1. A curve has parametric equations $x = t + \frac{2}{t}$ and $y = t - \frac{2}{t}$, for $t \neq 0$.

(a) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [4]

(b) Explain why the curve has no stationary points. [2]

(c) By considering $x + y$, or otherwise, find a cartesian equation of the curve, giving your answer in a form not involving fractions or brackets. [4]

2. The parametric equations of a curve are

$$x = 2 + 3 \sin \theta \text{ and } y = 1 - 2 \cos \theta \text{ for } 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi.$$

i. Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$.

[5]

ii. Find the cartesian equation of the curve.

[2]

3. A curve has parametric equations $x = 1 - \cos t$, $y = \sin t \sin 2t$, for $0 \leqslant t \leqslant \pi$.

i. Find the coordinates of the points where the curve meets the x-axis.

[3]

ii. Show that $\frac{dy}{dx} = 2\cos 2t + 2\cos^2 t$. Hence find, in an exact form, the coordinates of the stationary points.

[7]

iii. Find the cartesian equation of the curve. Give your answer in the form $y = f(x)$, where $f(x)$ is a polynomial.

[3]

iv. Sketch the curve.

[2]

4. The parametric equations of a curve are given by
$x = 2\cos\theta$ and $y = 3\sin\theta$ for $0 \le \theta < 2\pi$.

(a) Find $\frac{dy}{dx}$ in terms of θ. [2]

The tangents to the curve at the points P and Q pass through the point (2, 6).

(b) Show that the values of θ at the points P and Q satisfy the equation
$2\sin\theta + \cos\theta = 1$. [4]

(c) Find the values of θ at the points P and Q. [5]

5. A curve is defined by the parametric equations $x = \frac{2t}{1+t}$ and $y = \frac{t^2}{1+t}$, $t \neq -1$.

(a) (i) Show that the curve passes through the origin. [1]

(ii) Find the y-coordinate when $x = 1$. [1]

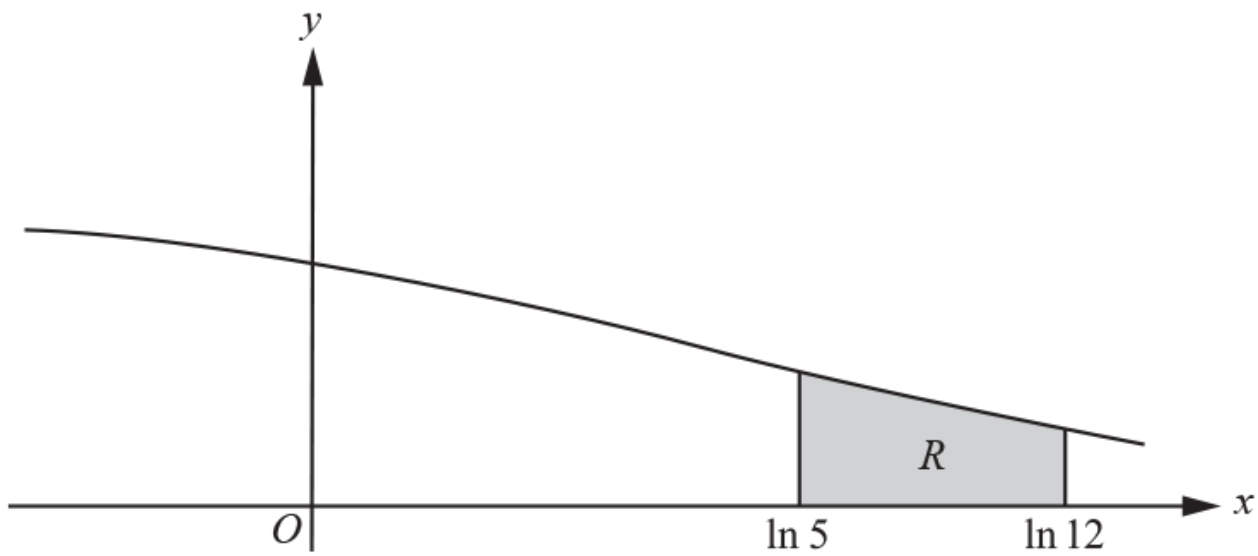
(b) Show that the area enclosed by the curve, the x-axis and the line $x = 1$ is given by

$$\int_0^1 \frac{2t^2}{(1+t)^3}\,dt.$$ [5]

(c) **In this question you must show detailed reasoning.**

Hence use an appropriate substitution to find the exact area enclosed by the curve, the x-axis and the line $x = 1$. [6]

6.



The diagram shows the curve with parametric equations $x = \ln(t^2 - 4), \quad y = \dfrac{4}{t^2}$, where $t > 2$.

The shaded region R is enclosed by the curve, the x-axis and the lines $x = \ln 5$ and $x = \ln 12$.

(a) Show that the area of R is given by

$$\int_a^b \frac{8}{t(t^2-4)}\,\mathrm{d}t,$$

where a and b are constants to be determined. [4]

(b) **In this question you must show detailed reasoning.**

Hence find the exact area of R, giving your answer in the form $\ln k$, where k is a constant to be determined. [8]

(c) Find a cartesian equation of the curve in the form $y = \mathrm{f}(x)$. [3]

7. A curve has parametric equations $x = \frac{1}{t} - 1$ and $y = 2t + \frac{1}{t^2}$.

i. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *t*, simplifying your answer.

[3]

ii. Find the coordinates of the stationary point and, by considering the gradient of the curve on either side of this point, determine its nature.

[4]

iii. Find a cartesian equation of the curve.

[2]

8.

i. Express $\frac{x+8}{x(x+2)}$ in partial fractions.

