1. A sequence u_1, u_2, u_3, … is defined by

$$u_1 = 7 \text{ and } u_{n+1} = u_n + 4 \text{ for } n \geqslant 1.$$

i. Show that $u_{17} = 71$.

[2]

ii. Show that $\sum_{n=1}^{35} u_n = \sum_{n=36}^{50} u_n$.

[4]

2. A sequence u_1, u_2, u_3, … is defined by $u_n = 3n - 1$, for $n \geqslant 1$.

i. Find the values of u_1, u_2 and u_3.

[2]

ii. Find $\sum_{n=1}^{40} u_n$.

[3]

3. In an arithmetic progression the first term is 5 and the common difference is 3. The nth term of the progression is denoted by u_n.

i. Find the value of u_{20}.

[2]

ii. Show that $\sum_{n=10}^{20} u_n = 517$.

[3]

iii. Find the value of N such that $\sum_{n=N}^{2N} u_n = 2750$.

[6]

4. **In this question you must show detailed reasoning.**

A sequence S has terms u_1, u_2, u_3 … defined by $u_1 = 500$ and $u_{n+1} = 0.8u_n$.

(a) State whether S is an arithmetic sequence or a geometric sequence, giving a reason for your answer. [1]

(b) Find u_{20}. [2]

(c) Find $\sum_{n=1}^{20} u_n$. [2]

(d) Given that $\sum_{n=k}^{\infty} u_n = 1024$, find the value of k. [5]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$7 + 16 \times 4 = 71$ **AG**	M1	Attempt to find 17th term in the given AP	Attempt to use $u_n = a + (n - 1)d$ with $a = 7$ and $d = 4$ Allow a more informal method, including writing out the sequence with $a = 7$ and $d = 4$ Could also attempt u_{17} from attempt at $u_n = 4n + 3$ – must be seen explicitly If listing terms, 71 must either be last number in list or clearly identified eg underlined
		i		A1	Show clear detail to obtain $u_{17} = 71$	**Examiner's Comments** Most candidates gained both of the marks available. The most common approach was to use the formula for the nth term of an arithmetic progression and the majority did so successfully, showing enough detail to be convincing. Some candidates first derived $u_n = 4n + 3$ and then used this to show the given result, and a few resorted to writing out the sequence term by term.
		ii	$S_{35} = {}^{35}/_2\,(2 \times 7 + 34 \times 4) = 2625$	M1	Attempt sum of first 35 terms of given AP	Must use correct formula, with $a = 7$ and $d = 4$ If using $^1/_2 n(a + l)$ then must be valid attempt at l Could use $4\Sigma n + \Sigma 3$, but M0 for $4\Sigma\ n + 3$
		ii		A1	Obtain 2625	Must be evaluated Allow M1A1 for 2625 from no working
		ii	either $S_{50} = {}^{50}/_2\,(2 \times 7 + 49 \times 4)$ $= 5250$ $5250 - 2625 = 2625$ **AG** or $S_{36-50} = {}^{15}/_2\,(2 \times 147 + 14 \times 4)$ $= 2625$ **AG**	M1	Attempt a correct method to show given relationship	Must show explicit calculation so M0 for just stating eg $S_{50} = 5250$ Could sum first 50 terms of AP **and** find the difference between this and the sum of the first 35 terms, or equiv Could attempt to sum terms from u_{36} to u_{50} but M0 if summing from u_{35} (= 143)
		ii		A1	Show given equality convincingly	No need for explicit conclusion once both sums shown to be 2625

						Examiner's Comments Most candidates gained the first two marks with ease, using one of the two relevant formulae to sum the first thirty-five terms. The next two marks proved a little more problematical for many and they struggled to identify an appropriate strategy, with a mismatch between the values used for the number of terms and the first term. Those who attempted the sum of fifty terms from which they subtracted the sum of the first thirty-five terms were usually successful, and the working suggested that some were aided in their attempts by the given answer. A number of candidates attempted to sum from the 36th to the 50th term, but many struggled to identify that they were summing fifteen terms, and errors were also made in determining the first term. One of the more successful methods was to find values for the 36th and the 50th terms and then use $\frac{1}{2}n(a+l)$. Whilst most candidates gained at least two marks on this question, there were a number who seemed unfamiliar with sigma notation, or used incorrect values for *a* and/or *n* despite the hint given in part (i).
			Total	6		
2		i	2, 5, 8	B1	Obtain at least one correct value	Either stated explicitly or as part of a longer list, but must be in correct position eg −1, 2, 5 is B0
		i		B1	Obtain all three correct values **Examiner's Comments** Virtually all of the candidates were able to write down the required terms, gaining the two marks available. A few candidates mistakenly treated it as a recursive definition rather than an *n*th term definition.	Ignore any subsequent values, if given

