1. The curve $y = (1 - x)(x^2 + 4x + k)$ has a stationary point when $x = -3$.

i. Find the value of the constant k.

[7]

ii. Determine whether the stationary point is a maximum or minimum point.

[2]

iii. Given that $y = 9x - 9$ is the equation of the tangent to the curve at the point A, find the coordinates of A.

[5]

2. A curve has equation $y = (x + 2)^2 (2x - 3)$.

(i) Sketch the curve, giving the coordinates of all points of intersection with the axes.

[3]

(ii) Find an equation of the tangent to the curve at the point where $x = -1$. Give your answer in the form $ax + by + c = 0$.

[9]

3. A curve has equation $y = 3x^3 - 7x + \frac{2}{x}$.

(i) Verify that the curve has a stationary point when $x = 1$.

[5]

(ii) Determine the nature of this stationary point.

[2]

(iii) The tangent to the curve at this stationary point meets the y-axis at the point Q. Find the coordinates of Q.

[2]

4. The curve $y = 2x^3 - ax^2 + 8x + 2$ passes through the point B where $x = 4$.

i. Given that B is a stationary point of the curve, find the value of the constant a.

[5]

ii. Determine whether the stationary point B is a maximum point or a minimum point.

[2]

iii. Find the x-coordinate of the other stationary point of the curve.

[3]

5. The curve $y = 4x^2 + \frac{a}{x} + 5$ has a stationary point. Find the value of the positive constant *a* given that the y-coordinate of the stationary point is 32.

[8]

6. The curve $y = 2x^3 + 3x^2 - kx + 4$ has a stationary point where $x = 2$.

(a) Determine the value of the constant *k*. [5]

(b) Determine whether this stationary point is a maximum or a minimum point. [2]

7. A curve has equation $y = kx^{\frac{3}{2}}$ where *k* is a constant. The point *P* on the curve has *x*-coordinate 4. The normal to the curve at *P* is parallel to the line $2x + 3y = 0$ and meets the *x*-axis at the point *Q*. The line *PQ* is the radius of a circle centre *P*.

Show that $k = \frac{1}{2}$. Find the equation of the circle. [10]

8. A curve has equation $y = x^5 - 5x^4$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Verify that the curve has a stationary point when $x = 4$. [2]

(c) Determine the nature of this stationary point. [2]

9. **In this question you must show detailed reasoning.**

A curve has equation $y = f(x)$, where f(*x*) is a quadratic polynomial in *x*. The curve passes through (0, 3) and (4, –13). At the point where $x = 3$ the gradient of the curve is –2. Find f(*x*). [8]

10. A curve has equation $y = \frac{1}{4}x^4 - x^3 - 2x^2$.

(a) Find $\frac{dy}{dx}$. [1]

(b) Hence sketch the gradient function for the curve. [4]

(c) By considering the x-intercepts of the graph drawn in part (b), determine the coordinates of the maximum point on the curve with equation $y = \frac{1}{4}x^4 - x^3 - 2x^2$. [2]

11. (i) Find the x values of the stationary points of the curve $y = 2x^4 - x^2$. [3]

(ii) Determine, in each case, whether the stationary point is a maximum point or a minimum point. [2]

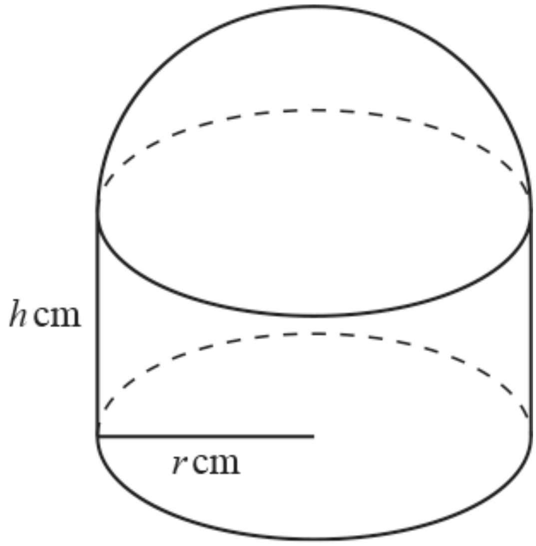
(iii) Hence state the set of values of x for which curve $2x^4 - x^2$ is a decreasing function. [2]

12. A curve has equation $y = 2x^2 + x - 10$.

(i) Determine the set of values of x for which the graph of the curve lies above the x-axis. [4]

(ii) The line $3x + y = c$ is a tangent to the curve. Find the value of c. [5]

13.



