}}$	A1	ignore omission of "$e^{2x} = 0$ has no solution"	or $\sqrt{5}\cos(x + \tan^{-1}½) = 0$
			$x = 1.11$ and -2.03 cao	M1	(1.11, 4.09) and / or (– 2.03, – 0.00765)	if **A0A0**, **SC1** for all 4 values to greater precision 1.107…, – 2.034…, 4.094…., – 0.0076457… (or – 0.007646)
			$y = 4.09$ and -0.00765 cao	A1	or **A1** for each correct pair of co-ordinates: mark to benefit of candidate	NB $x = 1.107148718$ and -2.034443936 $y = 4.094229238$ and -0.007645738
					extra values within range incur a penalty of one mark; **or** any finite value for x obtained from $e^{2x} = 0$ incurs a penalty of one mark <u>Examiner's Comments</u> The differentiation was very well done by nearly all candidates, and an overwhelming majority set the derivative equal to zero and successfully identified $\tan x = 2$. Thereafter many lost accuracy or omitted either the y- values or one of the x-value. Only a few candidates found incorrect finite values from $e^{2x} = 0$, rather more failed to recognise that tanx was available, and worked with $\sin^2 x$ or $\cos^2 x$, thus nearly always introducing incorrect extra values in the specified range. A very small number of candidates integrated instead of differentiating.	ignore extra values outside range

			Total	6		
10			$\frac{dy}{dx} = 2x + k + 4x^{-2}$	M1(AO 1.1a)	Attempt to differentiate	Power decreases by 1 for at least 2 terms
			$2(-2) + k + 4(-2)^{-2} = 0$	M1(AO3.1a)	Substitute $x = -2$, equate to 0 and attempt to solve	
			$k = 3$	A1(AO1.1)		
			$\frac{d^2y}{dx^2} = 2 - 8x^{-3}$	M1(AO3.1a)	Equate second derivative to 0 and attempt to solve	
			$2 - 8x^{-3} = 0$			
			$x = 4^{\frac{1}{3}}$	A1(AO1.1)		
			for $x < 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} < 0$ for $x > 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} > 0$	E1(AO2.1)	Consider convex / concave either side of $x = 4^{\frac{1}{3}}$ and conclude	
			when $x = 4^{\frac{1}{3}}, \frac{dy}{dx} \neq 0$ hence not a stationary point	E1(AO2.1) [7]	Consider gradient at $x = 4^{\frac{1}{3}}$, or justify that $x = -2$ is the only stationary point	
			Total	7		
11			Differentiate to obtain form $e^{x^2}(px^3 + qx)$	M1		

			Obtain $\frac{dy}{dx} = 2xe^{x^2}(ax^2+b) + 2axe^{x^2}$ Obtain $\frac{d^2y}{dx^2} = e^{x^2}(4ax^4 + 10ax^2 + 4bx^2 + 2a + 2b)$ Equate coefficient of $x^2e^{x^2}$ to zero Confirm $5a + 2b = 0$	A1 A1 M1 A1 [5]	Or equiv Or equiv Provided second derivative involves $e^{x^2}x^4$, $e^{x^2}x^2$ and e^{x^2} terms and no others AG – necessary detail needed **Examiner's Comments** A significant number of candidates made no progress with part (b) and some were unable to devise a strategy. Many others did earn the first two marks with accurate work in finding the first derivative. Indeed it was pleasing that many were able to differentiate the awkward e^{x^2} correctly. Further success usually depended on candidates organising their first derivative into a form suitable for further differentiation. Candidates who tried to differentiate a term such as $2xe^{x^2}(ax^2 + b)$ occasionally succeeded but generally were unable to cope, either failing to use the product rule appropriately or making careless slips. Candidates who organised their first derivative into a form such as $e^{x^2}(2ax^3 + 2ax + 2bx)$ were then faced with a manageable task to find the second derivative. Some of the candidates who reached an expression for the second derivative did not realise the implication provided by the form of the second derivative given in the question and were unable to conclude successfully. But it is pleasing to record the fact that approximately one-fifth of the candidates did manage to prove the result.	
			Total	9		
12		a	DR Attempt product rule for y $y' = (4x - 3)e^{-x} - (2x^2 - 3x)e^{-x}$	M1 (AO 3.1a)	Attempt must be of the form $(ax + b)e^{-x} \pm (cx^2 + dx)e^{-x}$	

