1. i. The line joining the points (–2, 7) and (–4, p) has gradient 4. Find the value *of p.*

[3]

ii. The line segment joining the points (–2, 7) and (6, q) has mid-point (m, 5). Find m and q.

[3]

iii. The line segment joining the points (–2, 7) and (d, 3) has length $2\sqrt{13}$. Find the two possible values of d.

[4]

2. A is the point (–2, 6) and B is the point (3, –8). The line l is perpendicular to the line $x - 3y + 15 = 0$ and passes through the mid-point of AB. Find the equation of l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

[7]

3. A is the point (5, 7) and B is the point (–1, –5).

(i) Find the coordinates of the mid-point of the line segment AB.

[2]

(ii) Find an equation of the line through A that is perpendicular to the line segment AB, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

[5]

4. The points A and B have coordinates (2, 1) and (5, –3) respectively.

i. Find the length of AB.

[2]

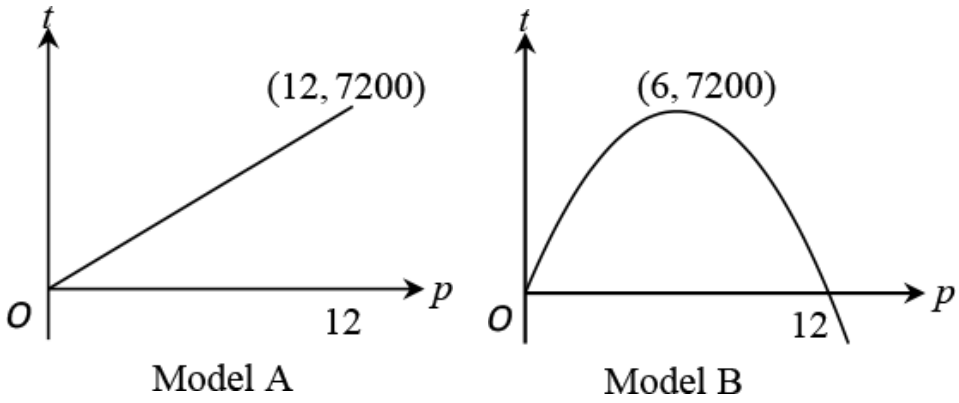
ii. Find an equation of the line through the mid-point of AB which is perpendicular to AB, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

[7]

5. A publisher has to choose the price at which to sell a certain new book. The total profit, £t, that the publisher will make depends on the price, £p. He decides to use a model that includes the following assumptions.

- If the price is low, many copies will be sold, but the profit on each copy sold will be small, and the total profit will be small.
- If the price is high, the profit on each copy sold will be high, but few copies will be sold, and the total profit will be small.

The graphs below show two possible models.



(a) Explain how model A is inconsistent with one of the assumptions given above. [1]

(b) Given that the equation of the curve in model B is quadratic, show that this equation is of the form $t = k(12p - p^2)$, and find the value of the constant k. [2]

(c) The publisher needs to make a total profit of at least £6400. Use the equation found in part (b) to find the range of values within which model B suggests that the price of the book must lie. [4]

(d) Comment briefly on how realistic model B may be in the following cases. [2]

- $p = 0$
- $p = 12.1$

6. The points A and B have coordinates (1, 5) and (4, 17) respectively. Find the equation of the straight line which passes through the point (2, 8) and is perpendicular to AB. Give your answer in the form $ax + by = c$, where a, b and c are constants.

[4]

7. In this question you must show detailed reasoning.

Andrea is comparing the prices charged by two different taxi firms.
Firm **A** charges £20 for a 5 mile journey and £30 for a 10 mile journey, and there is a linear relationship between the price and the length of the journey.
Firm **B** charges a pick-up fee of £3 and then £2.40 for each mile travelled.

[4]

Find the length of journey for which both firms would charge the same amount.

END OF QUESTION paper

Mark scheme

Question	Answer/Indicative content	Marks	Part marks and guidance	
1 i	$\frac{p-7}{-4-^{-}2}$ or $\frac{7-p}{-2-^{-}4}$	M1	uses $\frac{y_2-y_1}{x_2-x_1}$ (at least 3 out of 4 correct)	**Alternative method:** Equation of line through one of the given points with gradient 4 **M1** Substitutes **other point** into their equation **M1**
i	$\frac{p-7}{-4-^{-}2}=4$ or $\frac{7-p}{-2-^{-}4}=4$	A1	Correct, unsimplified **equation**	
i	$p=-1$	A1	Obtains $p=-1$ (Accept $y=-1$)**A1** **Note:** Other "informal" methods can score full marks provided **www**	
ii	$\frac{^{-}2+6}{2}=m, \quad \frac{7+q}{2}=5$	M1	Correct method (may be implied by one correct coordinate)	Use the same marking principle for candidates who add / subtract half the difference to an end point or use similar triangles or other valid "informal" methods
ii	$m=2$	A1		
ii	$q=3$	A1	Again informal methods were often more successful than formal ones; use of the mid-point formula was more successful in finding m than q.	
iii	$\sqrt{(-2-d)^2+(7-3)^2}$	*M1	Correct method to find line length / square of line length using Pythagoras' theorem (at least 3 out of 4 correct)	
iii	$d^2+4d+20=52$	B1	$\left(2\sqrt{13}\right)^2=52$ or $2\sqrt{13}=\sqrt{52}$	