The diagram shows a container which consists of a cylinder with a solid base and a hemispherical top. The radius of the cylinder is r cm and the height is h cm. The container is to be made of thin plastic. The volume of the container is $45\pi \text{cm}^3$.

(a) Show that the surface area of the container, A cm^2, is given by

$$A = \frac{5}{3}\pi r^2 + \frac{90\pi}{r}.$$

[The volume of a sphere is $V = \frac{4}{3}\pi r^3$ and the surface area of a sphere is $S = 4\pi r^2$.] [4]

(b) Use calculus to find the minimum surface area of the container, justifying that it is a minimum. [4]

(c) Suggest a reason why the manufacturer would wish to minimise the surface area. [1]

14. A curve has equation $y = ax^4 + bx^3 - 2x + 3$.

(a) Given that the curve has a stationary point where $x = 2$, show that $16a + 6b = 1$. [3]

(b) Given also that this stationary point is a point of inflection, determine the values of a and b. [3]

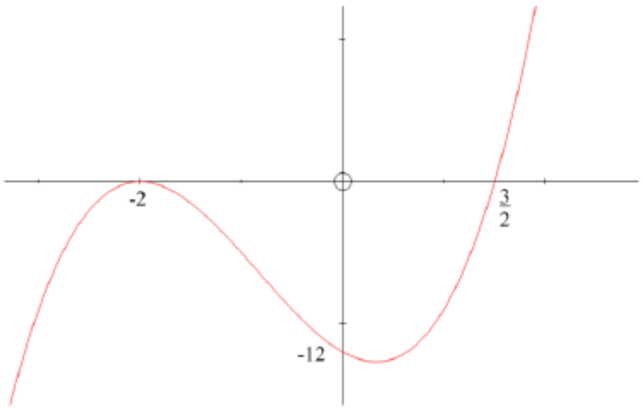
END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$y = -x^3 - 3x^2 + 4x - kx + k$	M1	Attempt to multiply out brackets	Must have $\pm x^3$ and 5 or 6 terms
		i		A1	Can be unsimplified	
		i	$\frac{dy}{dx} = -3x^2 - 6x + 4 - k$	M1	Attempt to differentiate **their** expansion	**If using product rule:**
		i		A1	(**M0** if signs have changed throughout)	Clear attempt at correct rule **M1***
		i	When $x = -3$, $\frac{dy}{dx} = 0$	M1*	Sets $\frac{dy}{dx} = 0$	Differentiates both parts correctly **A1**
		i		DM1*		Expand brackets of both parts ***DM1**
		i			Substitutes $x = -3$ into their $\frac{dy}{dx} = 0$	
		i	$-27 + 18 + 4 - k = 0$			Then as main scheme
		i	$k = -5$	A1	**www** **Examiner's Comments** More than half of candidates secured all seven marks available for this question and many clear, compact solutions were seen. Others also scored highly producing solutions marred only by arithmetical error. The conceptual problems that arose came from difficulties in differentiating *kx* or differentiating *k* to give 1; most knew to set their derivative to zero and substitute $x = -3$. Only a few candidates substituted into either the original expression or its expanded form without any attempt at differentiation.	

		ii	$\frac{d^2y}{dx^2} = -6x - 6$	M1	Evaluates second derivative at $x = -3$ or other fully correct method	**Alternate valid methods include:** 1) Evaluating gradient at either side of −3 2) Evaluating y at either side of −3 3) Finding other turning point and stating "negative cubic so min before max"
		ii	When $x = -3$, $\frac{d^2y}{dx^2}$ is positive so min point	A1	No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 12. (Ignore errors in k value) **Examiner's Comments** Most candidates found the second derivative and considered the sign at $x = -3$; only a few equated to zero in error. As his result was independent of k, this was by far the easiest route to success; candidates considering signs or using other methods rarely made any progress.	
		iii	$-3x^2 - 6x + 9 = 9$	M1	Sets their gradient function from (i) (or from a restart) to 9	Allow first **M** even if k not found but look out for correct answer from wrong working.
		iii	$3x(x + 2) = 0$ $x = 0$ or $x = -2$	A1	Correct x-values	<u>**Alternative Methods:**</u>
		iii	When $x = 0$, $y = -9$ for line $y = -5$ for curve	M1	One of their x-values substituted into both curve and line / substituted into one and verified to be on the other	**Note:** Putting a value into $x^3 + 3x^2 - 4 = 0$ (where the line and curve meet) is equivalent
		iii	When $x = -2$, $y = -27$ for line $y = -27$ for curve	M1	Conclusion that $x = -2$ is the correct value **or** Second x-value substituted into both curve and line / verified as above	If curve equated to line before differentiating: **M0 A0**, can get **M1M1** but **A0 ww**
		iii	$x = -2$, $y = -27$	A1	$x = -2$, $y = -27$ **www** *(Check k correct)*	Maximum mark **2/5**