[3]

ii. By first using division, express $\frac{7x^2+16x+16}{x(x+2)}$ in the form $P + \frac{Q}{x} + \frac{R}{x+2}$.

[3]

A curve has parametric equations $x = \frac{2t}{1-t}$, $y = 3t + \frac{4}{t}$.

iii. Show that the cartesian equation of the curve is $y = \frac{7x^2+16x+16}{x(x+2)}$.

[4]

iv. Find the area of the region bounded by the curve, the *x*-axis and the lines $x = 1$ and $x = 2$. Give your answer in the form $L + M\ln 2 + N\ln 3$.

[4]

9. The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3 - 3t \text{ for } -2 < t \le 0.$$

(i) Find $\frac{dy}{dx}$ in terms of t. [3]

(ii) Find the coordinates of the stationary point on the curve and determine its nature. [4]

(iii) State the range of values of x and the range of values of y. [2]

(iv) Sketch the curve. [1]

10. A curve is defined, for $t \ge 0$, by the parametric equations

$$x = t^2, \qquad y = t^3.$$

(a) Show that the equation of the tangent at the point with parameter t is

$$2y = 3tx - t^3.$$ [4]

(b) **In this question you must show detailed reasoning.**

It is given that this tangent passes through the point $A\left(\frac{19}{12}, -\frac{15}{8}\right)$ and it meets the x-axis at the point B.

Find the area of triangle OAB, where O is the origin. [7]

11. A curve has parametric equations

$$x = 2\sin t,\ y = \cos 2t + 2\sin t$$

for $-\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi$

i. Show that $\frac{dy}{dx} = 1 - 2\sin t$ and hence find the coordinates of the stationary point.

[5]

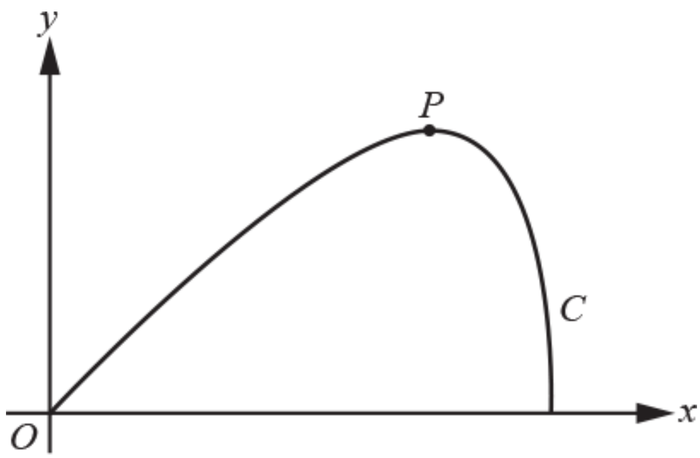
ii. Find the cartesian equation of the curve.

[3]

iii. State the set of values that x can take and hence sketch the curve.

[3]

12.



The diagram shows the curve C with parametric equations

$$x = \frac{1}{4}\sin t,\ y = t\cos t,$$

where $0 \leq t \leq k$.

(a) Find the value of k. [2]

(b) Find $\frac{dy}{dt}$ in terms of t. [2]

The maximum point on C is denoted by P.

(c) Using your answer to part (b) and the standard small angle approximations, find an approximation for the x-coordinate of P.

[4]

(d) (i) Show that the area of the finite region bounded by C and the x–axis is given by

$$b\int_0^a t(1+\cos 2t)\,dt,$$

where a and b are constants to be determined. [3]

(ii) **In this question you must show detailed reasoning.**

Hence find the exact area of the finite region bounded by C and the x–axis. [5]

END OF QUESTION paper

Question			Answer/Indicative content	Marks	Part marks and guidance		
1		a	$\frac{dx}{dt}=1-2t^{-2}$ $\frac{dy}{dt}=1+2t^{-2}$ $\frac{dy}{dx}=\frac{1+2t^{-2}}{1-2t^{-2}}=\frac{\frac{t^2+2}{t^2}}{\frac{t^2-2}{t^2}}$ $\frac{dy}{dx}=\frac{t^2+2}{t^2-2}$	B1 (AO 1.1) B1 (AO 1.1) M1 (AO 1.1a) A1 (AO 1.1) [4]	Correct $\frac{dx}{dt}$ Correct $\frac{dy}{dt}$ Attempt correct method to combine their derivatives Obtain correct derivative	Any equivalent form Any equivalent form Division must be correct way around Allow any simplified equivalent such as $1+\frac{4}{t^2-2}$	
					<u>Examiner's Comments</u> Most candidates gained at least 3 marks for obtaining a correct expression for the derivative, but the subsequent simplification proved to be more challenging. Candidates who worked with fractions tended to be more successful than those who worked with negative indices. The most common error was 'cancelling' just a single term in the numerator and denominator.		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		b	$\frac{dy}{dx}=0 \Rightarrow t^2+2=0$	E1ft (AO 2.2a)	Justify $t^2 + 2 = 0$ for stat point	Must state that gradient ($\frac{dy}{dx}$) = 0 (or t (cannot be implied by method) then equate their numerator to 0
			$t^2 \geq 0$, hence $t^2 + 2 = 0$ has no solutions, hence curve has no stationary points	E1 (AO 2.4)	Justify no stationary points	Allow use of a gradient that is no longer a fraction Explain why there are no solutions eg referring to $t^2 + 2 \geq 2$ eg t^2 is always positive (as $t \neq 0$ given) eg $t^2 + 2 = 0$ has no real roots and conclude with 'no stationary points' Must now be from a fully correct derivative only
				[2]	**Examiner's Comments** Whilst most candidates were able to attempt the correct method, many did	

