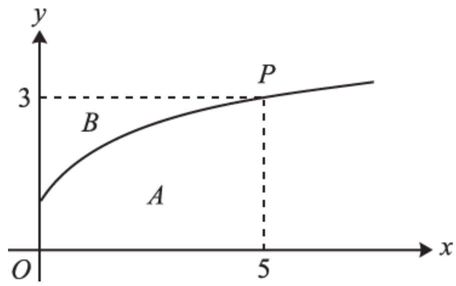
1. Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

$$\int_5^{11} \frac{8}{x}\, dx.$$

[4]

2.




The diagram shows part of the curve $y = -3 + 2\sqrt{x+4}$. The point $P$(5, 3) lies on the curve. Region $A$ is bounded by the curve, the $x$-axis, the $y$-axis and the line $x = 5$. Region $B$ is bounded by the curve, the $y$-axis and the line $y = 3$.

i. Use the trapezium rule, with 2 strips each of width 2.5, to find an approximate value for the area of region $A$, giving your answer correct to 3 significant figures.

[3]

ii. Use your answer to part (i) to deduce an approximate value for the area of region $B$.

[2]

iii. By first writing the equation of the curve in the form $x = f(y)$, use integration to show that the exact area of region $B$ is $\frac{14}{3}$.

[7]

3. i. Use the trapezium rule, with 4 strips each of width 1.5, to estimate the value of

$$\int_4^{10} \sqrt{2x-1}\, dx,$$

giving your answer correct to 3 significant figures.

[4]

ii. Explain how the trapezium rule could be used to obtain a more accurate estimate.

[1]

4. i. The curve $y = 3^x$ can be transformed to the curve y $3^{x-2}$ by a translation. Give details of the translation.

[2]

ii. Alternatively, the curve $y = 3^x$ can be transformed to the curve $y = 3^{x-2}$ by a stretch. Give details of the stretch.

[2]

iii. Sketch the curve $y = 3^{x-2}$, stating the coordinates of any points of intersection with the axes.

[2]

iv. The point $P$ on the curve $y = 3^{x-2}$ has $y$-coordinate equal to 180. Use logarithms to find the $x$-coordinate of $P$, correct to 3 significant figures.

[3]

v. Use the trapezium rule, with 2 strips each of width 1.5, to find an estimate for $\int_1^4 3^{x-2}\,dx$. Give your answer correct to 3 significant figures.

[3]

5. (a) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for [3]

$$\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$$
.

(b) Explain how the trapezium rule might be used to give a better approximation to the integral given in part (a). [1]

6. Use the trapezium rule, with 4 strips each of width 0.2, to find an estimate for

(a) $\int_0^{0.8} \cos x\,dx$, where $x$ is in radians. Give your answer correct to 3 significant figures. [4]

(b) Explain, with the aid of a sketch, why the value from part (a) is an under-estimate. [2]

7. Fig. 1 shows a garden that is to be designed to include a lawn and a flowerbed.

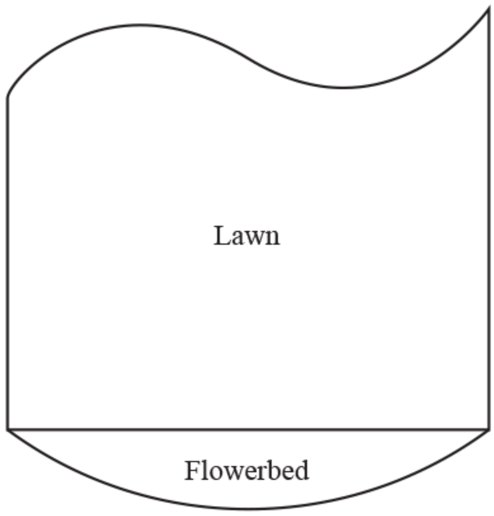



Fig. 1

The lawn can be modelled using four trapezia, as shown in Fig. 2. Each trapezium has a width of 1.5 m, and the lengths of the parallel sides are 8.0 m, 8.5 m, 8.2 m, 8.4 m and 8.6 m respectively.

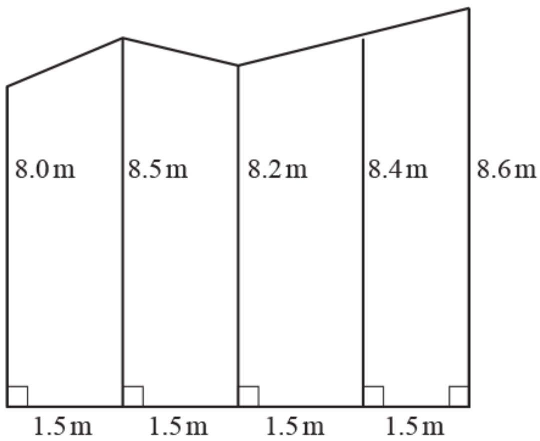



Fig. 2

(a) (i) Use the trapezium rule with 4 strips to estimate the area of the lawn. [2]

(ii) Given that lawn seed costs £0.49 per square metre, estimate the total cost of the lawn seed required. [1]

(b) Suggest two limitations of this model. [2]

(c) Suggest one possible refinement of this model. [1]

The flowerbed can be modelled as the segment of a circle with radius 3.2 m. Fertiliser costs £0.17 per square metre.