				A1 (AO 1.1)	Correct derivative, in any form	
			$y' = 0 \Rightarrow (4x - 3) - (2x^2 - 3x) = 0$	M1 (AO 2.1)	Set $y' = 0$ and eliminate exponentials	
			Obtain quadratic in x and attempt to solve	M1 (AO 1.1)	Dependent on both previous M marks	$2x^2 - 7x + 3 = 0$
			$x = \frac{1}{2}, \quad x = 3$	A1 (AO 1.1)	Correct values from correct equation	
			$-e^{-\frac{1}{2}} \le y \le 9e^{-3}$	A1 (AO 2.5) [6]	Correct range, including correct inequality signs and either y, f or f(x) used for range notation (not x)	Allow 'closed interval' notation $[-e^{-\frac{1}{2}}, 9e^{-3}]$
		b	DR $k = 3$	B1ft (AO 2.3) [1]	FT their larger value of x from **(a)**	
		c	Use integration by parts with $u = 2x^2 - 3x$ and $v' = e^{-x}$	M1 (AO 1.1)	Must obtain result f(x) ± ∫ g(x) dx	
			$\int(2x^2 - 3x)e^{-x}\,dx = -(2x^2 - 3x)e^{-x} + \int(4x - 3)e^{-x}\,dx$	A1 (AO 1.1)		
			Attempt parts again with $u = ax + b$ and $v' = e^{-x}$	M1 (AO 1.1)	Dependent on previous M mark	
			$\int(2x^2 - 3x)e^{-x}\,dx = -(2x^2 + x + 1)e^{-x}$ $(+c)$	A1 (AO 1.1)		

			Answer	Marks	Guidance
			$2x^2-3x=0 \Rightarrow x=\frac{3}{2}$ (and $x=0$)	B1 (AO 3.1a)	oe; accept unsimplified (but all bracketing must be correct)
			Correct use of correct limits	M1 (AO 1.1	
			Integral is $1-7e^{-\frac{3}{2}}<0$ so area is $7e^{-\frac{3}{2}}-1$	A1 (AO 2.2a) [7]	Dependent on both previous M marks
			Total	14	
13		a	DR $\frac{\ln x}{x}=0$		
			$\Rightarrow \ln x = 0$ or $\frac{\ln 1}{1}=0$	M1 (AO 1.1a)	May not be seen
			$\Rightarrow x = 1$	A1 (AO 1.1) [2]	May be implied **Examiner's Comments** This question was well answered.
		b	y-coordinates are $\frac{\ln 2}{2}$ and $\frac{\ln 4}{4}$		
			$\frac{\ln 4}{4}=\frac{2\ln 2}{4}=\frac{\ln 2}{2}$ oe	B1* (AO 1.1)	Allow Both = 0.346... B0B0