		iii			**Examiner's Comments** This proved, appropriately, the most challenging question on the paper with only about half of candidates making any progress. Even high scoring candidates rarely produced a complete solution as it was necessary to identify a point here both the gradient of the line was equal to the gradient of the curve and such that the point was on both the line and the curve. Most commonly, candidates equated their derivate to 9 and tried to solve the resulting quadratic. Often the solution $x = 0$ was ignored or omitted; others substituted their solution(s) into the line only, sometimes offering both solutions. Many tried to equate the line and curve but were unable to solve the resulting cubic equation. A few who equated using the original form of the equation noticed that $x - 1$ was a common factor and the resulting quadratic had a repeated root, thus implying tangency but this approach was rarely rigorously explained.	Alternative method Attempt to solve equations of curve and tangent simultaneously **and** uses valid method to establish at least one root of the resulting cubic ($x^3 + 3x^2 - 4 = 0$ oe) **M1** All roots found **A1** Either 1) States $x = -2$ is repeated root so tangent **M2** (If double root found but not explicitly stated that repeated root implies tangent then **M0** but **B1** if (−2, −27) found) Or 2) Substitutes one x value into their gradient function to determine if equal to gradient of the line **M1** Substitutes other x value into their gradient function to determine if equal to gradient of the line or conclusion that −2 is the correct one **M1** $x = -2$, $y = -27$ **A1 www** **SC** Trial and Improvement Finds at least one value at which the gradient of the curve is 9 **B1** Verifies on both line and curve **B1 2/5**
			Total	14		

2		i		B1	Positive cubic with max and min	For first mark must clearly be a cubic — must not stop at either axis, do not allow straight line sections / tending to extra turning points etc.
		i		B1	Correct y intercept — graph must be drawn	
		i		B1	Double root shown at $x = -2$ and single root at $x = \frac{3}{2}$ with no extras — graph must be drawn **Examiner's Comments** The sketching of this cubic graph was generally done well, with the majority recognising the need for a double root at $x = -2$. There were some errors such as the inversion of the positive root and the occasional negative cubic was seen. Those who sketched a quadratic were only able to score a mark if they correctly identified the intercept on the y axis.	
		ii	$x^2 + 4x + 4$ or $2x^2 + x - 6$	B1	Obtain one quadratic factor	**Check for working for this in (i)**
		ii		M1	Multiply their three term quadratic by linear factor to obtain at least 5 term cubic	
		ii	$2x^3 + 5x^2 - 4x - 12$	A1	If simplified, must be correct	Alternative using product rule: Clear attempt at product rule **M1***

		ii	$\frac{dy}{dx} = 6x^2 + 10x - 4$	M1*	Attempt to differentiate (power of at least one term involving x reduced by one)	Differentiates $(x + 2)^2$ correctly **A1** Both expressions fully correct **A2 (1 each)**, then as main scheme
		ii		M1dep*	Substitutes to find gradient at $x = -1$	
		ii	When $x = -1$, gradient $= -8$	A1ft	Correct gradient found **ft** their derivative, differentiation of their expression must be fully correct to earn this mark	
		ii	When $x = -1$, $y = -5$	B1	Correct y value	y must have been found, do not allow use of gradient of normal instead of tangent
		ii	$y + 5 = -8(x + 1)$ $8x + y + 13 = 0$	M1	Correct equation of straight line through (-1, their y), their gradient from differentiation	
		ii	$8x + y + 13 = 0$	A1	Correct answer in correct form **Examiner's Comments** Just over half of candidates scored full marks for this final unstructured question, with many others scoring highly. Indeed, most candidates structured their solutions very well and the vast majority of errors were arithmetical rather than conceptual. These included errors in the initial expansion and in the substitution of $x = -1$ into the derivative. A few candidates set their derivative to zero. Another fairly common error was to find the equation of the normal rather than that of the tangent as required.	i.e. $k(8x + y + 13) = 0$. Must have "=0". Note If $x = 1$ used instead of $x = -1$, then max possible from last 5 marks is **M1 M1** only
			Total	12		
3		i	$\frac{dy}{dx} = 9x^2 - 7 - 2x^{-2}$	M1*	Attempt to differentiate, any term correct	
		i		A1	Two correct terms	
		i		A1	Fully correct	Alternative for the last two marks:

		i	When $x = 1$, $\frac{dy}{dx} = 9 - 7 - 2 = 0$	M1dep	Substitute $x = 1$ into their derivative Correctly obtain zero **www** and state conclusion **AG** **Examiner's Comments**	Sets derivative to zero and makes valid attempt to solve resulting quartic **M1dep**
		i	Therefore a stationary point	A1	Most candidates approached this very sensibly, differentiating and substituting in $x = 1$ to show the gradient is zero. Many, however, then failed to link this to question and state that this was why there was a stationary point. Some candidates equated their derivative to zero and solved the resulting quartic to show was a solution, again some omitted to explain the significance of this. In all cases, differentiation was generally accurate with some errors with the negative term.	Correctly establishes $x = 1$ as solution and draws clear conclusion **A1www**
		ii	$\frac{d^2y}{dx^2} = 18x + 4x^{-3}$ When $x = 1$, $\frac{d^2y}{dx^2} > 0$ so minimum	M1	Correct method to find nature of stationary point e.g. substituting $x = 1$ into second derivative (at least one term correct from their first derivative in (i)) No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 22. **Examiner's Comments**	**Alternate valid methods include:** 1) Evaluating gradient at either side of 1 ($x > 0$) 2) Evaluating y at 1 and either side of 1 ($x > 0$)
		ii		A1	Most candidates used the second derivative to determine that this was a minimum point, although there were a number of arithmetical errors at this stage. A few candidates found the second derivative to be 22 and then said "increasing", showing apparent confusion over the purpose and meaning of this method. Some candidates tried to find the gradient at either side of $x = 1$ but many of these chose $x = 0$ as the point to the left; as the function was undefined at this point, this invalidated this approach.	If using alternatives, working must be fully correct to obtain the **A** mark
		iii	When $x = 1$, $y = -2$	B1	Finding $y = -2$ at $x = 1$	

		iii	(0, −2)	B1	Correct coordinate **www** **Examiner's Comments** Less than half of candidates were successful on this part. Many realised the need to find the value of the function when $x = 1$ but then struggled to relate this to where the tangent would cut the axis. Q = − 2 was a common incorrect answer.	
			Total	9		
4		i	$\frac{dy}{dx} = 6x^2 - 2ax + 8$	M1	Attempt to differentiate, at least two non-zero terms correct	
		i		A1	Fully correct	
		i	When $x = 4$, $\frac{dy}{dx} = 104 - 8a$	M1	Substitutes $x = 4$ into their $\frac{dy}{dx}$	These Ms may be awarded in either order
		i	$\frac{dy}{dx} =$ 0 gives $a = 13$	M1	Sets their $\frac{dy}{dx}$ to 0. Must be seen **Examiner's Comments**	
		i		A1	Differentiating and setting to zero and substituting x = 4 was the obvious strategy and, although the arithmetic proved troublesome for some, many candidates were able to secure full marks for this part.	
		ii	$\frac{d^2y}{dx^2} = 12x - 26$	M1	Correct method to find nature of stationary point e.g. substituting $x = 4$ into secondderivative (at least one term correct from their first derivative in (i)) and consider the sign	**Alternate valid methods include:** 1) Evaluating gradient at either side of $4 (x > \frac{1}{3})$ e.g. at 3, −16 at 5, 28