(d) Estimate the total cost of fertiliser required to cover the entire area of the flowerbed. [5]

8. (i) Use Simpson's rule with four strips to find an approximation to

$$\int_1^9 \ln x \ln(x+4)\,dx,$$

giving your answer correct to 4 significant figures. [4]

(ii) Deduce an approximation to

$$\int_1^9 \ln(x^{-1})\ln(x^2+8x+16)\,dx,$$

giving your answer correct to 4 significant figures. [2]

9. (a) Use the trapezium rule, with four strips each of width 0.5, to estimate the value of

$$\int_0^2 e^{x^2}\,dx$$

giving your answer correct to 3 significant figures. [3]

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate. [1]

10. (a) Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \approx \frac{11}{4} - \sqrt{2} \ .$$ [5]

(b) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,dx \ .$$ [6]

(c) Using your answers to parts **(a)** and **(b)**, show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4} \ ,$$

where $k$ is a rational number to be determined. [2]

END OF QUESTION paper

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| 1 | | | $\frac{1}{2} \times 2 \times (^8/_5 + 2(^8/_7 + ^8/_9) + ^8/_{11})$ | M1 | Attempt the 4 correct $y$-coordinates, and no others | M0 if other $y$-values also found (unless not used)<br>Allow decimal equivs |
| | | | | M1 | Attempt correct trapezium rule, any $h$, to find area between $x = 5$ and $x = 11$ | Correct structure required, including placing of $y$-values<br>The 'big brackets' must be seen, or implied by later working – this may not be clear if using $h = 2$ so allow BOD for eg $\frac{1}{2} \times 2 \times (^8/_5 + ^8/_{11}) + 2(^8/_7 + ^8/_9)$<br>Could be implied by stating general rule in terms of $y_0$ etc, as long as these have been attempted elsewhere and clearly labelled<br>Could use other than 3 strips as long as of equal width Using $x$-values is M0<br>Can give M1, even if error in evaluating $y$-values as long correct intention is clear<br>Allow BoD if first or last $y$-value incorrect, unless clearly from an incorrect $x$-value |
| | | | | M1 | Use correct $h$ (soi) for their $y$-values (must have equally spaced $x$-values), to find area between $x = 5$ and $x = 11$ | Must be in attempt at trap rule, not Simpson's rule<br>As $h = 2$, allow BOD if ½ × 2 not seen explicitly<br>Allow if muddle over placing $y$-values, including duplication (but M0 for $x$-values)<br>Allow if ½ missing<br>Allow other than 3 strips, as long as $h$ is consistent |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | | = 6.39 | A1 | Obtain 6.39, or better<br><br>Examiner's Comments<br><br>This was a straightforward start to the paper and most candidates gained full marks on the question. Some candidates lost the final mark by failing to work to the required degree of accuracy throughout. Candidates are strongly advised to use exact values in their calculations rather than truncated decimals. Candidates also lost marks through using two strips of width 3, rather than the required three strips of width 2, but they were still able to gain some credit. There were very few candidates who first attempted integration, which is an improvement on previous series. | Allow answers in the range [6.390, 6.391] if > 3sf<br><br>Answer only is 0/4<br>Using 2 strips of width 3 is M0M1M1 and not a misread<br>Using the trap rule on result of an integration attempt is 0/4 – this includes functions such as $8x$ even if integration is not explicit<br>Using 3 separate trapezia can get full marks – if other than 3 trapezia then mark as above<br>However, using only one trapezium is 0/4 |
| | | | Total | 4 | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| 2 | | i | $0.5 \times 2.5 \times (1 + 2(-3 + 2\sqrt{6.5}) + 3)$ | M1* | Attempt *y*-values at $x = 0$, 2.5, 5 only | M0 if additional *y*-values found, unless not used $y_1$ can be exact or decimal (2.1 or better)<br>Allow M1 for using incorrect function as long as still clearly *y*-values that are intended to be the original function eg $-3 + 2\sqrt{x} + 4$ (from $\sqrt{(x + 4)} = \sqrt{x} + \sqrt{4}$) |
| | | i | = 10.2 | M1d* | Attempt correct trapezium rule, inc $h = 2.5$ | Fully correct structure reqd, including placing of *y* -values<br>The 'big brackets' must be seen, or implied by later working<br>Could be implied by stating general rule in terms of $y_0$ etc, as long as these have been attempted elsewhere and clearly labelled<br>Using *x*-values is M0<br>Can give M1, even if error in evaluating *y*-values as long correct intention is clear |
| | | i | | A1 | Obtain 10.2, or better<br><br>**Examiner's Comments**<br><br>Candidates were familiar with the trapezium rule and the majority were able to apply it accurately to the given situation. A surprising minority rounded the final answer to 10.3 rather than 10.2. This could be ignored provided that a correct, more accurate, answer had been seen previously. Slips in calculating the *y* values were condoned, as long as there was sufficient working to convey the correct intent. | Allow answers in the range [10.24, 10.25] if > 3sf<br>A0 if exact surd value given as final answer<br><br>Answer only is 0/3<br>Using 2 separate trapezia can get full marks<br>Using anything other than 2 strips of width 2.5 is M0<br>Using the trapezium rule on result of an integration attempt is 0/3 |
| | | ii | $(5 \times 3) - 10.2 = 4.8$ | M1 | Attem pt area of r ectangle - their **(i)** | As long as 0 < their **(i)** < 15 |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | ii | | A1FT | Obtain 4.8, or better<br>**Examiner's Comments**<br>The overwhelming majority of candidates gained both of the marks available, especially as full credit could still be awarded following an incorrect result to part (i). | Allow for exact surd value as well<br>Allow answers in range [4.75, 4.80] if > 2sf |