		ii	$S_{40} = \frac{40}{2}(2 \times 2 + 39 \times 3)$	B1*	Identify AP with $a = 2$, $d = 3$	Could be stated, listing of further terms linked by '+' sign or by recognisable attempt at any formula for AP including attempt at u_{40}
		ii	= 2420	M1d*	Attempt to sum first 40 terms of the AP	Must use correct formula, with $a = 2$ and $d = 3$ If using ½n ($a + l$) then must be valid attempt at l Could use 3Σ n – Σ 1, but M0 for 3Σ n – 1 If summing manually then no need to see all middle terms explicitly as long as intention is clear
		ii		A1	Obtain 2420 **Examiner's Comments** Despite having written out the first three terms of the sequence in part (i), a number of candidates struggled to identify the correct values of a and d, with $a = 1$ being the most common error. The sight of the sigma sign resulted in other candidates attempting to use one of the summation formulae, but this was only occasionally done correctly. However, the majority of candidates could quote the relevant formula, substitute the correct values and obtain the required final answer to gain all of the marks available.	Either from formula or from manual summing of 40 terms
			Total	5		
3		i	$u_{20} = 5 + 19 \times 3$	M1	Attempt u_{20}	Must be using correct formula, with $a = 5$ and $d = 3$ Could use $u_n = 3n + 2$ Could attempt to list terms
		i	= 62	A1	Obtain 62 **Examiner's Comments** This question was invariably correct, with most candidates using the formula for the nth term of an AP. Other methods included firstly generating an nth term expression for the sequence, and some just resorted to manually listing the terms.	If listing terms then need to indicate that 62 is the required answer

		ii	$S_{20} = {}^{20}/_2\,(10 + 57)$ $S_9 = {}^{9}/_2\,(10 + 24)$	M1	Explicitly attempt either S_{20} or S_9	Must be using correct formula with $a = 5$ and $d = 3$ Use of formula must be explicit, so M0 for eg $S_{20} = 670$ with no other evidence Could use $½n\,(a + l)$, with l obtained from $a + (n - 1)d$ – expect to see ${}^{20}/_2\,(5 + 62)$ and / or ${}^{9}/_2\,(5 + 29)$ Could use $\Sigma(3n + 2)$, with correct formulae for Σn and $\Sigma 1$
		ii	${}^{20}/_2\,(10 + 57) - {}^{9}/_2\,(10 - 24)$	M1	Attempt $S_{20} - S_9$, where both summations have been shown explicitly	Can get M1 if formulae have not yet been evaluated M0 for $S_{20} - S_{10}$ (see below for one exception)
		ii	$= 670 - 153$ $= 517$ **AG**	A1	Evaluate both summations and hence obtain 517 CWO	**AG** so detail is required – only award A1 if both unsimplified sums are seen, as well as both evaluated sums **SR** Allow **B1** if only $670 - 153 = 517$ seen Explicitly detailing only one summation will get M1M0A0 Allow 3/3 for $S_{20} - S_{10} + u_{10}$ as long as all explicit Allow 3/3 for manually summing terms as long as all terms are shown and are all correct, but no partial credit if wrong
		ii	**OR** $u_{10} = 5 + 9 \times 3 = 32$	M1	Attempt u_{10}	Must be shown explicitly
		ii	$S = {}^{11}/_2\,(32 + 62)$	M1	Attempt required sum	Must have $n = 11$ Or $S = {}^{11}/_2\,(2 \times 32 + 10 \times 3)$
		ii	$= 517$ **AG**	A1	Obtain 517 <u>Examiner's Comments</u> The purpose of this part of the question was to assist candidates in finding an appropriate strategy with which to attempt the final part. Because the answer was given, candidates were expected to show full detail of their method and too many solutions did not address this. Most candidates gained at least the first mark for attempting the sum of the first twenty terms, but a number then struggled to make any further	Detail reqd – award M0M1A0 if no evidence for $u_{10} = 32$