Question			Answer/Indicative content	Marks	Part marks and guidance		
					not give sufficient detail of their reasoning. Rather than just equating the derivative to 0 examiners expected to see some justification for this, such as a statement that the gradient is 0 at a stationary point. Candidates were then expected to explain why the equation $t^2 + 2 = 0$ had no real roots before concluding that this meant there were no stationary points.		
		c	$x + y = 2t$ hence $t = \frac{1}{2}(x+y)$	B1 (AO 1.1)	Correct expression for t	Any correct equation involving t along with x and/or y where t only appears once	
			$x = \frac{1}{2}(x+y) + \frac{2}{\frac{1}{2}(x+y)}$	M1 (AO 1.2)	Substitute for t into either equation	Expression for t must be correct	
			$2x(x + y) = (x + y)^2 + 8$ $2x^2 + 2xy = x^2 + 2xy + y^2 + 8$	M1 (AO 3.1a)	Attempt rearrangement	Could be using attempt (possibly no longer correct) at a rearranged parametric equation eg $xt - t^2 = 2$	
			$x^2 - y^2 = 8$	A1 (AO 1.1)	Correct equation	As far as requested form	
				[4]		Any correct three term	

Question			Answer/Indicative content	Marks	Part marks and guidance
					equivalent Allow A1 for eg $y = \pm\sqrt{x^2 - 8}$, but A0 if not ± __Examiner's Comments__ Many candidates were able to use the hint given in the question to produce $x + y = 2t$ but a number were then unsure how to make any further progress. Others appreciated that they could now use this equation to eliminate t from one of the given parametric equations, but errors when rearranging to the requested from were common, demonstrating a lack of confidence when dealing with algebraic fractions. A minority of candidates ignored the hint given in the question, and attempted to use an alternative method such as rearranging to $t^2 - xt + 2 = 0$, solving for t and substituting into the other equation. Whilst some progress was usually made, these attempts were rarely fully correct. The most elegant solution, provided by a few candidates, was to consider both $x + y$ and $x - y$ to produce two equations from which $2t$ could then be easily eliminated.
			Total	10	

Question			Answer/Indicative content	Marks	Part marks and guidance	
2		i	their $\dfrac{dy}{d\theta} \Big/ \dfrac{dx}{d\theta}$	M1		
		i	$\dfrac{dy}{dx} = \dfrac{2\sin\theta}{3\cos\theta}$	A1		
		i	their $\dfrac{dy}{dx} = \dfrac{1}{2}$	M1		
		i	$\tan\theta = \dfrac{3}{4}$	A1	If $\tan\theta = \dfrac{3}{4}$ not seen, award this A1 only if coords are correct **Examiner's Comments** This was generally done well though a few were unable to manipulate the equation $\frac{2}{3}\tan\theta = \frac{1}{2}$ into its simpler version $\tan\theta = \frac{3}{4}$. Apart from rounding errors, the actual coordinates were then relatively easy to find.	
		i	$(3.8, -0.6)$ or $\left(\frac{19}{5}, -\frac{3}{5}\right)$ or $x = 3.8, y = -0.6$	A1		
		ii	Manipulating equations into form sin θ = f (x) and cos θ = g(y) and then using $\sin^2\theta + \cos^2\theta = 1$	M1	If part (ii) is attempted first, and then part (i), allow B1 for obtaining $\dfrac{dy}{dx} = \dfrac{4(x-2)}{9(y-1)}$ M1 for equating their $\dfrac{dy}{dx}$ to $\dfrac{1}{2}$	the following marks in part (i): –

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$\frac{(x-2)^2}{9}+\frac{(1-y)^2}{4}=1$ oe www ISW Accept e.g. $\left(\frac{x-2}{3}\right)^2$ $4x^2 + 9y^2 - 16x - 18y - 11 = 0$	A1	A1 for obtaining $9y - 8x = -7$ M1 for eliminating x or y from above eqn… A1 for (3.8,–0.6) **Examiner's Comments** A large number of candidates assumed that the required cartesian equation had to be linear, or that inverse trigonometrical functions would be acceptable in the answer. A few said that $\cos\theta = \frac{y-1}{2}$ and then used it in the equation $\cos^2\theta + \sin^2\theta = 1$; although $\left(\frac{y-1}{2}\right)^2$ and $\left(\frac{1-y}{2}\right)^2$ were equivalent at that stage, an earlier mistake had been seen and was consequently penalised.	…. and their Cartesian equation
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance	
3		i	$\sin t \sin 2t = 0$ oe seen	M1		**NB** $t = 0$, ½π, π
		i	(0, 0) (1, 0) and (2, 0) or $x = 0$, $x = 1$, $x = 2$ cao	A2	**A1** for two of three correct	deduct 1 mark if all three correct plus extra values if **A0**, allow **SC1** for $t = 0$, ½π, π if unsupported, full marks for all three values correct **Examiner's Comments** Most candidates set $y = 0$, but few went on to successfully find all three values. Surprisingly, (0, 0) was almost as commonly omitted as (1, 0) and (2, 0).
		ii	$\left[\frac{dy}{dt}\right] = 2\sin t\cos 2t + \cos t\sin 2t$	B1	or $4\sin t\cos^2 t - 2\sin^3 t$	
		ii	$\frac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ or $\frac{(4\sin t\cos^2 t - 2\sin^3 t)}{\sin t}$	M1	allow sign errors and/or one incorrect coefficient	
		ii	substitution of $\sin 2t = 2\sin t \cos t$ in their $\frac{(2\sin t\cos 2t + \cos t\sin 2t)}{\sin t}$ and completion to	M1	may be seen before differentiation	
		ii	$2\cos 2t + 2\cos^2 t$ www **NB AG**	A1	at least one correct intermediate step needed	
		ii	eg $2(2\cos^2 t - 1) + 2\cos^2 t = 0$ or $2\cos 2t + 2\times½(1 + \cos 2t) = 0$	M1	use of double angle formula to obtain quadratic equation in eg $\cos t$ or linear equation in $\cos 2t$; may be seen before differentiation	mark intent: allow sign error, bracket error, omission of one coefficient
		ii	$(1 + \frac{1}{\sqrt{3}}, \frac{-4}{3\sqrt{3}})$ oe isw	A1		eg $(\frac{\sqrt{3}+3}{3}, -\frac{4\sqrt{3}}{9})$