| | | iii | $x = \frac{1}{4}\ (y^2 + 6y - 7)$ | M1 | Attempt to write as $x = f(y)$ | Must be correct order of operations, but allow slip with inverse operations eg + / –, and omitting to square the $\frac{1}{2}$<br>Allow $y^2 + 9$ from an attempt to square $y + 3$, even if $(y + 3)^2$ is not seen explicitly first<br>Allow maximum of 1 error |
| | | iii | | A1 | Obtain $x = \frac{1}{4}(y^2 + 6y - 7)$ aef | Allow A1 as soon as any correct equation seen in format $x = f(y)$, eg<br>$x = \frac{1}{4}(y + 3)^2 - 4$or<br>$x = \frac{1}{4}(y^2 + 6y + 9) - 4$, and isw subsequent error |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | iii | area = $\left[\frac{1}{12}y^3+\frac{3}{4}y^2-\frac{7}{4}y\right]_1^3$ | M1* | Attempt integration of f($y$) | Expand bracket and increase in power by 1 for at least two terms (allow if constant term disappears)<br>Independent of rearrangement attempt so M0M1 is possible<br>Can gain M1 if their f($y$) has only two terms, as long as both increase in power by 1<br>Allow M1 for $k(y + 3)^3$, any numerical $k$, as the integral of $(y + 3)^2$ or M1 for $k$ $(\frac{1}{2}(y+3))^3$from $(\frac{1}{2}(y+3))^2$oe if their power is other than 2 |
| | | iii | | A1 | Obtain $\frac{1}{12}y^3+\frac{3}{4}y^2-\frac{7}{4}y$ aef | Or $\frac{1}{12}(y+3)^3-4y$<br>A0 if constant term becomes $-\frac{7}{4}x$ not $-\frac{7}{4}y$ |
| | | iii | | B1 | State or imply limits are $y$ = 1, 3 | Stated, or just used as limits in definite integral<br>Allow B1 even if limits used incorrectly (eg wrong order, or addition)<br>Allow B1 even if constant term is $-\frac{7}{4}x$(or their $cx$) |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | iii | $= \frac{15}{4} - (-\frac{11}{12})$ | M1d* | Attempt correct use of limits | Correct order and subtraction<br>Allow M1 (BOD) if $y$ limits used in $-\frac{7}{4}x$ (or their $cx$), but M0 if $x$ = 0, 5 used<br>Minimum of two terms in $y$<br>Only term allowed in $x$ is their $c$ becoming $cx$<br>Allow processing errors eg $(\frac{1}{12} \text{ x } 3)^3$fo $\frac{1}{12} \text{ x } 3^3$<br>Answer is given so M0 if $\frac{14}{3}$ appears with no evidence of use of limits<br>Minimum working required is $\frac{15}{4} + \frac{11}{12}$<br>Allow M1 if using decimals (0.92 or better fo $\frac{11}{12}$)<br>M0 if using lower limit as $y$ = 0, even if $y$ = 3 is also used Limits must be from attempt at $y$-values, so M0 if using 0 and 5 |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | iii | $= \frac{14}{3}$ **AG** | A1 | Obtain $\frac{14}{3}$<br><br>Examiner's Comments<br><br>This proved to be a suitably challenging end to the paper, with the most able candidates gaining full credit but the weaker ones struggling to make any kind of progress. Most candidates could attempt to change the subject of the equation but weak algebra skills meant that the result was not always the required equation. Most candidates were then able to make a reasonable attempt at the integration, with the most common error being for the constant term to be integrated with respect to *x* rather than *y*. There was a straightforward mark for identifying that the limits were 1 and 3, and candidates then had to use this to attempt the required area. Only the most able candidates were able to provide a convincing demonstration that the area of the region was indeed $^{14}/_{3}$, as given on the paper. | Must come from exact working ie fractions or recurring decimals - correct notation required so A0 for 0.9166…<br><br>A0 if $-\frac{7}{4}x$ seen in solution<br><br>**SR** for candidates who find the exact area by first integrating onto the *x*-axis:<br>**B4** obtain area between curve and *x*-axis as $10\frac{1}{3}$<br>**B1** subtract from 15 to obtain $^{14}/_{3}$<br>And, if seen in the solution, **M1A1** for $x = f(y)$ as above |
| | | | Total | 12 | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| 3 | | i | $0.5 \times 1.5 \times (\sqrt{7} + 2(\sqrt{10} + \sqrt{13} + \sqrt{16}) + \sqrt{19})$ | B1 | State the 5 correct $y$-values, and no others | B0 if other $y$-values also found (unless not used)<br>Allow for unsimplified, even if subsequent error made<br>Allow decimal equivs |
| | | i | | M1* | Attempt to find area between<br>$x = 4$ and $x = 10$, using $k(y_0 + y_n + 2(y_1 + \dots + y_{n-1}))$ | Correct placing of $y$-values required $y$-values may not necessarily be correct, but must be from attempt at using correct $x$-values (allow 7, 10 etc ie no √)<br>The 'big brackets' must be seen, or implied by later working<br>Could be implied by stating general rule in terms of $y_0$ etc, as long as these have been attempted elsewhere and clearly labelled<br>Could use other than 4 strips as long as of equal width (but M0 for just one strip) |
| | | i | | M1d* | Use $k = 0.5 \times 1.5$ soi | Or $k = 0.5 \times h$, where $h$ is consistent with the number of strips used |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | i | = 21.4 | A1 | Obtain 21.4, or better<br><br>**Examiner's Comments**<br><br>This question was generally very well done, with the majority of the candidates gaining full marks. Candidates generally showed their method clearly, though the brackets were omitted in some solutions, resulting in an incorrect evaluation of their intended expression. Candidates also need to be careful when copying their work from one line to the next; it was not uncommon to see √19 become √9. Another common slip was for √16 to be evaluated as 4 but then used as √4. In an improvement from previous sessions, there were very few candidates who first attempted to integrate the function before applying the trapezium rule. | Allow answers in the range [21.40, 21.41] if > 3sf<br>Answer only is 0/4<br>Using the trap rule on result of an integration attempt is 0/4, even if integration is not explicit<br>Using 4 separate trapezia can get full marks<br>Using other than 4 separate trapezia (but not just 1) can get<br>M2, if done correctly |