			$\Rightarrow AB$ is / to x-axis AG	B1dep* (AO 3.1a) [2]	$\frac{\ln 4}{4} = \ln 4^{\frac{1}{4}} = \ln\sqrt{2} = \frac{\ln 2}{2}$ Show that $\frac{\ln 4}{4} = \frac{2\ln 2}{4}$ and conclusion use of $\frac{\ln 4}{4} - \frac{\ln 2}{2} = 0$ unjustified B0B0 Examiner's Comments Many candidates just stated or implied that $\frac{\ln 2}{2} - \frac{\ln 4}{4} = 0$, either without proof, or by using their calculator and decimals. These candidates scored no marks, because of the "**detailed reasoning**" instruction.	
		c	$\frac{dy}{dx} = \frac{x \times \frac{1}{x} - 1 \times \ln x}{x^2}$ or $\frac{1}{x} \times \frac{1}{x} + \ln x \times (-\frac{1}{x^2})$ oe $\frac{1}{x^2} - \frac{\ln x}{x^2} = 0$ or $\frac{1 - \ln x}{x^2} = 0$ $1 - \ln x = 0$ oe $x = e$ or 2.72 or 2.7 (2 sf) Coordinates are $(e, \frac{1}{e})$	M1 (AO 3.1a) M1 (AO 1.1) A1 (AO 1.1) A1 (AO 1.1) [4]	Attempt diff, ≥ one term correct oe, their $\frac{dy}{dx} = 0$ Allow (e, 0.368) or (e, 0.37) or (2.7, 0.37) (2 sf)	

					Examiner's Comments This question was well answered. A few candidates did not find the y-coordinate. Some made mistakes in the differentiation.	
	d		Attempt $\frac{d^2y}{dx^2}$ $= \frac{x^2(-\frac{1}{x})-2x(1-\ln x)}{x^4}$ or $\frac{-3+2\ln x}{x^3}$ oe Substitute x = e (or 2.72) into $\frac{d^2y}{dx^2}$ $\frac{d^2y}{dx^2} = -\frac{1}{e^3}$ oe or –0.0498 $\frac{d^2y}{dx^2} < 0$, hence maximum	M1 (AO 2.1) A1 (AO 1.2) M1 (AO 1.1) A1 (AO 1.1) B1f (AO 3.2a) [5]	Attempt diff their $\frac{dy}{dx}$ All correct, not necessarily simplified cao Sub their x from (c) into their $\frac{d^2y}{dx^2}$ cao Allow or – 0.0497 or –0.05 ft their result of sub their x into their $\frac{d^2y}{dx^2}$ dep see result	Example of grad method Sub 2.7 & 2.8 in $\frac{dy}{dx}$ M1 0.00093, –0.0038 A1A1 State grad +ve & –ve or show on diag dep A1A1 M1 Hence max B1f dep M1A1A1 No proof, no marks

					Examiner's Comments Most candidates attempted a correct method. Some made mistakes in the differentiation. Others made numerical errors when substituting $x = e$ into $\frac{d^2y}{dx^2}$ Some considered the gradient on either side of the turning point, generally correctly. In both this part and part (c), candidates who used "e" throughout, rather than its approximate decimal value, produced neater and more efficient solutions.	
			Total	13		
14		a	$f'(x) = -12x(x^2 + a)^{-2}$ for $x > 0$, $-12x < 0$ and $(x^2 + a)^2 > 0$ negative divided by positive is always negative, hence function is decreasing	M1 (AO 3.1a) A1(AO 2.1) M1 (AO 2.1) E1(AO 2.4) [4]	Attempt differentiation to obtain $kx(x^2 + a)^{-2}$ Obtain fully correct derivative Attempt to show that $f'(x) < 0$ Fully convincing argument	
		b	$f''(x) = -12(x^2 + a)^{-2} + 48x^2(x^2 + a)^{-3}$ $f''(x) = 0$ (and $f'(x) \neq 0$ since $f'(x) = 0$ only when $x = 0$)	M1 (AO 3.1a) A1 (AO 1.1) B1 (AO 1.2)	Attempt use of product, or quotient, rule Obtain correct expression Identify condition for a	Allow unsimplified

			$-12(x^2 + a) + 48x^2 = 0$ $36x^2 = 12a$ $x^2 = \frac{a}{3}$ $x = \sqrt{\frac{a}{3}}$, $y = \frac{9}{2a}$	M1 (AO 3.1a) A1 (AO 1.1) [5]	point of inflection. Attempt correct process to solve for x Obtain correct coordinates	Seen or implied Accept non-stationary condition omitted A0 if $x = \pm\sqrt{\frac{a}{3}}$
			Total	9		