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$(1-\frac{1}{\sqrt{3}}, \frac{4}{3\sqrt{3}})$ oe isw	A1	if **A0**, **A0**, allow **A1** for both x values correct	**Examiner's Comments** Many candidates knew what to do to obtain the required result, and there were many examples of clear, well-structured solutions. Most realised the need to resolve the double angle for the next part of the question, and many made no further progress. Only the best candidates went on to obtain both pairs of coordinates correctly.
		iii	$y = 2(1 - \cos^2 t)\cos t$ oe may be implicit equation, may be implied by partial substitution for $\cos t$ eg $(1-x)^2 + \frac{y}{2\cos t} = 1$	M1	or $\frac{dy}{dx} = 6\cos^2 t - 2$	use of double angle formula (and Pythagoras) to obtain expression for y or $\frac{dy}{dx}$ in terms $\cos t$ only;
		iii	$y = 2(1 - (1 - x)^2)(1-x)$	M1	or $\frac{dy}{dx} = 6(1-x)^2 - 2$	substitution of $\cos t = \pm 1 \pm x$ to obtain expression in terms of x only allow sign errors, bracket errors or minor slips in arithmetic eg omission of 2 for these method marks

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$y = 2x^3 - 6x^2 + 4x$ or $y = 2x(x^2 - 3x + 2)$ or $y = 2x(x - 1)(x - 2)$ oe cao	A1	integration and substitution of eg (0, 0) to obtain correct answer must see $y =$ at some stage for **A1**	Examiner's Comments A significant number of candidates worked immediately with inverse trig functions and failed to score - not realising that they could not achieve a polynomial expression by this route. Many candidates appreciated the need to use the double angle formula and Pythagoras, but mistakes in expanding brackets were common and the “2” was frequently omitted following substitution. Expressing cost in terms of x often went wrong and the high frequency of algebraic slips prevented many candidates from achieving full marks.
		iv iv	cubic with two turning points and of correct orientation through (0, 0) x-intercepts correct and only for $0 \leq x \leq 2$	M1 A1		Examiner's Comments Not many candidates made the connection between this part of the question and earlier work. Cubics of the right orientation and with the right intercepts were sometimes seen, but very few candidates appreciated the restriction on the x -values. Nevertheless, a few candidates who had made little progress in earlier parts of the question reached for their graphical calculators and achieved both marks.
			Total	15		

Question			Answer/Indicative content	Marks	Part marks and guidance	
4		a	Use $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{d\theta}{dx}$ Obtain $\frac{-3\cos\theta}{2\sin\theta}$	M1(AO1.1a) A1(AO1.1) [2]		
		b	$(y-3\sin\theta)=\frac{-3\cos\theta}{2\sin\theta}(x-2\cos\theta)$ $2y\sin\theta-6\sin^2\theta=-3x\cos\theta+6\cos^2\theta$ $2y\sin\theta+3x\cos\theta=6$ $12\sin\theta+6\cos\theta=6\Rightarrow 2\sin\theta+\cos\theta=1$	M1(AO3.1a) M1(AO1.1) A1FT(AO1.1) E1(AO2.1) [4]	Attempt equation of straight line in any unsimplified form Accept *x, y* confusion Simplify their equation and use $\cos^2\theta+\sin^2\theta=1$ Substitute (2, 6) and simplify to AG	**OR** **M1** When $\theta=\theta_Q$, gradient of curve is given by $\frac{-3\cos\theta_Q}{2\sin\theta_Q}$ **M1** The gradient of the line through (2,6) and ($2\cos\theta_Q$,$3\sin\theta_Q$)is **M1** Equate and clear fractions **E1** Obtain AG	

Question			Answer/Indicative content	Marks	Part marks and guidance		
		c	Use $R\sin(\theta + \alpha)$ on $2\sin\theta + \cos\theta$ $R\sin\alpha = 1$, $R\cos\alpha = 2$ Obtain $\alpha = 0.4636$ and $R = \sqrt{5}$ Use correct order of operations to solve $\sqrt{5}\sin(\theta + 0.4636) = 1$ Obtain 0 Obtain 2.21	M1(AO3.1a) A1(AO1.1) M1(AO1.1) B1(AO2.2a) A1(AO1.1) [5]	Should go as far as finding R and α Allow alternative forms Attempt to solve their $R\sin(\theta + \alpha)$ Or better (2.214345)	**OR** **M1** Square and use $\sin^2\theta + \cos^2\theta = 1$ **A1** $4\sin^2\theta + 4\sin\theta(1 - 2\sin\theta) + (1 - \sin^2\theta) = 1$ **M1** Simplify and solve $5\sin^2\theta - 4\sin\theta = 0$	
			Total	11			