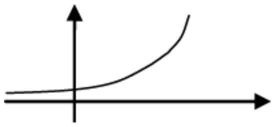
| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | ii | Use more strips / narrower strips | B1 | Any reference to increasing no of strips or reducing width of strips<br><br>**Examiner's Comments**<br><br>The vast majority of candidates gained a mark for either identifying that more strips could be used, or identifying that the width of the strips could be reduced. Benefit of doubt was given to those candidates who gave both reasons, but seemed to think that they were mutually exclusive. The most common error was for candidates to justify why it was inaccurate, referring to it being an underestimate, rather than focusing on the actual question posed. | No need to explicitly state that it is over the same interval<br>Ignore any reference to under- / over-estimate<br>Ignore any attempts at sketching the curve<br>Ignore any irrelevant comments, but penalise contradictory statements eg use more strips, which are wider<br>Could give numerical example eg 'use 6 strips', but if giving both width and no of strips then must give total width of 6 |
| | | | Total | 5 | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| 4 | | i | 2 (units) in the positive $x$ -direction | M1 | Correct direction | Identify that the translation is in the $x$-direction (either positive or negative, so M1 for eg '2 in negative $x$ -direction')<br>Allow any terminology as long as intention is clear, such as in/on/along the $x$ -axis<br>Ignore the magnitude |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | i | | A1 | Fully correct description | Must have correct magnitude and correct direction, using precise language - such as ‘in the *x*-direction’, ‘parallel to the *x*-axis’, ‘horizontally’ or ‘to the right’<br>A0 for in/on/along the *x*-axis etc<br>Allow M1A1 for ‘2 in the *x* -direction’ as positive is implied<br>A0 for ‘factor 2’<br>‘Units’ is not required, but A0 for ‘places’, ‘spaces’, ‘squares’ etc<br>Allow in vector notation as well, so M1 for $\binom{k}{0}$ and M1A1 for $\binom{2}{0}$<br><br>**Examiner's Comments**<br><br>Most candidates could correctly identify that it would be a horizontal translation, thus gaining one mark. To gain the second mark, candidates had to state the correct direction and magnitude. The most efficient and convincing method was to use a vector to describe the translation, and a number of candidates did so. However, most candidates opted to use a worded description instead and a number of those failed to gain the final mark as they did not use mathematically precise language. Phrases such as ‘in the *x*-axis’ were condoned for the first mark, but were not given full credit. |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | ii | sf $\frac{1}{9}$ in the $y$-direction | M1 | Correct direction, with sf of $\frac{1}{9}$ or 9 | Identify that the stretch is in the $y$-direction, with a scale factor of either $\frac{1}{9}$ or 9 (or equiv in index notation)<br>Allow just $\frac{1}{9}$ or 9, with no mention of 'scale factor'<br>Allow exact decimal equiv for $\frac{1}{9}$<br>Allow any terminology as long as the intention is clear, such as in/on/along the $y$-axis |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | ii | | A1 | Fully correct description | Must have correct scale factor and correct direction, using precise language - such as ‘in the *y*-direction’, ‘parallel to the *y*-axis’ or ‘vertically’<br>A0 for in/on/along the *y*-axis etc<br>Must now have ‘scale factor’ or ‘factor’<br>Allow ‘positive *y*-direction’ (not incorrect as graph is wholly above *x*-axis)<br><br>**Examiner's Comments**<br><br>Describing the transformation by means of a stretch proved to be much more challenging and only the most able candidates gained any credit at all on this part. Whilst some candidates used index manipulation to rewrite the equation in the form $y = k \times 3^x$ others generated a table of values in an attempt to deduce the effect of the stretch. Otherwise correct solutions were sometimes spoiled by the careless use of language, including both omitting to describe 1/9 as a scale factor and also an imprecise description of the direction. |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | iii |  | B1* | Correct sketch, in both quadrants | Curve must tend towards the negative $x$-axis, but not touch or cross it, nor a significant flick back upwards<br>If from plotted points then there must be enough of the graph shown to demonstrate the correct general shape, including the negative $x$-axis being an asymptote<br>Ignore any numerical values given |