		ii	When $x = 4$, $\frac{d^2y}{dx^2} > 0$ so minimum	A1	www **Examiner's Comments** Considering the sign of the second derivative was by far the most common approach for this part and was generally successful. Some candidates equated their second derivative to zero, a confusion that has been common for many sessions.	2) Evaluating $y = -46$ at 4 and either side of $4\ (x > \frac{1}{3})$ e.g. (3, −37), (5, −33) If using alternatives, working must be fully correct to obtain the **A** mark
		iii	$6x^2 - 26x + 8 = 0$	M1	Sets their derivative to zero	
		iii	$(3x - 1)(x - 4) = 0$	M1	Correct method to solve quadratic **(appx 1)** oe	Could be $(6x - 2)(x - 4) = 0$
		iii	$x = \frac{1}{3}$	A1	**Examiner's Comments** A small minority omitted this question, but most candidates were comfortable in returning to their expression in (i), equating to zero and finding the other root. An alternative method not seen before by several markers was to equate the second derivative to the negative of the value found in (ii); this is perfectly valid for cubics and was usually successful.	or $(3x - 1)(2x - 8) = 0$
			Total	10		
5			$y = 4x^2 + ax^{-1} + 5$	B1	ax^{-1} **soi**	
			$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct	
				A1	Fully correct	
			At stationary point, $8x - ax^{-2} = 0$	M1	Sets their derivative to 0	

$a = 8x^3$ oe	A1	Obtains expression for a in terms of x, or x in terms of a www	$x = \frac{\sqrt[3]{a}}{2}$ oe, $a = 18x$ oe also fine
When $a = 8x^3$, $y = 32$ $32 = 4x^2 + 8x^2 + 5$	M1	Substitutes their expression and 32 into equation of the curve to form single variable equation	
$x = \frac{3}{2}$ **oe**	A1	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	or expression for a e.g. $a^{\frac{2}{3}} = 9$
$a = 27$	A1	Obtains correct value for a. Ignore −27 given as well.	
OR			
$y = 4x^2 + ax^{-1} + 5$	B1	ax^{-1} soi	
$\frac{dy}{dx} = 8x - ax^{-2}$	M1	Attempt to differentiate – at least one non-zero term correct	
	A1	Fully correct	
$32 = 4x^2 + ax^{-1} + 5$	M1	Substitutes 32 into equation of the curve to find expression for a	
$a = 27x - 4x^3$	A1	Obtains expression for a in terms of x www	
At stationary point, $8x - ax^{-2} = 0$ $8x - (27x - 4x^3)x^{-2} = 0$	M1	Sets derivate to zero **and** forms single variable equation	
$x = \frac{3}{2}$ **oe**	A1	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	
$a = 27$	A1	Obtains correct value for a. Ignore −27 given as well.	

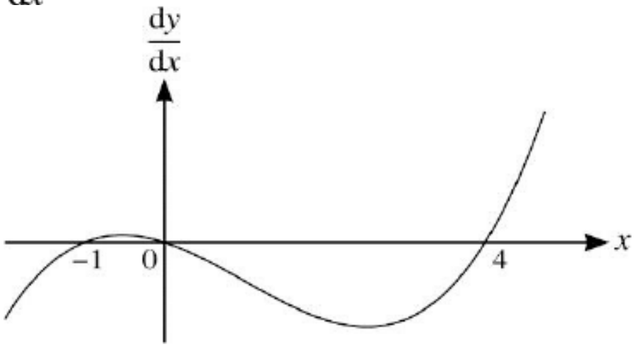
						Examiner's Comments Many candidates obtained at least the first four marks for this demanding final question, by correctly differentiating and setting equal to zero; the most common errors at this stage were to equate to 32 or to leave the constant term 5 in the derivative. Thereafter, a significant proportion candidates went on to secure at least 7 of the 8 marks by finding an expression for a and correctly substituting this and 32 into the equation of the curve, or other equivalent methods. Some did not spot this way forward and others lost marks due to incorrect simplification of algebra or arithmetical slips. Many did not spot the factor of 3 in $x^2 = \frac{27}{12}$ and so were then unable to finish the question. Occasionally candidates appear to consider a to be a variable passing through the point (*x*, 32); often these attempts were unclear and involved the (often unrealised) creation of functions of the form *xy* and the subsequent attempts at implicit differentiation were incorrect.
			Total	8		
6		a	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 + 6x - k$	M1 (AO3.1a) A1 (AO1.1) E1 (AO2.1)	Attempt differentiation	