Question			Answer/Indicative content	Marks	Part marks and guidance		
5		a	(a) when $x = 0$, $t = 0$ and hence $y = 0$	E1(AO2.4) [1]	Justify (0, 0) convincingly		
		a	(b) when $x = 1$, $t = 1$ and hence $y = 0.5$	B1(AO1.1) [1]	Obtain $y = 0.5$		
		b	$\frac{dx}{dt} = \frac{2}{(1+t)^2}$ $\int \frac{t^2}{1+t} dx = \int \frac{t^2}{1+t} \times \frac{2}{(1+t)^2} dt$ $= \int \frac{2t^2}{(1+t)^3} dt$	M1(AO2.1) A1(AO2.1) M1(AO2.1) A1(AO2.1) B1(AO2.4) [5]	Attempt $\frac{dx}{dt}$ Obtain correct derivative Use $\int y dx = \int y \frac{dx}{dt} dt$ Obtain given answer Justify t -limits from $x = 0, 1$	Using quotient rule, or other valid method $x = 0$: $\frac{2t}{1+t} = 0$ so $t = 0$ $x = 1$: $\frac{2t}{1+t} = 1$ *$2t = 1 + t$* *so $t = 1$*	

Question			Answer/Indicative content	Marks	Part marks and guidance		
		c	DR use $u = 1 + t$ giving du = dt	E1(AO1.1a)	Must be stated explicitly		
			$\int \frac{2t^2}{(1+t)^3}\,\mathrm{d}t = \int \frac{2(u-1)^2}{u^3}\,\mathrm{d}u$	M1(AO1.1a)	Attempt to change integrand to function of u		
			$= \int 2u^{-1} - 4u^{-2} + 2u^{-3}\,\mathrm{d}u$	A1(AO1.1)	Obtain correct integrand	Any equivalent form	
			$= \left[2\ln u + 4u^{-1} - u^{-2}\right]_1^2$	M1(AO1.1a)	Attempt integration		
			= (2ln2 + 2 – 0.25) – (2ln1 + 4 – 1)	M1(AO1.1a)	Attempt use of limits u = 1, 2		
			$= 2\ln 2 - \frac{5}{4}$	A1(AO1.1) [6]	Obtain correct exact area	Allow any exact equiv	
			Total	13			

Question			Answer/Indicative content	Marks	Part marks and guidance	
6		a	$x = \ln(t^2 - 4) \Rightarrow \frac{dx}{dt} = \frac{2t}{t^2 - 4}$	M1 (AO 1.1)	Attempt differentiation of *x* using chain rule – must be of the form $\frac{kt}{t^2 - 4}$	
			$\text{Area} = \int \frac{4}{t^2}\left(\frac{2t}{t^2 - 4}\right) dt$	M1 (AO 1.1a)	Use $\int y \frac{dx}{dt} dt$ with their $\frac{dx}{dt}$	
			$= \int \frac{8}{t(t^2 - 4)} dt$	A1 (AO 2.2a)	AG	
			a = 3, *b* = 4	B1 (AO 2.2a)	Correct limits	
				[4]		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		b	DR $\frac{8}{t(t^2-4)} \equiv \frac{A}{t} + \frac{B}{t-2} + \frac{C}{t+2}$ $8 = A(t-2)(t+2) + Bt(t+2) + Ct(t-2)$ $A = -2$, $B = 1$, $C = 1$ $\int\left(-\frac{2}{t}+\frac{1}{t-2}+\frac{1}{t+2}\right)\mathrm{d}t = -2\ln t + \ln(t-2) + \ln(t+2)$ (–2ln4 + ln2 + ln6) – (–2ln3 + ln1 + ln5) l $\left(\frac{27}{20}\right)$ n	B1 (AO 3.1a) M1 (AO 1.1a) A2 (AO 1.1,1.1) M1* (AO 1.1) M1dep* (AO 1.1) M1 (AO 2.1) A1 (AO 2.2a) [8]	Correct form of partial fractions Cover up, substituting or equating coefficients – must be a complete method for finding one of A, B or C **A1** for one correct Attempt to integrate all three terms – must be of the form $\alpha \ln t + \beta \ln(t-2) + \gamma \ln(t+2)$ Applying their limits correctly Correctly combining all their log terms – dependent on both previous M marks $k = \frac{27}{20}$	

Question			Answer/Indicative content	Marks	Part marks and guidance		
		c	$t^2 = \frac{4}{y} \Rightarrow x = \ln\left(\frac{4}{y} - 4\right)$ $e^x = \frac{4}{y} - 4 \Rightarrow y = \text{K}$ $y = \frac{4}{e^x + 4}$ Alternative solution $e^x = t^2 - 4$ $t^2 = e^x + 4 \Rightarrow y = \text{K}$ $y = \frac{4}{e^x + 4}$	M1* (AO 3.1a) M1dep* (AO 1.1) A1 (AO 1.1) M1* M1dep* A1 [3]	Re-arrange and eliminate t Remove logs and attempt to make y the subject Remove logs Rearrange and eliminate t		
			Total	15			

Question			Answer/Indicative content	Marks	Part marks and guidance	
7		i	$\frac{dy}{dt} = 2(+) - \frac{2}{t^3}$; $\frac{dx}{dt} = -\frac{1}{t^2}$ oe soi ISW $\frac{2}{t} - 2t^2$ or $-2\left(t^2 - \frac{1}{t}\right), \frac{2t^3 - 2}{-t}, -t^2\left(2 - \frac{2}{t^3}\right)$ oe	B1, B1 B1	ISW. Must not involve (implied) 'triple-deckers' e.g. fractions with neg powers…	… e.g. $\frac{2-2t^{-3}}{-t^2}$
		i			**Examiner's Comments** In general, apart from the derivative of $\frac{1}{t}$ being Int in some cases, the differentiation was handled competently. The question asked for the answer to be simplified and many alternatives were accepted — though not fractions with negative powers involved in numerator and denominator.	
		ii	(Any of their expressions for $\frac{dy}{dx}$) = 0 or their $\frac{dy}{dt} = 0$	M1		
		ii	$t = 1$ →(stationary point) = (0, 3)	A1	Not awarded if $\frac{dy}{dx}$ is wrong in (i) and used here BUT allow recovery if only explicitly considering $\frac{dy}{dt} = 0$	
		ii	Consider values of x on each side of their critical value of x which lead to finite values of $\frac{dy}{dx}$	M1		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	Hence (0, 3) is a minimum point www	A1	Totally satis; values of x must be close to 0 & not going below or equal to $x = -1$ **Examiner's Comments** The stationary point was relatively easy to find; having found t, the question directed candidates to find x and y. It was hoped that this would focus attention on the value of x, as is normal in the classifying of stationary points. However, some considered points on either side of the critical value of t, not realising that this would not indicate which side of the stationary point they were considering.	
		iii iii	Attempt to find t from $x = \frac{1}{t} - 1$ and substitute into the equation for y $y = \frac{2}{x+1} + (x+1)^2$ oe (can be unsimplified) ISW	M1 A1	**Examiner's Comments** Apart from careless errors, this part of the question was well done.	
			Total	9		