| | | iii | intersect at $(0, \frac{1}{9})$ | B1d* | State $(0, \frac{1}{9})$ | Condone $x = 0$, $y = \frac{1}{9}$ as an alternative, but $x = 0$ must be stated explicitly rather than implied<br>Allow no brackets around the coordinates<br>Allow exact decimal equiv for $\frac{1}{9}$<br>Allow just $\frac{1}{9}$ as long as marked on the $y$-axis<br>Allow BOD for ($\frac{1}{9}$, 0) on $y$-axis, but not if just stated<br>Just being seen in a table of values is not sufficient<br>Ignore any other labelled coordinates<br>**Examiner's Comments**<br>There were a number of carefully drawn and correct exponential graphs, and many of those with an acceptable graph also correctly gave the $y$-intercept. However too many candidates, who clearly knew the general shape of the curve, were unable to convey their intention in a sufficiently convincing manner. A |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | | | | | number of attempts had neither ruled nor labelled axes. A lack of care when sketching the curve resulted in errors such as vertical asymptotes to the right, horizontal asymptotes nowhere near the *x*-axis and curves that had a distinct minimum point in the second quadrant. All of these errors resulted in the mark not being awarded. Several candidates did not extend their sketched curve into the second quadrant, and many candidates did not know the shape at all. |
| | | iv | $\log 3^{x-2} = \log 180$ (or $x - 2 = \log_3 180$)<br>$(x - 2)\log 3 = \log 180$<br>$x - 2 = 4.7268\ldots$<br>$x = 6.73$ | M1* | Introduce logs and drop power | Can use logs to any base, as long as consistent on both sides, and allow no explicit base as well<br>The power must also be dropped for the M1<br>Brackets must be seen around the $(x - 2)$, or implied by later working<br>If taking $\log_3$ then base must be explicit |
| | | iv | | M1d* | Attempt to solve for $x$ | Correct order of operations, and correct operations so M0 for $\log_3 180 - 2$<br><br>M0 if logs used incorrectly<br><br>eg $x - 2 = \log(\frac{180}{3})$ |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | iv | | A1 | Obtain 6.73, or better | If > 3sf, allow answer rounding to 6.727 with no errors seen<br>0/3 for answer only or T&I<br>If rewriting eqn as $3^{x-2} = 3^{4.73}$ then 0/3 unless evidence of use of logs to find the index of 4.73<br><br>**SR** If using index rules first then **B1** for $3^x = 1620$<br>**M1** for attempting to use logs to solve $3^x = k$<br>**A1** for 6.73<br><br>**Examiner's Comments**<br><br>Candidates continue to demonstrate proficiency when solving straightforward equations involving logarithms and this was true on this question, with the vast majority of candidates gaining all of the available marks with ease. The most common approach was to use logarithms to base 3, although solutions involving base 10, or even some unspecified base, were also seen. There are still a number of candidates who do not make effective use of brackets, and it was relatively common to see $x - 2 \log 3$ rather than $(x - 2) \log 3$. Some candidates retrieved this by subsequently using their invisible brackets correctly, whereas others continued as if they were never intended. |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | v | $0.5 \times 1.5 \times \{3^{-1} + 2 \times 3^{0.5} + 3^2\}$<br>= 9.60 | B1 | State the 3 correct *y*-values, and no others | B0 if other *y*-values also found (unless not used)<br>Allow for unsimplified, even if subsequent error made<br>Allow decimal equivs |
| | | v | Enter text here. | M1 | Attempt use of correct trapezium rule to attempt area between $x = 1$ and $x = 4$ | Correct placing of *y*-values required<br>*y*-values may not necessarily be correct, but must be from attempt at using correct *x*-values<br>The 'big brackets' must be seen, or implied by later working<br>Could be implied by stating general rule in terms of $y_0$ etc, as long as these have been attempted elsewhere and clearly labelled<br>Could use other than 2 strips as long as of equal width (but M0 for just one strip)<br>Must have *h* as 1.5, or a value consistent with the number of strips used if not 2 |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | v | | A1 | Obtain 9.60, or better (allow 9.6) | Allow answers in the range [9.595, 9.600] if > 3sf<br><br>Answer only is 0/3<br>Using the trap. rule on the result of an integration attempt is 0/3, even if integration is not explicit<br>Using two separate trapezia can get full marks<br>Using other than 2 trapezia (but not just 1) can get M1 only<br><br>**Examiner's Comments**<br><br>This final part of the question was also very well answered, with many fully correct solutions being seen. Candidates generally showed their method clearly, and were able to identify the three relevant *x*-values and attempt the corresponding *y*-values. The trapezium rule was then usually correctly attempted, although some candidates committed the common error of omitting the necessary brackets. Very few candidates attempted to integrate the function before applying the trapezium rule. |
