1. The acceleration of a particle P moving in a straight line is $(t^2 - 9t + 18)$ ms^{-2}, where t is the time in seconds.

i. Find the values of t for which the acceleration is zero.

[2]

ii. It is given that when $t = 3$ the velocity of P is 9 ms^{-1}. Find the velocity of P when $t = 0$.

[4]

iii. Show that the direction of motion of P changes before $t = 1$.

[2]

2. A particle P moves in a straight line. The displacement of P from a fixed point on the line is $(t^4 - 2t^3 + 5)$m, where t is the time in seconds. Show that, when $t = 1.5$,

i. P is at instantaneous rest,

[3]

ii. the acceleration of P is 9 m s^{-2}.

[3]

3. A particle P moves in a straight line. At time t s after passing through a point O of the line, the displacement of P from O is x m. Given that $x = 0.06t^3 - 0.45t^2 - 0.24t$, find

i. the velocity and the acceleration of P when $t = 0$,

[6]

ii. the value of x when P has its minimum velocity, and the speed of P at this instant,

[5]

iii. the positive value of t when the direction of motion of P changes.

[3]

4. A particle P travels in a straight line. The velocity of P at time t seconds after it passes through a fixed point A is given by $(0.6t^2 + 3)\text{ms}^{-1}$. Find

i. the velocity of P when it passes through A,

[1]

ii. the displacement of P from A when $t = 1.5$,

[4]

iii. the velocity of P when it has acceleration 6 ms^{-2}.

[3]

5. A particle P moves in a straight line on a horizontal surface. P passes through a fixed point O on the line with velocity 2 m s^{-1}. At time ts after passing through O, the acceleration of P is $(4 + 12t)$ m s^{-2}.

i. Calculate the velocity of P when $t = 3$.

[4]

ii. Find the distance OP when $t = 3$.

[4]

A second particle Q, having the same mass as P, moves along the same straight line. The displacement of Q from O is $(k - 2t^3)$ m, where k is a constant. When $t = 3$ the particles collide and coalesce.

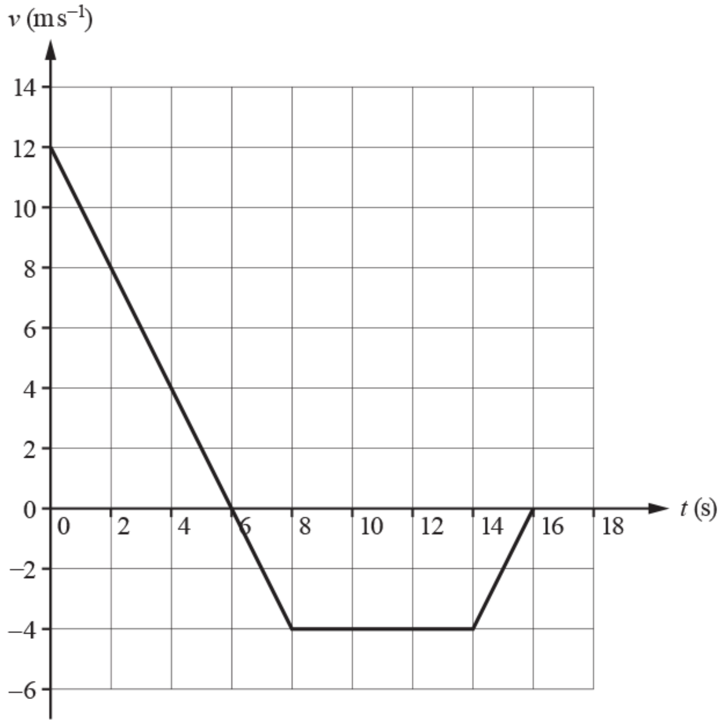
iii. Find the value of k.

[1]

iv. Find the common velocity of the particles immediately after their collision.

[5]

6.



A particle is moving along a straight line. The motion of the particle is modelled by the velocity-time graph shown above, where v m s^{-1} is the velocity of the particle at time t s after it passes through a point A.

(a) Describe the motion of the particle between times $t = 0$ and $t = 8$. [2]

(b) Calculate the acceleration of the particle at time $t = 3$. [1]

(c) Find the displacement of the particle from A at time $t = 16$. [3]

A second model for the motion of the particle is given by $v = at^2 + bt + 12$, where a and b are constants. It is given that the two models agree on the value of v at times $t = 0$, $t = 6$ and $t = 16$.