Question			Answer/Indicative content	Marks	Part marks and guidance	
8		i	$\frac{A}{x}+\frac{B}{x+2}$	B1		award if only implied by answer
		i	$x + 8 = A(x + 2) + Bx$ soi	M1	allow one sign error Examiner's Comments☐☐☐ Most recognised the correct form of partial fractions and successfully cleared the fractions to produce a fully correct solution.	clearing fractions successfully
		i	$A = 4$ *and* $B = –3$	A1		if **M0**, **B1** for each value www
		ii	quotient (P) is 7	B1		
		ii	$2x + 16$ seen	B1	if **B0**, **B1** for $Q = 8$ and **B1** for $R = – 6$ www Examiner's Comments Most candidates used long division and successfully found the quotient and the remainder. Many then used their answer to part (i) to produce a correct solution. A variety of other approaches were also successful, but a significant minority of those who equated coefficients went astray in the algebra. A small number of candidates tried to divide by x and x + 2 separately, and were rarely successful.	eg as remainder or in division chunking
		ii	$7+\frac{8}{x}-\frac{6}{x+2}$	B1		or allow $P = 7$, $Q = 8$, $R = – 6$
		iii	$t = f(x)$	M1*	from $x=\frac{2t}{1-t}$; **M0** for $t = g(y)$	

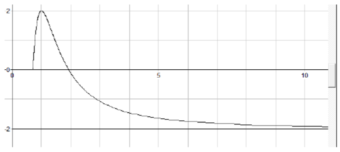
Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$t = \dfrac{x}{x+2}$	A1	or **B2** if unsupported <u>Examiner's Comments</u> There were many well laid out, perfectly correct responses to this question. However, it proved to be surprisingly difficult for many. Sometimes a formula for t in terms of x and t was substituted in, which didn't lead anywhere. In other cases the expression for t contained a sign error or an algebraic slip. Often candidates persisted with a clearly incorrect formula, instead of checking the early part of their work. A few candidates verified the result by substitution, which was a convoluted approach and did not earn full marks.	
		iii	$y = 3 \times \text{their}\,\dfrac{x}{x+2} + \dfrac{4}{\text{their}\,\dfrac{x}{x+2}}$ eg $\dfrac{3x^2 + (8+4x)(x+2)}{x(x+2)}$ and completion to	M1dep*		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	$y = \dfrac{7x^2 + 16x + 16}{x(x+2)}$	A1		at least one correct, constructive, intermediate step shown if **M0M0**, **SC2** for substitution of $x = \dfrac{2t}{1-t}$ in RHS of given equation and completion with at least two correct, constructive intermediate steps to $y = 3t + \dfrac{4}{t}$ www
		iv	$\int$ their $(P + \dfrac{Q}{x} + \dfrac{R}{x+2})[\mathrm{d}x]$	M1*	where *P, Q* and *R* are constants obtained in (ii)	allow omission of d*x*
		iv	F[*x*] = 7*x* + 8ln*x* – 6ln(*x* + 2)	A1ft	allow recovery from omission of brackets in subsequent working	if **M0**, **SC1** for *Px* + *Q*ln*x* + *R*ln(*x* + 2) where constants are unspecified or arbitrary
		iv	F[2] – F[1]	M1dep*		
		iv	7 – 4ln2 + 6ln3	A1	<u>Examiner's Comments</u> There were many excellent responses to this part of the question. Most candidates spotted the link with part (ii) and went on to earn three or four marks. Those who started from scratch were almost never successful.	
			Total	14		

Question			Answer/Indicative content	Marks	Part marks and guidance	
9		i	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) = 3t^2 - 3$	B1		
			$\left(\frac{\mathrm{d}x}{\mathrm{d}t} = \right) k\left(2+t\right)^{-\frac{3}{2}}$	M1	$k \neq 0$	
			$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3t^2 - 3}{-\frac{1}{2}(2+t)^{-\frac{3}{2}}}$ oe isw	A1 [3]	do not allow bracket errors in marked answer	
			Alternatively			
			$[y =](x^{-2} - 2)^3 - 3x^{-2} + 6$ oe	B1		
			$\left[\frac{\mathrm{d}y}{\mathrm{d}x} = \right] 3(x^{-2} - 2)^2 \times (-2x^{-3}) + 6x^{-3}$ oe	M1		
			$3\left[\left((2+t)^{-\frac{1}{2}}\right)^{-2} - 2\right]^2 \times -2\left((2+t)^{-\frac{1}{2}}\right)^{-3}$ $+6\left((2+t)^{-\frac{1}{2}}\right)^{-3}$ oe isw	A1 [3]	allow sign errors and/or one coefficient error	
					Examiner's Comments This was done well by most. A few slipped up with $\frac{\mathrm{d}x}{\mathrm{d}t}$ or made bracket or sign errors when combining the two derivatives.	