			At $x = 2$ there is a stationary point, so $\frac{dy}{dx} = 0$ $6 \times 2^2 + 6 \times 2 - k = 0$ $k = 36$	M1 (AO1.1a) A1FT (AO1.1) [5]	Explain the substitution step Substitute $x = 2$ in their $\frac{dy}{dx} = 0$ FT their $\frac{dy}{dx} = 0$	
		b	$\frac{d^2y}{dx^2} = 12x + 6$ and $12 \times 2 + 6 (= 30)$ $\frac{d^2y}{dx^2} > 0$ hence minimum	M1 (AO1.1) A1FT (AO2.2a) [2]	Attempt differentiation again and substitute x = 2, FT their $\frac{dy}{dx}$ Correct conclusion FT www from their $\frac{d^2y}{dx^2}$ at $x = 2$	OR **M1** Attempt to evaluate gradient or *y* either side **A1** Correct values and conclusion **M1** For a complete sketch (all intercepts and both turning points identified) **A1** for

						conclusion given.	
			Total	7			
7			$2x+3y=0$ $\Rightarrow y=-\frac{2}{3}x$ and gradient $-\frac{2}{3}$ Hence, gradient of the tangent is $\frac{3}{2}$ $\frac{dy}{dx}=\frac{3}{2}kx^{\frac{1}{2}}$ At $x = 4$, $\frac{3}{2}k(4)^{\frac{1}{2}}=3k$ Hence $3k=\frac{3}{2}$, so $k=\frac{1}{2}$ At P, $y=\frac{1}{2}(4)^{\frac{3}{2}}=4$ so $P = (4, 4)$ so equation of normal through P is $(y-4)=-\frac{2}{3}(x-4)$ When $y = 0$, $x = 10$ so $Q = (10, 0)$	M1(AO3.1a) A1FT(AO1.1) M1(AO1.1a) A1(AO1.1) M1(AO1.1) E1(AO1.1) M1(AO3.1a) A1(AO1.1) M1(AO1.1) A1FT(AO1.1) [10]	Identify gradient of line $\left(=-\frac{2}{3}\right)$ anywhere Use $m_1m_2 = -1$ anywhere $\left(=\frac{3}{2}\right)$ FT their gradient Attempt differentiation Obtain $\frac{3}{2}kx^{\frac{1}{2}}$ Substitute $x = 4$ and equate to the normal gradient AG Identify coordinates, gradient of normal and form equation with their coordinates Substitute $y = 0$ and obtain $x = 10$ Use Pythagoras to	Allow sign slip The power must be seen to decrease Tangent gradients may also be used i.e. $-\frac{1}{3k}=-\frac{2}{3}$ Accept $y = 4$	

			Using $P(4, 4)$ and $Q(10, 0)$ $PQ^2 = (10 - 4)^2 + (0 - 4)^2$ Circle equation is $(x - 4)^2 + (y - 4)^2 = 52$		obtain length $PQ2$ Accept equivalent forms FT their coordinates for P and Q	
			Total	10		
8		a	$\frac{dy}{dx} = 5x^4 - 20x^3$ oe $\frac{d^2y}{dx^2} = 20x^3 - 60x^2$ oe	M1(AO1.1a) A1(AO1.1) A1FT(AO1.1) [3]	For attempt at differentiation FT their $\frac{dy}{dx}$	Both indices decrease
		b	When $x = 4$, $\frac{dy}{dx} = 5x^4 - 20x^3 = 5 \times 4^4 - 20 \times 4^3$ = 0 hence there is a stationary point	M1(AO1.1) A1(AO2.1) [2]	Substitute into their $\frac{dy}{dx}$	
		c	When $x = 4$, $\frac{d^2y}{dx^2} = 20x^3 - 60x^2 = 20 \times 4^3 - 60 \times 4^2$ > 0 hence the stationary point is a minimum	M1(AO1.1) E1FT(AO2.2a) [2]	FT from their $\frac{d^2y}{dx^2}$ in part **(a)**	
			Total	7		