| | | | Total | 12 | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| 5 | | a | $\frac{0.25}{2}(1 + 0.7071 + 2(0.970 + 0.8944 + 0.8))$<br><br>0.880 | B1(AO 1.1)<br><br>M1(AO1. 1a)<br><br>A1(AO1. 1)<br><br>[3] | Obtain all five ordinates and no others: 0.7071, 0.8944, 1, 0.8, 0.970<br><br>Use correct structure for trapezium rule with $h$ = 0.25<br><br>0.880 or better (0.87 953077) | Accept exact values: 1, $\frac{4}{\sqrt{17}}$, $\frac{2}{\sqrt{5}}$, $\frac{4}{5}$, $\frac{1}{\sqrt{2}}$<br><br>$x$ -coordinates used **M0.** Omission of large brackets unless implied by correct answer **M0** Accept 0.88 (0.879 53077) | |
| | | b | "Use smaller intervals" or "use more trapezia" | B1(AO 2.4)<br><br>[1] | | | |
| | | | Total | 4 | | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| 6 | | i | 0.5×0.2{cos0 + cos0.8 + 2(cos0.2 + cos0.4 + cos0.6)}<br><br>= 0.715 | B1<br><br>M1* | State the 5 correct *y* -values, and no others<br><br>Attempt to find area between *x* = 0 and *x* = 0.8, using $k\{y_0 + y_n + 2(y_1 + \ldots + y_{n-1})\}$ | B0 if other *y*-values also found (unless not used)<br>Allow for exact values seen, even if subsequent error made (including evaluating in degree mode)<br>Allow decimal equivs (2dp or better) (1, 0.980, 0.921, 0.825, 0.697); if using 2dp then allow 0.7 rather than 0.70 for final *y* value<br><br>Correct placing of *y*-values required *y* -values may not necessarily be correct, but must be from attempt at using correct *x*-values in *y* = cos*x* (in radian mode or degree | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| | | | | M1d*<br><br><br><br><br>A1<br><br><br>**[4]** | Use $k$ = 0.5 × 0.2 soi<br><br><br>Obtain 0.715, or better | mode) The 'big brackets' must be seen, or implied by later working Could be implied by stating general rule in terms of $y_0$ etc, as long as these have been attempted elsewhere and clearly labelled Could use other than 4 strips as long as of equal width (but M0 for just one strip)<br><br>Or $k$ = 0.5 × $h$, where $h$ is consistent with the number of strips used<br><br>Allow answers rounding to 0.715 if >3sf Using 4 separate trapezia can get full | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| | | | | | | marks<br>Must see evidence of trapezium rule or 0/4 (integration gives 0.717 to 3sf)<br><br>Working in degrees:<br>B1 if exact values seen (ie cos0.2 etc), but B0 if straight into decimals<br>M1 M1 is then possible as long as it is clear where each value is being placed<br><br>0.5×0.2 {1.00 + 1.00 + 2(1.00 + 1.00 + 1.00)} = 0.800 will be 0/4 unless more detail shown | |
| | | | | | **Examiner's Comments**<br>This question was very well answered, with many candidates gaining full marks. The most effective method was to write out the trapezium rule using exact values and then evaluate this on the calculator. Using decimal equivalents often resulted in a loss of accuracy in the final | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| | | | | | answer. The most common error was for candidates to use their calculator in degree mode rather than radian mode. | | |
| | | ii | Graph of $y = \cos x$, with 4 trapezia drawn | B1 | Correct $y = \cos x$ graph, with exactly 4 trapezia of roughly equal width | Trapezia must be plausibly [0, 0.8], allow BOD as long as final trapezium ends before $\pi/2$ Curve may be shown beyond $x = 0.8$, but B0 if clearly of the incorrect shape beyond $x = 0.8$ No need for scale on either axis Exactly four trapezia must be shown, of roughly equal widths, with top vertices on the curve. | |
| | | | Tops of the trapezia are below the curve | B1<br>**[2]** | Any valid explanation | Not dependent on previous B1 Must refer to the tops of the trapezia so B0 for ‘trapezia | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| | | | | | | are below curve' (ie 'top' not used) Allow 'trapezium' rather than 'trapezia' Concave / convex is B0<br>B0 if comparing to exact area<br>B1 for decreasing gradient (but B0 for decreasing curve)<br>Candidates could also use their diagram as part of their explanation – as long as there was an intention to draw trapezia then they are eligible for the second B1 even if B0 for the diagram. This could include a single trapezium (even if labelled 0 – 0.8), several trapezia whose tops are | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | | | | | collinear, an incorrect $y$ = cosx graph (including $y$ = sin$x$) and similar. Use of rectangles to support their explanation however is B0.<br>They could shade gaps on their diagram but some text also required<br>B0 for 'some area not calculated' unless clear which area - could be described or shaded<br>ISW any irrelevant comments, but B0 if contradictory comments |