Question			Answer/Indicative content	Marks	Part marks and guidance		
		ii	their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1	allow eg $3t^2 - 3 = 0$	allow one transcription error	
			(1, 2) oe identified as only stationary point	A1	NB $t = -1$		
			eg $t = -0.5$, $x = \sqrt{2/3}$ and gradient = 8.27 eg $t = -1.5$, $x = \sqrt{2}$ and gradient = –2.65	M1	consideration of gradient either side of *their* $x = 1$	ignore work with other points for the last two marks ignore work with other points for the last two marks	
			or eg $t = -0.5$ and $y = 1.375$, $t = -1.5$ and $y = 1.125$		**or** consideration of y -values either side of their $y = 2$		
			hence maximum value at (1, 2)	A1			
			Alternatively, for last two marks				
			evaluation of second derivative at *their* $t = -1$ or *their* $x = 1$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -18x^{-4} + 24x^{-8} - 48x^{-6} + 18x^{-4}(x^{-2} - 2)^2$ or oe $6(2 + t)^2(7t^2 + 8t - 3)$	M1	second derivative must be obtained from correct method; allow sign errors		
			convincing justification that second derivative < 0 [NB –24] so maximum	A1 [4]			
					Examiner's Comments		

Question			Answer/Indicative content	Marks	Part marks and guidance	
					Most were able to start this part, but often went stray in finding the correct value of *t*. A significant minority of candidates worked with $t = -2$ and / or $t = 1$, which was pointless as both values are outside the specified range. Even those who did work with $t = -1$ only often went astray in finding *x* and *y*. Only a few candidates realised the need to check the x-values as well as the values of the gradient when determining the nature of the stationary point, and those who tried to use the second derivative almost invariably went wrong.	
		iii	$x \geq \frac{1}{\sqrt{2}}$ $-2 < y \leq 2$	B1 B1 [2]	**Examiner's Comments** Very few candidates were able to state both ranges correctly.	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iv		B1 [1]	curve with maximum in 1st quadrant and horizontal asymptote in 4th quadrant drawn for $x \geq k$, where $k > 0$ **Examiner's Comments** Only a very small minority sketched the curve successfully. A significant proportion of those who were successful had not managed full marks in the previous parts of the question.	
			Total	10		

Question			Answer/Indicative content	Marks	Part marks and guidance	
10		a	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$, where $\frac{dy}{dt} = 3t^2$, $\frac{dx}{dt} = 2t$	*M1 (AO 1.1a)	Correct application of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ with their $\frac{dy}{dt}$ and $\frac{dx}{dt}$ (with at least one correct)	
			$\frac{dy}{dx} = \frac{3t^2}{2t} \quad \left(= \frac{3}{2}t\right)$	A1 (AO 1.1)	Correct derivative (need not be simplified at this stage)	
			$y - t^3 = \frac{3t^2}{2t}(x - t^2)$	dep*M1 (AO 1.1)	Use of $y - t^3 = m(x - t^2)$ with their m in terms of t	
			$2y = 3tx - t^3$	A1 (AO 2.2a)	AG	
				[4]		

Question			Answer/Indicative content	Marks	Part marks and guidance		
		b	DR Substitute A giving $t^3 - 3t\left(\frac{19}{12}\right) + 2\left(-\frac{15}{8}\right) = 0,$ and attempt factor theorem with $f(t) = 4t^3 - 19t - 15$, oe $f(-1) = 0 \Rightarrow (t + 1)$ is a factor $f(t) = (t + 1)(4t^2 - 4t - 15)$ $f(t) = (t + 1)(2t - 5)(2t + 3)$ $t = \frac{5}{2}$ only, as $t \geq 0$ $y = 0 \Rightarrow B\left(\frac{25}{12}, 0\right)$ and area $= \frac{1}{2} \times \frac{15}{8} \times \frac{25}{12}$ area $= \frac{125}{64}$	M1 (AO 3.1a) A1 (AO 1.1) M1 (AO 1.1) A1 (AO 1.1) A1 (AO 3.2a) M1 (AO 1.1) A1 (AO 2.2a) [7]	Correctly substitute A into given tangent and attempt to find a factor Attempt to obtain a quadratic factor Use of their t to find B and attempt to find area using their B	By any correct method Their value of t must be positive	
			Total	11			

Question			Answer/Indicative content	Marks	Part marks and guidance	
11		i	$\frac{dy}{dt} = -2\sin 2t + 2\cos t$ soi	B1	NB $\frac{dx}{dt} = 2\cos t$	if B0M0A0 SC3 fo $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of *x* = 2sin*t*
		i	$\frac{dy}{dx} = \text{their}\,\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ oe	M1		
		i	$\frac{-2\sin 2t + 2\cos t}{2\cos t}$ soi	A1		
		i	$\frac{-4\sin t\cos t + 2\cos t}{2\cos t}$ or $\frac{2\cos t(-2\sin t + 1)}{2\cos t}$ and completion to 1 – 2sint www	A1	or equivalent intermediate step	
		i	(1, 1½)	B1	NB $t = \frac{\pi}{6}$ **Examiner's Comments** This proved accessible to most, with a good number of candidates achieving full marks. However, in many cases "2" went missing from the double angle, and $= \frac{-2\sin t + 2\cos t}{2\cos t} = 1 = 2\sin t$, in an attempt to achieve the given answer. Surprisingly, a large number of candidates simply omitted to find the co-ordinates of the turning point, or stopped at $t = \frac{\pi}{6}$.	from 1 – 2sin*t* = 0