(d) Find the values of a and b. [2]

(e) Hence find, according to this second model,

- an expression in terms of t for the displacement of the particle from A,
- the distance travelled by the particle from its position when $t = 0$ to its position when $t = 16$. [5]

(f) Calculate the time when the two models agree on the acceleration of the particle in the interval $0 \le t \le 8$. [2]

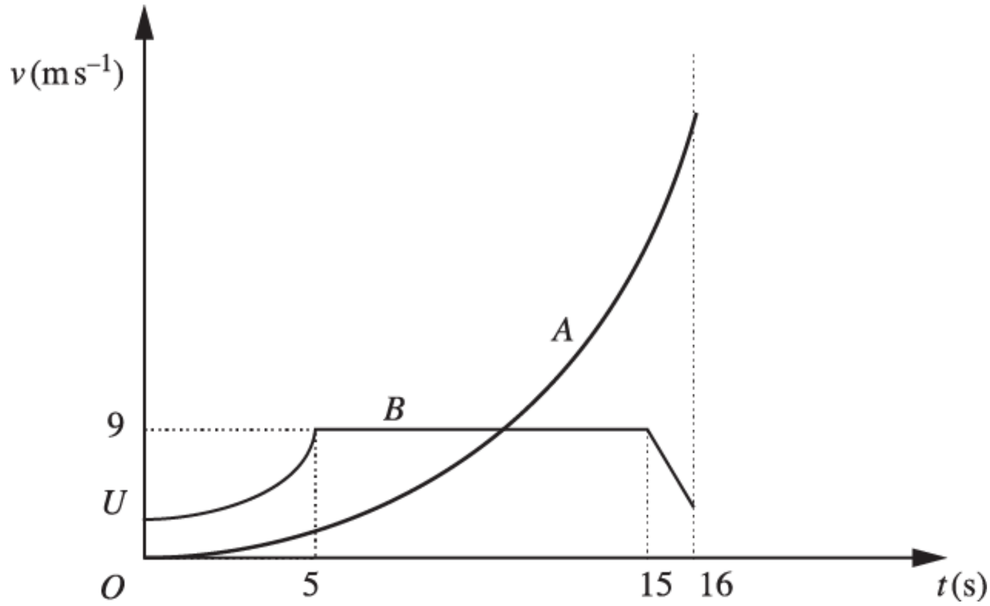
7. A particle moves in a straight line on a horizontal surface. At time t s after being released from rest at a point O on the line, the particle has a velocity v m s^{-1} and a displacement from O of x m. It is given that

$$v = 0.8t^3 - 4t^2 + 5.6t.$$

(i) Find the positive values of t at which the particle has its maximum and minimum velocities, and calculate the values of these velocities. [5]

(ii) Express x in terms of t, and hence find the distance travelled by the particle while it is decelerating. [6]

8.



The diagram shows the (t, v) graphs for two particles A and B which move on the same straight line. The units of v and t are m s^{-1} and s respectively. Both particles are at the point S on the line when $t = 0$. The particle A is initially at rest, and moves with acceleration $0.18t$ m s^{-2} until the two particles collide when $t = 16$. The initial velocity of B is U m s^{-1} and B has variable acceleration for the first five seconds of its motion. For the next ten seconds of its motion B has a constant velocity of 9 m s^{-1}; finally B moves with constant deceleration for one second before it collides with A.

i. Calculate the value of t at which the two particles have the same velocity.

[4]

For $0 \leqslant t \leqslant 5$ the distance of B from S is ($Ut + 0.08t^3$) m.

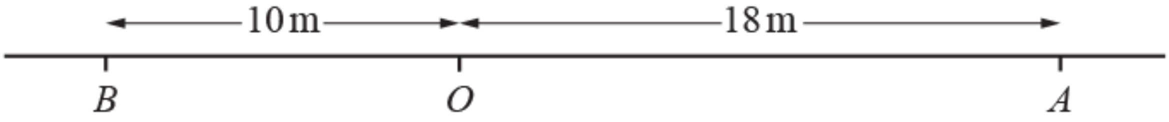
ii. Calculate U and verify that when $t = 5$, B is 25 m from S.

