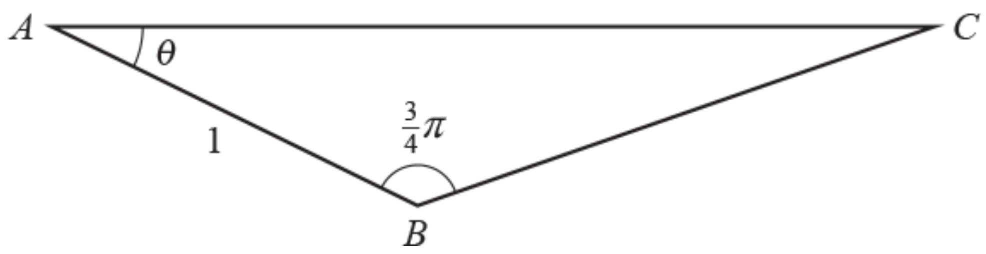
1. For a small angle θ, where θ is in radians, show that $1+\cos\theta-3\cos^2\theta \approx -1+\frac{5}{2}\theta^2$.

[4]

2.



The diagram shows triangle ABC, in which angle $A = \theta$ radians, angle $B = \frac{3}{4}\pi$ radians and $AB = 1$ unit.

(a) Use the sine rule to show that $AC = \dfrac{1}{\cos\theta - \sin\theta}$. [3]

(b) Given that θ is a small angle, use the result in part (a) to show that

$$AC \approx 1 + p\theta + q\theta^2,$$

where p and q are constants to be determined. [4]

3. Use small angle approximations to estimate the solution of the equation $\dfrac{\cos\frac{1}{2}\theta}{1+\sin\theta} = 0.825$, if θ is small enough to neglect terms in θ^3 or above. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance	
1			When θ is small $1 + \cos\theta - 3\cos^2\theta$ $\approx 1+\left(1-\frac{1}{2}\theta^2\right)-3\left(1-\frac{1}{2}\theta^2\right)^2$	M1(AO 1.1a)	Attempt to use cos $\approx 1-\frac{1}{2}\theta^2$ or $=1+\left(1-\frac{1}{2}\theta^2+\ldots\right)$ $-3\left(1-\frac{1}{2}\theta^2+\ldots\right)^2$	**OR** **M1** Attempt to use $\cos\theta \approx 1-\frac{1}{2}\theta^2$
			$=1+\left(1-\frac{1}{2}\theta^2\right)-3\left(1-\theta^2+\frac{1}{4}\theta^4\right)$	M1(AO1.1)	Multiply out	**M1** use trigonometric identity $1 + \cos\theta - 3\cos^2\theta$ $=1+\cos\theta-\frac{3}{2}-\frac{3}{2}\cos 2\theta$
			$=1+1-\frac{1}{2}\theta^2-3+3\theta^2-\frac{3}{4}\theta^4$	E1(AO2.5)		
			Since θ is small, we can neglect the higher order terms	E1(AO2.1)	For explanation of loss of θ^4 term and consistent use of notation throughout (Working need not be fully correct)	**E1** For showing clearly which identity has been used and consistent use of notation throughout
			so $1+\cos\theta - 3\cos^2\theta \approx -1+\frac{5}{2}\theta^2$ as required	[4]	AG Clearly obtained www Condone θ^4 term missing without explanation and inconsistent notation	**E1** AG Clearly obtained www Condone inconsistent notation
			Total	4		
2		a	$\frac{AC}{\sin\frac{3}{4}\pi}=\frac{1}{\sin\left(\pi-\frac{3}{4}\pi-\theta\right)}$	M1(AO2.1)	Attempt sine rule	

			$AC = \dfrac{\sin\frac{3}{4}\pi}{\sin\frac{1}{4}\pi\cos\theta - \cos\frac{1}{4}\pi\sin\theta}$ $\sin\frac{3}{4}\pi = \sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi$ so $AC = \dfrac{1}{\cos\theta - \sin\theta}$	M1(AO2.1) E1(AO2.2a) [3]	For expanding $\sin\left(\frac{1}{4}\pi - \theta\right)$ AG, so must show sufficient working; e.g. stating $\sin\frac{3}{4}\pi = \sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi$ or using $\frac{1}{\sqrt{2}}$ oe for each
		b	$AC = \left(1 + (-\theta - \frac{1}{2}\theta^2)\right)^{-1}$ $AC = 1 + (-1)\left(-\theta - \frac{1}{2}\theta^2\right) + \dfrac{(-1)(-2)}{2}\left(-\theta - \frac{1}{2}\theta^2\right)^2 + \ldots$ $AC \approx 1 + \theta + \frac{3}{2}\theta^2$	B1(AO1.1) M1(AO3.1a) A1(AO1.1) A1(AO1.1) [4]	Using both small angle approximations Attempt binomial expansion of *AC*, with at least the first two terms present $p = 1$ $q = \frac{3}{2}$
			Total	7	

3			$\dfrac{1-\frac{1}{8}\theta^2}{1+\theta}=0.825$ $0.125\theta^2 + 0.825\theta - 0.175 = 0$ $\theta = 0.206$ or -6.81 (3 sf) Discard -6.81 as not small. $\theta = 0.206$ (3 sf)	M1 (AO1.1a) A1 (AO1.1) A1 (AO1.1) A1 (AO2.3) [4]	BC Statement needed and $\theta =$ 0.206 alone	
			Total	4		