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	(y =) $1 - 2\sin^2 t + 2\sin t$	B1	may be awarded after correct substitution for x eg (y =) $1 - {}^{x^2}/_4 - \sin^2 t + 2\sin t$	or (y =) $x + \cos 2t$
		ii	substitution of $\sin t = \frac{1}{2}x$ to eliminate t	M1		substitution of $t = \sin^{-1}({}^x/_2)$ to eliminate t
		ii	$y = 1 + x -$ $\frac{1}{2}x^2$ oe isw	A1	or B3 www Examiner's Comments Those who attempted to find a polynomial equation often went astray in the substitution: $x^2 = (2\sin t)^2 = 2\sin^2 t$ was a common error, leading to $y = 1 - x^2 + x$. It was not always clear what substitution candidates were making: in cases where it went wrong a method mark was sometimes lost. Weaker candidates opted for an equation involving arcsin($\frac{x}{2}$); this nearly always resulted in zero in part (iii).	$y = x + \cos 2(\sin^{-1}({}^x/_2))$ oe isw
		iii	$-2 \le x \le 2$ **or** $x \ge -2$ (and) $x \ge 2$ or $\mid x \mid \le 2$	B1	cao	
		iii	sketch of negative quadratic with endpoints in 1st and 3rd quadrants	M1	RH point must be to the right of the maximum	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		iii	positive y-intercept and one distinguishing feature isw	A1	<u>Examiner's Comments</u> Some candidates were able to deduce the range of values for x, but more often than not did not take the hint and relate this to their sketch. Only the best candidates produced a graph of the correct shape with endpoints in the correct quadrants, and only a handful identified a correct distinguishing feature for the third mark.	one from: endpoints (–2, –3) and (2, 1), vertex at (1, 1½), y - intercept is (0, 1), x-intercept is $(1 – \sqrt{3}, 0)$
			Total	11		

Question			Answer/Indicative content	Marks	Part marks and guidance	
12		a	$y = 0 \Rightarrow (t = 0$ or$)$ $\cos t = 0$ $k = \frac{1}{2}\pi$	M1 (AO1.1a) A1 (AO2.2a) [2]	Setting $y = 0$	
		b	$\frac{\mathrm{d}y}{\mathrm{d}t} = \cos t - t\sin t$	M1 (AO1.1) A1 (AO1.1) [2]	Attempt at product rule – allow sign errors	
		c	$\cos t - t\sin t = 0 \Rightarrow \left(1 - \frac{1}{2}t^2\right) - t(t) = 0$ $\frac{3}{2}t^2 = 1 \Rightarrow t = \ldots$ $t = \sqrt{\frac{2}{3}}$ x $\approx$ 0.2	M1* (AO2.1) dep*M1 (AO1.1) A1 (AO1.1) A1 (AO2.2a) [4]	Setting $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ and substituting small angle approximations for both sine and cosine Simplify and attempt to solve for t (with correct order of operations) Condone 0.18	Allow ± 0.1821878 …

Question			Answer/Indicative content	Marks	Part marks and guidance		
		d	(i) $I=\int t\cos t\left(\frac{1}{4}\cos t\right)\,\mathrm{d}t$	M1 (AO1.2)	Attempted use of $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$	Ignore limits for first two marks	
			$=\frac{1}{4}\int t\left(\frac{1}{2}(1+\cos 2t)\right)\,\mathrm{d}t$	M1 (AO3.1a)	Use of $\cos^2 t\equiv\frac{1}{2}(1+\cos 2t)$	Allow sign errors in identity	
			$=\frac{1}{8}\int_0^{\frac{1}{2}\pi}t(1+\cos 2t)\,\mathrm{d}t$ d	A1FT (AO2.2a) [3]	$a=\frac{1}{2}\pi$ FT their k from part (a)		
			(ii) DR		$b=\frac{1}{8}$		
			$\int t\cos 2t\,\mathrm{d}t=\frac{1}{2}t\sin 2t-\frac{1}{2}\int\sin 2t\,\mathrm{d}t$	M1* (AO2.1)	$\int t\cos 2t\,\mathrm{d}t=\alpha t\sin 2t\pm\beta\int\sin 2t\,\mathrm{d}t$	For any non–zero α, β	
			$\int t\cos 2t\,\mathrm{d}t=\frac{1}{2}t\sin 2t+\frac{1}{4}\cos 2t$	A1 (AO1.1)			
			$\int t\,\mathrm{d}t=\frac{1}{2}t^2$	B1 (AO1.1)	Must be seen		
			$I=\frac{1}{16}\left[t^2\right]_0^{\frac{1}{2}\pi}+\frac{1}{8}\left[\frac{1}{2}t\sin 2t+\frac{1}{4}\cos 2t\right]_0^{\frac{1}{2}\pi}$	dep*M1 (AO1.1)			
			$=\frac{1}{64}(\pi^2-4)$	A1 (AO2.2a)			
			Alternative method				
			$\int t(1+\cos 2t)\,\mathrm{d}t=t\left(t+\frac{1}{2}\sin 2t\right)-\int\left(t+\frac{1}{2}\sin 2t\right)\mathrm{d}t$	M1*	Use of 0 and their k in their integrated expression		
			$=t\left(t+\frac{1}{2}\sin 2t\right)-\left(\frac{1}{2}t^2-\frac{1}{4}\cos 2t\right)$	A1	Or exact equivalent		
			$I=\left[t\left(t+\frac{1}{2}\sin 2t\right)\right]_0^{\frac{1}{2}\pi}-\left[\frac{1}{2}t^2-\frac{1}{4}\cos 2t\right]_0^{\frac{1}{2}\pi}$	A1			
			$=\frac{1}{64}(\pi^2-4)$	dep*M1	For attempt at integration by parts		

Question			Answer/Indicative content	Marks	Part marks and guidance		
				A1 [5]	Correct first application Complete integration correct Use of 0 and their *k* in their integrated expression Or exact equivalent		
			Total	16			