		iii	$d^2 + 4d - 32 = 0$ $(d + 8)(d - 4) = 0$	DM1	Correct method to solve 3 term quadratic, must involve their "52"	**SC: B1** for each value of d found or "spotted" from correct working
					Examiner's Comments	
		iii	$d = -8$ or 4	A1	In this question candidates again used a variety of methods, with many spotting that the difference between d and –2 had to be $\sqrt{36}$; commonly the negative root was missed here leading to a score of 3 out of 4. Most candidates approached this more formally, using Pythagoras' theorem and again weak algebra and difficulties with negative values led to the loss of marks; some struggled to square $2\sqrt{13}$ accurately.	**Note:** Other "informal" methods can score full marks provided **www**
			Total	**10**		
2			Midpoint of AB is $\left(\frac{-2+3}{2}, \frac{6+-8}{2}\right)$	M1	Correct method to find midpoint – can be implied by one correct value	**NB – "correct" answer can be found with wrong mid-pt. Check working thoroughly.**
			$\left(\frac{1}{2}, -1\right)$	A1		
			$\text{Gradient of given line} = \frac{1}{3}$	B1	Must be stated or used – just rearranging the equation is not sufficient	
			Gradient of $l = -3$	B1FT	Use of $m_1 m_2 = -1$ (may be implied), allow for any initial non-zero numerical gradient	
			$y + 1 = -3\left(x - \frac{1}{2}\right)$	M1	Correct equation for line, any non-zero numerical gradient, through their $\left(\frac{1}{2}, -1\right)$	
				A1	Correct equation in any three-term form $k(6x + 2y - 1) = 0$ for integer k **www**	
					Examiner's Comments	
			$6x + 2y - 1 = 0$	A1	This familiar question was an opportunity for candidates to demonstrate proficiency in some basic techniques. Around two-fifths secured all seven marks and the vast majority at least three marks. The main errors were arithmetical, usually sign errors, when finding the mid-point and/or when simplifying the final equation of the line. Most candidates both found the gradient of the given line and the associated negative reciprocal accurately. Other errors that lost marks were lack of attention to detail in the question; some lines went through A or B and using the	Must include "= 0"

					negative reciprocal of the gradient of line segment *AB* was common. Others failed to notice the instruction to use integer coefficients in the final answer.	
			Total	**7**		
3		i	$\left(\frac{5+-1}{2}, \frac{7+-5}{2}\right)$	M1	Correct method to find midpoint of line Examiner's Comments	At least 3 out of 4 terms correctly substituted
		i	(2, 1)	A1	This was another area of improvement from previous sessions with very few candidates quoting or using an ncorrect formula to find the mid-point. Thus the vast majority scored both marks.	
		ii	Gradient of AB = $\frac{7--5}{5--1} = 2$	B1	Gradient of AB correctly found as 2	
		ii	Perpendicular gradient = $-\frac{1}{2}$	B1ft	Fully processed $\frac{-1}{\text{their gradient}}$	
		ii	$y - 7 = -\frac{1}{2}(x - 5)$	M1	Equation of straight line through A or B, any non-zero gradient	
		ii	$x + 2y - 19 = 0$	A1ft	Equation of straight line through A **only**, their perpendicular gradient, in any form Correct equation in given form Examiner's Comments	
		ii		A1	Around two-thirds of candidates provided fully correct solutions to find the equation of the required line. For those who were not successful, errors occurred at all stages. Some failed to find the correct gradient whilst others omitted to find the negative reciprocal to give the gradient of the perpendicular. More commonly, candidates did not read the question carefully and found the equation of the line through the mid-point rather than the required point, or did not give their final answer in the correct form.	i.e. $k(x + 2y - 19) = 0$ for integer k. Must have "=0".
			Total	**7**		
4		i	$AB = \sqrt{(5-2)^2 + (-3-1)^2}$	M1	Attempt to use Pythagoras' theorem – 3/4 numbers substituted correctly **and attempt to square root** Final answer correct, must be fully processed. ±5 is A0.	
		i	*AB = 5*	A1	**Examiner's Comments** Candidates were generally successful in applying	