9			DR $f(x) = ax^2 + bx + c$ $c = 3$ $a \times 4^2 + b \times 4 + 3 = -13$ $16a + 4b = -16$ $f'(x) = 2ax + b$ $6a + b = -2$ eg $8a = 8$ or $16a + 4(-2 - 6a) = -16$ $a = 1$ or $b = -8$ $f(x) = x^2 - 8x + 3$	B1(AO1.1) M1(AO3.1a) A1(AO1.1a) M1(AO1.1a) A1(AO1.1) M1(AO2.2a) A1(AO1.1) A1(AO3.2a) [8]	Attempt sub (4, −13) in f(*x*) oe, correct equn Attempt diff f(*x*) Correct equn Solve & obtain a correct equn in *a* or *b*	
			Total	8		
10		a	$\frac{dy}{dx} = x^3 - 3x^2 - 4x$	B1 (AO1.1) [1]	Correct differentiation	

		b	$\frac{dy}{dx} = x(x+1)(x-4)$ 	M1 (AO1.1a) A1 (AO1.1) B1 (AO1.1) B1FT (AO1.1) [4]	Attempt to factorise (3 linear factors) Factorisation all correct Good curve – correct shape, positive cubic FT their x-intercepts labelled correctly	
		c	Consider when function sketched in part (b) is increasing/decreasing at the x-intercepts The only x-intercept at which the gradient function is decreasing is at $x = 0$, so the maximum point on the original (quartic) curve is (0, 0)	M1 (AO2.1) A1 (AO2.2a) [2]	Correct conclusion – dependent on correct curve in (b)	
			Total	7		
11		i	$\frac{dy}{dx} = 8x^3 - 2x$ At stationary points $8x^3 - 2x = 0$	B1 M1	Correct differentiation Sets their derivative	B0 M0 if expression is integrated and equated to zero.

			$x = \frac{1}{2}, x = -\frac{1}{2}, x = 0$	A1 [3]	to zero Correctly obtains **all three** roots. | Do not accept $\pm\sqrt{\frac{1}{4}}$. **Examiner's Comments** Almost all candidates secured the first two marks in this part, but a significant proportion then failed to obtain all three roots of the cubic, either neglecting zero or $-\frac{1}{2}$.	
		ii	$\frac{d^2y}{dx^2} = 24x^2 - 2$ When $x = \pm\frac{1}{2}$, $\frac{d^2y}{dx^2} > 0$ so mimimum, maximum when $x = 0$	M1 A1 [2]	Uses correct method to find nature of **at least one** stationary point e.g. substitution into second derivative (at least one term correct from their first derivative in (i)) and consider sign. Correct conclusions for all three points **www** | **Alternate valid methods include:** 1) Determining sign of gradient at either side of stationary point 2) Evaluating y at, and either side of, stationary point 3) Correct sketch Working must be fully correct to obtain the **A** mark **Examiner's Comments** Most candidates successfully used the second derivative to find the nature of their roots from part (i), gaining the method mark, but the	

					accuracy mark was withheld if they did not complete the process for all three roots. Alternative successful methods included drawing a sketch of the quartic.	
		iii	$x < -\frac{1}{2}, 0 < x < \frac{1}{2}$	B2 [2]	Both regions correct (allow B1 for one correct region) — Condone use of ≤ instead of <. Condone e.g. $\sqrt{\frac{1}{4}}$ here. **Examiner's Comments** Even those successful in the previous parts seemed unsure of how to use their answers to determine where the function was decreasing. Many made no serious attempt, and those that scored often only had one region correct because of previous errors. Some gave single values of *x* other than regions, and others appeared to be identifying the region where the curve lay below the *x*-axis.	
			Total	7		
12		i	$(2x+5)(x-2) = 0$ $-\frac{5}{2}, 2$ $x < -\frac{5}{2}, x > 2$	M1 A1 M1 A1 [4]	Correct method to find roots. **See appendix 1.** Roots correct Chooses the "outside region" for their roots Allow "$x < -\frac{5}{2}, x > 2$", — **NB** e.g. $-\frac{5}{2} > x > 2$ scores **M1A0** if correct	