					progress. Subtracting the sum of the first ten terms was the most common error; some candidates gave up at this point, whereas others made a valiant, but not always convincing, attempt to obtain the given 517. Some candidates listed, and summed, the relevant eleven terms. This approach gained full credit in this part of the question, but was not a method that could then be replicated in part (iiii).	
		iii	$S_{2N} = \frac{2N}{2}(10 + 3(2N - 1))$	B1	Correct (unsimplified) S_{2N} soi	Or $\frac{2N}{2}(5 + 5 + 3(2N - 1))$, or equiv, from $\frac{1}{2}n(a + l)$ Or $\frac{3}{2}(2N)(2N + 1) + 2(2N)$, or equiv, from $\Sigma(3n + 2)$
		iii	$S_{N-1} = \frac{N-1}{2}(10 + 3(N - 2))$	B1	Correct (unsimplified) S_{N-1} soi Or $S_N - u_N$ soi	Or $\frac{N-1}{2}(5 + 5 + 3(N - 2))$, or equiv, from $\frac{1}{2}n(a + l)$ Or $\frac{3}{2}(N - 1)(N) + 2(N - 1)$, or equiv, from $\Sigma(3n + 2)$
		iii	$N(6N + 7) - \frac{N-1}{2}(3N + 4) = 2750$	M1*	Subtract attempt at S_{N-1} from S_{2N} equate to 2750	Expressions could still be unsimplified Must have attempted to use correct formula, with $a = 5$, $d = 3$ and correct n each time Allow sign errors, resulting from lack of essential brackets M0 for $S_{2N} - S_N$ but M1 for $S_{2N} - S_N + u_N$
		iii	$9N^2 + 13N - 5496 = 0$	A1	Rearrange to obtain $9N^2 + 13N - 5496$ (= 0)	aef not involving brackets and with like terms combined
		iii	$(9N + 229)(N - 24) = 0$	M1d*	Attempt to solve 3 term quadratic	Any valid attempt to solve quadratic (see guidance) to obtain at least the positive root If solving an incorrect quadratic then method **must** be shown for M1 to be awarded
		iii	$N = 24$	A1	Obtain $N = 24$ only CWO	No need to consider the negative root, but A0 if found but not discarded Answer only gains full credit
		iii	OR $\frac{N+1}{2}(2(5 + 3(N - 1)) + 3N) = 2750$	M1*	Attempt sum from u_N to u_{2N}	Correct formula, $a = 5 + 3(N - 1)$, $d = 3$, and $n = N$ or $N + 1$
		iii		M1d*	Use $n = N + 1$	Use $n = N + 1$ only
		iii		A1	Correct unsimplified sum = 2750	Just equate to 2750, no need to rearrange

		iii	$9N^2 + 13N - 5496 = 0$	A1	Obtain correct quadratic	Or $^{N+1}/_2$ ((5 + 3(N − 1)) + (5 + 3(2N − 1))) from ½n (a + l)
		iii	(9N + 229)(N − 24) = 0	M1dd*	Attempt to solve 3 term quadratic	Quadratic must have come from sum = 2750
		iii	N = 24	A1	Obtain N = 24 only **Examiner's Comments** This final part of the question proved to be challenging for even the most able candidates, and fully correct solutions were in the minority. It was disappointing that so few candidates made the link between what they had been asked to do in the previous part of the question and what was now required of them. The first mark was available for finding the sum of the first 2N terms, and this was gained by just over half of the candidates. To make any further progress candidates now had to consider the sum of the first N - 1 terms, and then equate the difference to 2750. Only a minority actually attempted this, with the most common error being to subtract the sum of the first N terms. A lack of care with brackets meant that some candidates could not obtain the correct, simplified, quadratic despite the initial part of the solution being correct. An elegant alternative method that was sometimes seen considered the sum of N + 1 terms, starting on the Nth term and finishing on the 2Nth term, and an equally efficient method used the nth term definition of the sequence.	
			Total	11		
4		a	Geometric sequence, as multiplying by a common ratio each time	B1(AO 2.2a) [1]	Identify geometric with reasoning | Allow GP or similar	
		b	$u_{20} = 500 \times 0.8^{19}$ = 7.21	M1(AO 2.1) A1(AO 1.1)	Attempt u_{20} using ar^{n-1}, with $a = 500$ | **DR** so method must be seen	

				[2]	and $r = 0.8$ Obtain 7.21 or better (7.205759)	
		c	$S_{20} = \frac{500(1-0.8^{20})}{1-0.8}$ $= 2471$	M1(AO 2.1) A1(AO 1.1) [2]	Attempt u_{20} using correct formula, with $a = 500$ and $r = \pm 0.8$ Obtain 2471 or better (2471.17696)	**DR** so method must be seen
		d	$\frac{u_k}{1-0.8} = 1024$ $u_k = \frac{1024}{5}$ $500 \times 0.8^{k-1} = \frac{1024}{5}$ $0.8^{k-1} = \frac{256}{625}$ $k - 1 = \log_{0.8}(\frac{256}{625}) = 4$ $k = 5$	M1(AO 2.1) A1(AO 1.1) M1(AO 2.1) M1(AO 1.1) A1(AO 1.1) [2]	Attempt to use correct S_∞ formula, equate to 1024 and attempt u^k Obtain correct value for first term in this sequence Equate $500 \times 0.8^{k-1}$ to their value for u_k and rearrange to $0.8^{k-1} = c$ Correct use of logs to attempt $k - 1$	OR attempt $S_\infty - S_{k-1} = 1024$ Could use other notation, OR obtain correct unsimplified equation OR rearrange to $0.8^{k-1} = c$ Allow M1 if using logs to solve $0.8^k = \frac{256}{625}$ **DR** so method must be seen

					Obtain $k = 5$	
			Total	10		