					Pythagoras' theorem. Use of a diagram was relatively rare. Some candidates gave ± 5 or $\sqrt{25}$ neither of which secured the accuracy mark.	
		ii	$\left(\frac{2+5}{2}, \frac{1+-3}{2}\right)$	M1	Correct method to find mid-point of line	Alternative using general point on the perpendicular
		ii	(3.5, –1)	A1		
		ii	Gradient of $AB = -\frac{4}{3}$	B1	Processed	**M2** States P (x, y) a point on the perpendicular and attempts PA = PB or $PA^2 = PB^2$
		ii	Perpendicular gradient $= \frac{3}{4}$	B1ft	$\frac{-1}{\text{their gradient}}$ processed	**A1** At least one of PA, PB correct **A1** Both correct **M1** Expands and simplifies
		ii	$y+1=\frac{3}{4}(x-\frac{7}{2})$	M1	Equation of straight line through their midpoint, any non-zero gradient in any form	**A1** Correct equation found **A1** Correct equation in required form
		ii		A1		
		ii	$6x - 8y - 29 = 0$	A1	**cao** Must be correct equation in required form i.e. $k(6x - 8y - 29) = 0$ for integer k. **Must have "= 0"** **Examiner's Comments** Candidates generally displayed good understanding of coordinate geometry with many securing all seven marks. Errors in the early stages of solutions were rare, with only occasional sign or calculation errors in finding the mid-point and/or the gradient and perpendicular gradient. It was noticeable, however, that even when candidates were successful in obtaining the equation $y+1=\frac{3}{4}(x-\frac{7}{2})$ the presence of two fractions then caused problems whatever approach was taken to try to simplify. Also, some candidates failed to give the answer in required form apparently misunderstanding the word 'integers'.	
			Total	**9**		
5		a	Total profit (or t) is large when price (or p) is high	**B1(AO3.5b)** **[1]**		

		b	Passes through (0, 0) and (12, 0) hence $t = kp(12 - p)$ $k = 200$	B1(AO3.1b) B1(AO3.3) [2]	Or $t = 200\ p(12 - p)$ Or $t = 200\ (12p - p^2)$		
		c	$6400 = 200\ p(12 - p)$ oe $p^2 - 12x + 32 = 0$ $p = 4$, $p = 8$ $4 \leq p \leq 8$ Price must be between £4 and £8	M1(AO3.4) A1FT(AO1.1) A1FT(AO1.1) A1(AO3.4) [4]	6400 = (their k) $p(12 - p)$ Any correct equation in form $ap^2 + bp + c = 0$ BC, but any method allowed Allow $4 < p < 8$	FT (b) FT (b)	
		d	E.g. $p = 0$ implies giving book for free. Unrealistic. oe E.g. When $p = 0$, $t = 0$; but t should be negative as would make a loss. Unrealistic. oe E.g. When $p = 12.1$, t is negative. Possibly realistic as could make a loss if p set too high. oe	E1(AO3.2b) E1(AO3.2b) [2]	Valid comment about $p = 0$ Valid comment about $p = 12.1$		
			Total	9			
6			$m = \frac{17-5}{4-1}\ (= 4)$ $m_{\text{perp}} = -\frac{1}{4}$ $y - 8 = -\frac{1}{4}(x - 2)$	M1 (AO 1.1a) M1 (AO 1.1a) M1 (AO 1.1a)	Attempt to find gradient of AB Attempt gradient of perpendicular line Attempt	Fraction must be correct way around, with coordinates used in a consistent order in the numerator and denominator Use $m_1 m_2 = -1$ with their numerical gradient Could be implied by the gradient that they use in the equation of the line Either substitute into $y - y_1 =$	

			$x + 4y = 34$	A1 (AO 1.1) [4]	equation of line through (2, 8), using their attempt at a perpendicular gradient Obtain $x + 4y = 34$	$m(x - x_1)$ or use $y = mx + c$, as far as attempting c If not correct, then their gradient must be either the negative or the reciprocal of their original gradient eg if $m_1 = 4$, then $m_2 = -4$ or $\frac{1}{4}$ would be allowed oe in required form ie with x and y terms on one side of the equation and a constant term on the other a, b and c do not have to be integers eg accept $\frac{1}{4}x + y = \frac{17}{2}$ (but not $\frac{34}{4}$)

Examiner's Comments

This question was very well answered, with most candidates gaining full credit. As with all solutions, it is important for candidates to show clear details of the method used; on this question some candidates made an error when finding the gradient. If the correct method was shown then a mark was credited, but if an incorrect value for the gradient was all that was shown then the method mark cannot be credited. This question requested that the final answer be given in a specified form, so candidates need to pay heed to this. Non-integer values for a, b and c were accepted but best practice would be to use integer coefficients when giving the equation of a line in this form.

			Total	**4**			
7			DR ***A***: $C = 2m + 10$ ***B***: $C = 2.4m + 3$ Where C is the total charge in pounds and m is the length of the journey in miles. $2.4m + 3 = 2m + 10$ $0.4m = 7$ $m = 17.5$ Same cost for a journey of 17.5 miles	M1(AO 3.1a) A1(AO 3.1b) M1(AO 1.1) A1(AO 3.2a) [4]	Either correct equation Both equations correct and variables defined. Attempt to solve simultaneously Obtain 17.5 miles	Attempt to solve their two equations Units needed	
			Total	**4**			