					"$x < -\frac{5}{2}$ **or** $x > 2$" but do not allow "$x < -\frac{5}{2}$ **and** $x > 2$" — answer not previously seen. Must be strict inequalities for **A** mark **Examiner's Comments** Most candidates used factorisation to start their solution to this quadratic inequality and chose the correct "outside" region securing all four marks. The notation used to describe the region was usually correct, although trying to describe two regions in a single inequality remains a fairly common error. Likewise, incorrect language such as joining the two sections with the word "and" still loses the accuracy mark. Sign errors in initial factorisation were not uncommon.	
		ii	Gradient of line = – 3 $\frac{dy}{dx} = 4x + 1$ $4x + 1 = -3$ $x = -1$ $y = -9$ $-9 = -3(-1) + c \Rightarrow c = -12$	B1 B1 M1 A1 A1	Stated or used. Correct differentiation Equates their derivative with their gradient of line x correct c correct. Could also obtain from substituting $x = -1$ Look out for using 3 instead of –3. This gives $x = \frac{1}{2}$ which also leads to $y = -9$. **B0B1M1A0A0** Max 2/5	

			OR $2x^2 + x - 10 = c - 3x$ $2x^2 + 4x - 10 - c = 0$ Tangent ⇒ $b^2 - 4ac = 0$ ⇒ $4^2 - 4.2.(-10 - c) = 0$ $c = -12$	OR M1 A1 M1 A1 A1 [5]	into $2x^2 + x - 10 = c - 3x$. Equates line and curve Obtains correct quadratic = 0 Uses tangency implies $b^2 - 4ac = 0$ Fully correct substitution c correct **Examiner's Comments** Close attention to detail was needed to ensure accuracy here, and many candidates produced clear full solutions. A large number of candidates differentiated the equation of the curve and equated this to the gradient of the line, although the use of 3 instead of –3 was a common error. Likewise there were sign slips in the subsequent attempts to find x, y and c. The other common approach was to equate the line and curve and use the fact that tangency implies one root and a zero discriminant. This was equally effective but similarly prone to sign error.	
			Total	9		

Question		Part	Answer	Marks	Guidance		
13		a	$\frac{2}{3}\pi r^3 + \pi r^2 h = 45\pi$	B1 (AO 3.1b)	Equate correct volume to 45π		
			$A = \pi r^2 + 2\pi r^2 + 2\pi rh$	B1 (AO 1.1)	Correct expression for surface area		
			$h = \frac{45 - \frac{2}{3}r^3}{r^2} = 45r^{-2} - \frac{2}{3}r$ $A = 3\pi r^2 + 2\pi r(45r^{-2} - \frac{2}{3}r)$	M1 (AO 1.1)	Attempt to make *h* the subject and hence eliminate *h*		
			$= 3\pi r^2 + 90\pi r^{-1} - \frac{4}{3}\pi r^2$ $A = \frac{5}{3}\pi r^2 + \frac{90\pi}{r}$ AG	A1 (AO 2.1) [4]	Simplify to obtain given answer		
		b	$\frac{dA}{dr} = \frac{10}{3}\pi r - 90\pi r^{-2}$	M1 (AO 1.1a)	Attempt differentiation		
			$\frac{10}{3}\pi r - 90\pi r^{-2} = 0$ ⇒ $r = 3$	M1 (AO 3.1b) A1 (AO 3.2a)	Equate to 0 and solve for *r*		
			$A = 45\pi$ cm^2 or 141 cm^2	A1FT (AO 2.2a)	Correct surface area, including units		

			$\frac{d^2A}{dr^2} = \frac{10}{3}\pi + 180\pi r^{-3} > 0$ hence minimum	[4]	FT their first derivative, provided it gives a minimum	Or using the sign-change of first derivative	
	c		E.g. Cheaper to manufacture as uses less material	E1 (AO 3.2b) [1]	Sensible reason based on surface area		
			Total	9			
14	a		$\frac{dy}{dx} = 4ax^3 + 3bx^2 - 2$ $4a(2)^3 + 3b(2)^2 = \Rightarrow 16a = 6b = 1$	M1 (AO1.1a) A1 (AO1.1) A1 (AO2.2a) [3]	Attempt to differentiate – all powers reduced by 1 Correct first derivative **AG** – sufficient working must be shown to establish given result		
	b		$\frac{d^2y}{dx^2} = 12ax^2 + 6bx$ $16a + 6b = 1$ and $4(a + b) = 0 \Rightarrow a = \ldots$ and $b = \ldots$	B1FT (AO1.1) M1 (AO2.1)	Correct second derivative following through from their first derivative Formulate two		

			$a=-\frac{1}{8}$, $b=\frac{1}{2}$	A1 (AO2.2a) [3]	equations in *a* and *b* and attempt to solve for both *a* and *b* Both values correct	No follow through for this mark
			Total	6		