[4]

iii. Calculate the velocity of B when $t = 16$.

[5]

9.



A particle P is moving along a straight line with constant acceleration. Initially the particle is at O. After 9 s, P is at a point A, where OA = 18 m (see diagram) and the velocity of P at A is 8 ms^{-1} in the direction $\overrightarrow{OA}$.

(a) (i) Show that the initial speed of P is 4 ms^{-1}. [2]

(ii) Find the acceleration of P. [2]

B is a point on the line such that OB = 10 m, as shown in the diagram.

(b) Show that P is never at point B. [4]

A second particle Q moves along the same straight line, but has variable acceleration. Initially Q is at O, and the displacement of Q from O at time t seconds is given by

$$x = at^3 + bt^2 + ct,$$

where a, b and c are constants.

It is given that

- the velocity and acceleration of Q at the point O are the same as those of P at O,
- Q reaches the point A when $t = 6$.

(c) Find the velocity of Q at A. [5]

10. The velocity v m s^{-1} of a car at time t s, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When t = 20 the acceleration of the car is 1.3 m s^{-2}. For $t > 20$ the car continues its journey with constant acceleration 1.3 m s^{-2} until its speed reaches 25 m s^{-1}.

(a) Find the value of k. [3]

(b) Find the total distance the car has travelled when its speed reaches 25 m s^{-1}. [7]

11. A particle P moves along the x-axis. At time t seconds the velocity of P is $v$ms^{-1}, where $v = 2t^4 + kt^2 - 4$.

The acceleration of P when $t = 2$ is 28ms^{-2}.

(a) Show that $k = -9$. [3]

(b) Show that the velocity of P has its minimum value when $t = 1.5$. [3]

When $t = 1$, P is at the point (–6.4125, 0).

(c) Find the distance of P from the origin O when P is moving with minimum velocity. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$(t-3)(t-6)=0$	M1	Solve 3 term QE, 2 correct coefficients if factorising, or using formula $9 +/- \sqrt{9}/2$ "By inspection" both values M1A1, one value M0A0	
		i	$t = 3, 6$	A1	**Examiner's Comments** Nearly all candidates gained both marks, getting their answers via factorisation.	
		ii	$v = \int(t^2 - 9t + 18)\mathrm{d}t$	M1*	Attempts integration of $a(t)\mathrm{d}t$, maximum one wrong term	
		ii	$v = t^3/3 - 9t^2/2 + 18t (+ c)$	A1	Accept omission of + c	
		ii	$3^3/3 - 9 \times 3^2/2 + 18 \times 3 + c = 9$	D*M1	Uses $v(3) = 9$	
		ii	$(v =) -13.5 \text{ m s}^{-1}$	A1	Must be negative, and goes beyond c = −13.5 **Examiner's Comments** Attempts to use constant acceleration were rare. Nearly all candidates, correctly, went beyond their value of the integration constant, explicitly finding v at time zero.	
		iii	$v(1) = 1/3 - 9/2 + 18 - 13.5 = 0.333$	M1	Finds $v(1)$ (= 1/3) Accurate values (v(0) = −13.5, v(0.5) = −5.58, v(0.9) = −0.702)	
		iii	Changed sign so direction of motion has changed	A1	**Examiner's Comments** While most candidates started well, many did not make explicit the link between the change of sign for v and a change in direction of	