| | | | | | **Examiner's Comments**<br>This part of the question was less well answered, due to a lack of precision in the explanations. Some candidates referred to the tops of the trapezia being under the curve, whereas others identified the areas that had not been included in the calculation. Either approach was condoned, | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | | | | as long as there was sufficient detail to be convincing. For the sketch graph, candidates were expected to provide a sketch of y = cosx with four trapezia shown. The most common error was to draw trapezia whose top vertices did not actually lie on the curve, and other errors included drawing just a single trapezium and even attempting to use y = sinx as the curve. Some precise and convincing solutions were seen,but these were in the minority. | |
| | | | Total | 6 | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| 7 | | a | (a) 0.5 × 1.5 × {8.0 + 8.6 + 2(8.5 + 8.2 + 8.40)} = 50.1 | M1(AO1.1a)<br><br>A1(AO1.1)<br><br>[2] | Attempt use of correct trapezium rule<br><br>Obtain correct area | | |
| | | a | (b) 50.1 × 0.49 = £24.55 | B1ft(AO1.1)<br><br>[1] | Obtain cost of £24.55, ft their area | | |
| | | b | Could be under-estimate as modeling tops of trapezia with straight lines<br><br>Lawn seed may only be sold in fixed volumes so may not be able to buy exact volume needed | B1(AO3.5b)<br><br>B1(AO3.5b)<br><br>[2] | Limitation based on use of trapezium rule<br><br>Limitation based on buying lawn seed | Accept any two sensible comments | |
| | | c | Any sensible refinement | B1(AO3.5c)<br><br>[1] | Eg Use trapezium rule with more strips | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| | | d | $2\sin^{-1}(0.9375) = 2.43$ rads | M1(AO3.1b) | Attempt to find angle (rads or degs) | Could use cosine rule | |
| | | | | A1(AO1.1) | Obtain 2.43 rads, or 139° | Allow 1.22 rads or 69.6° | |
| | | | $0.5 \times 3.2^2 \times (2.43 - \sin 2.43) = 9.10$ | M1(AO1.1a) | Attempt complete method to find area of segment | | |
| | | | | A1(AO1.1) | Obtain 9.10 ($m^2$) | Allow 9.1 | |
| | | | $9.10 \times 0.17 = £1.55$ | B1ft(AO3.2a)<br>[5] | Obtain cost of £1.55, ft their area | Must include units | |
| | | | Total | 11 | | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| 8 | | i | Attempt calculation of form $k(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$ | M1 | Any non-zero constant $k$ with attempts at $y$ values (in terms of ln or decimals); M0 if attempt does not involve exactly four strips; M0 if each $y$ value initially 'amended', to $\ln(2x + 4)$ for example | |
| | | | Obtain $k$(ln1ln 5 + 4ln 3ln 7 + 2ln 5ln 9 + 4ln 7ln11+ ln 9ln13) | A1 | Or equiv involving decimals indicating use of correct values | |
| | | | Use $k = \frac{2}{3}$ | A1 | | |
| | | | Obtain 26.62 | A1 | Allow greater accuracy 26.6159…; any value rounding to 26.62 with no errors seen | |
| | | | | [4] | **Examiner's Comments**<br><br>Part (i) was generally answered very well with candidates using the formula for Simpson's rule accurately. Some stated the expression to be evaluated using logarithms and proceeded to produce the answer. Others set out the values to be used in decimal form, sometimes with the values presented in a table. Errors did occur, presumably the result of careless use of calculator. There were very few instances of candidates using the wrong number of strips or of associating the coefficients 2 and 4 with the incorrect y values. There were a few instances of candidates trying to 'simplify' the y values where ln3ln7 , for example, became ln10 or ln 21. | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | ii | State or imply that integrand now involves –ln $x$ or 2ln($x$ + 4) or both<br><br>Obtain –53.23 or –53.24 as final answer | M1<br><br>A1ft<br><br>[2] | Following their Simpson rule answer from **(i)**, ie –2 times their answer; allow greater accuracy; correct answer with no working earns B2; second use of Simpson's rule leading to correct answer earns B2, but B0 if incorrect; concluding with 53.23 or 53.24 (perhaps with some reference to area below axis) is A0<br><br>**Examiner's Comments**<br><br>Part (ii) was a slightly more challenging request and fewer than half the candidates were able to note that logarithm properties mean that the answer to part (ii) is –2 times the answer to part (i). Answers such as 708.6 and 0.0014 were occasionally seen. Some candidates used Simpson's rule again and those doing so had to produce the correct answer to record both marks. Some candidates, having obtained the value –53.24 or –53.23 (either of which was acceptable) decided to drop the minus sign, often referring to an area below the<br>x-axis. But this question involves a purely numerical request and losing the minus sign meant the loss of the accuracy mark. | |
| | | | Total | 6 | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| 9 | | a | $0.5 \times 0.5\{(e^0 + e^{2^2}) + 2(e^{0.5^2} + e^{1^2} + e^{1.5^2})\}$ | B1 (AO 1.1) | Obtain all five ordinates and no others | Allow decimal equivs (1, 1.284, 2.718, 9.488, 54.598) – 3sf or better<br>B0 if other ordinates seen unless clearly not intended to be used | |
| | | | | M1 (AO 1.1a) | Use correct structure for trapezium rule with $h$ = 0.5 | Big brackets need to be seen or implied (40.9 is the result of no brackets)<br>$y$-values must be correctly placed<br>Must be using attempts at all 5 $y$-values<br>Must be attempting area between $x$ = 0 and $x$ = 2 | |
| | | | = 20.6 | A1 (AO 1.1)<br><br>[3] | Obtain 20.6, or better | Allow more accurate answers in the range [20.64, 20.65) | |
| | | | | | <u>Examiner's Comments</u> | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | | | | This question was also very well answered, with many fully correct solutions seen. The most successful candidates started by identifying the relevant *x* values, used exact *y* values rather than decimal approximations and made effective use of brackets. | |
| | | b | Use more trapezia, of a narrower width, over the same interval | **B1** **(AO 2.4)** **[1]** | Convincing reason — Condone just 'more trapezia' or 'narrower trapezia' Could refer to 'strips' not 'trapezia'<br><br><u>Examiner's Comments</u><br><br>The best explanations referred to using more trapezia, of a narrower width, over the same interval. Many candidates mentioned either using more trapezia or decreasing the width; giving just one reason was condoned in the mark scheme. | |
| | | | Total | 4 | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| 10 | | a | $h = 2$<br><br>$\frac{h}{2}\left[\frac{1}{2}+\frac{1}{4}+2\left(\frac{1}{2+\sqrt{2}}\right)\right]$<br><br>$I \approx \frac{3}{4}+\frac{2}{2+\sqrt{2}}$<br><br>$\frac{1}{2+\sqrt{2}}=\frac{\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)}=\frac{2-\sqrt{2}}{2}$<br><br>$I \approx \frac{3}{4}+\left(2-\sqrt{2}\right) = \frac{11}{4}-\sqrt{2}$ | B1(AO 1.1)E<br><br>M1(AO 2.1)E<br><br>A1(AO 1.1)C<br><br>M1(AO 3.1a)E<br><br>A1(AO 2.2a)A<br><br>[5] | Use of correct formula with correct (exact) *y* –values with their *h* — Condone one error<br><br>Correct method for rationalising the denominator of their surd together with correct simplification<br><br>AG – at least one step of intermediate working (from application of trapezium rule to given result) — Must be convincing as AG in values<br><br>**Examiner's Comments**<br>Candidates found this part extremely accessible and nearly all correctly derived | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | | |
|---|---|---|---|---|---|---|---|
| | | | | | the given result. Candidates are reminded though that in 'show that' questions suitable working must be shown and statements such as $\frac{2}{2}\left[\frac{1}{2}+\frac{1}{4}+2\left(\frac{1}{2+\sqrt{2}}\right)\right]=\frac{11}{4}-\sqrt{2}$ are generally not acceptable. | | |
| | | b | $x = u^2 \Rightarrow dx = 2u\,du$ | M1*(AO 3.1a)E | An attempt at integration by sub – allow any genuine attempt (as a minimum must differentiate their sub. and remove all $x$'s) | Limits not required for first four marks | |
| | | | $\int_0^4 \frac{dx}{2+\sqrt{x}} = \int_0^2 \frac{2u}{2+u}du$ | A1(AO 1.1)C | Correct integral in terms of $u$ | Or use $t = 2 + u$ to obtain integral of the form | |
| | | | $= 2\int_0^2 \frac{2+u-2}{2+u}du = 2\int_0^2 1-\frac{2}{2+u}du$ | Dep*M1(AO 2.1)C | Re–writes integral in the form $\int a+\frac{b}{1+u}du$ | $\int a+\frac{b}{t}dt$ <br> $\int 2-\frac{4}{t}dt$ | |
| | | | $= 2\left[u-2\ln(2+u)\right]_0^2$ | A1ft(AO 1.1)A | Correctly integrates their $\int a+\frac{b}{1+u}du$ | $= 2t - 4\ln t$ | |
| | | | = 2{(2 – 2ln(2 + 2)) – (0 – 2 ln (2 + 0))} | M1(AO 1.1)C | Uses correct limits correctly (dependent on both previous M | | |
| | | | = 2(2 – 2ln 2) | A1(AO 2.2a)A <br> [6] | | | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | | | | marks)<br>oe e.g.<br>$4 - 4 \ln 4 + 4 \ln 2$<br><br>**Examiner's Comments**<br>While nearly all candidates used the substitution correctly and re–wrote the integra $\int_0^2 \frac{2u}{2+u}\,du$ many could<br>l as not<br>deal with the resulting improper fraction in the integrand. The most successful<br>candidates $\frac{2u}{2+u}$ either by<br>re–wrote<br>using long division or realising that<br>$\frac{2u}{2+u} = \frac{4+2u-4}{2+u} = 2 - \frac{4}{2+u}$.<br><br>While examiners noted that some candidates employed more extreme methods (for example, further substitutions and the method of integration by parts) these were usually unsuccessful. | |

| Question | | | Answer/Indicative content | Marks | Part marks and guidance | |
|---|---|---|---|---|---|---|
| | | c | $\frac{11}{4}-\sqrt{2}\approx 2(2-2\ln 2)$<br><br>$\ln 2\approx \frac{5}{16}+\frac{\sqrt{2}}{4}$ | M1(AO 1.1a)C<br><br>A1(AO 2.1a)A<br><br>[2] | Setting the given result approx. equal to their (b)<br><br>$k=\frac{5}{16}$<br><br>**Examiner's Comments**<br>While some, who had struggled with part (b), left this part blank the majority of candidates equated their answers to parts (a) and (b) with nearly all who were successful in part (b) correctly determining tha $k=\frac{5}{16}$.<br>t | |
| | | | Total | 13 | | |