					motion. Finding a value of t when v was zero was not regarded as showing a change of direction.	
			Total	8		
2		i	$v = d(t^4 - 2t^3 + 5)/dt$	M1*	Differentiates displacement, one wrong term max, ignore +c	
		i	$v = 4 \times 1.5^3 - 6 \times 1.5^2$	D*M1	Substitutes $t = 1.5$ in $v(t)$ OR solves $4t^3 - 6t^2 = 0$ for a +ve root 0 + c is A0 unless c is discarded	
		i	$v = 0$ AG	A1	**Examiner's Comments** This question was very well done, with constant acceleration formulae almost entirely absent.	
		ii	$a = d(4t^3 - 6t^2)/dt$	M1*	Differentiates velocity, one wrong term max, ignore +c	
		ii	$a(1.5) = 12 \times 1.5^2 - 12 \times 1.5$	D*M1	Substitutes $t = 1.5$ in $a(t)$ OR solves $12t^2 - 12t = 9$ for a +ve root 9 + c is A0 unless c is discarded	
		ii	$a = 9 \text{ m s}^{-2}$ AG	A1	**Examiner's Comments** Again this was done well, with nearly all candidates demonstrating the appropriate substitution.	
			Total	6		
3		i	$V = d(0.06t^3 - 0.45t^2 - 0.24t)/dt$	M1	Differentiates displacement	
		i	$V = 0.18t^2 - 0.9t - 0.24$	A1	Accept with $+c$, unsimplified coefficients	
		i	$A = d(0.18t^2 - 0.9t - 0.24)/dt$	M1	Differentiates velocity	
		i	$A = 0.36t - 0.9$	A1	Accept with $+c$, unsimplified coefficients	
		i	$V(0) = -0.24 \text{ m s}^{-1}$	A1	cao, if coeffs in $V(t)$ wrong A0	

		i	$A(0) = -0.9$ m s^{-2}	A1ft	ft cv(−0.9), the constant in expression for *A*. Tolerate wrong coeff *t* **Examiner's Comments** Nearly all candidates adopted the correct methods, differentiating twice. Loss of marks mostly arose from leaving out the negative signs when giving the velocity and the acceleration for $t = 0$.	
		ii	Solves $A = 0$ for t	M1	Not if A(t) includes $+c$ in this section	
		ii	$0.36t - 0.9 = 0$	A1		
		ii	$t = 2.5$	A1		
		ii	$x(2.5) = -2.475$	A1	Final answer must be negative. Accept −2.47 and −2.48.	
		ii	Speed = $\lvert v(2.5) \rvert = 1.365$ m s^{-1}	A1	Final answer must be positive. Accept 1.36 or 1.37. **Examiner's Comments** About half of all candidates based their answer for minimum velocity on $v = 0$. Candidates who correctly used $a = 0$ to find the correct value of t might sometimes mis-calculate the corresponding value of x, but were more likely than not to give the corresponding velocity, instead of the <u>speed</u> as requested. There were also a significant proportion of scripts where only one of the two required quantities was found. (Some mark-worthy responses were based on candidates finding two values of t which had the same value of v. Candidates would then find the average these two values. As v is a quadratic function of t, the method was valid, and was marked as such.) About 10% of scripts contained a fully correct solution to Q6(ii). The value of t for the minimum velocity may be found by completing the square, M1. $0.18(t^2 - 5t - 4/3)$ gives $(t - 2.5)^2$ A1[$-$ 91/12] hence $t = 2.5$ A1. Candidates can either return to the formula for $v(t)$ or calculate $\lvert 0.18x - 91/12 \rvert$.	

					As $v(t)$ is a quadratic function, finding two values of t giving the same velocity identifies the mean of these t values as the time for the minimum velocity.	
		iii	Uses $v = 0$	M1		
		iii	$0.18t^2 - 0.9t - 0.24 = 0$	A1ft	Forms and offers solution of 3 term QE using cv(V(i))	
		iii	$t = 5.25$ s	A1	Must select +ve answer explicitly. Accept 5.3, not 5.2 **Examiner's Comments** Was frequently left out by candidates who had used $v = 0$ in part (ii), while others simply quoted the value found previously, which (if correct) would gain full marks. A significant number of solutions foundered because candidates could not solve accurately the quadratic equation $0.18t^2 - 0.9t - 0.24 = 0$. If the initial step in a solution was to convert the coefficients to integers, this was likely to yield $18t^2 - 9t - 24 = 0$.	
			Total	**14**		
4		i	3 ms^{-1}	B1	**Examiner's Comments** All three parts of this question were well answered by nearly all candidates.	MR $(0.6t^3 + 3)$, award B1 here
		ii	$x = \int(0.6t^2 + 3)dt$	M1*	Integrates v	MR $(0.6t^3 + 3)$
		ii	$x = 0.6t^3/3 + 3t (+ c)$	A1	Accept with / without + c	$0.6t^4/4 + 3t$ is A0
		ii	Substitutes 1.5 in expression for x	D*M1	Needs integration and 2 terms in t	
		ii	$x(1.5) = 5.175$ m	A1	Only without +c. Accept 5.17, 5.18 **Examiner's Comments**	MR 5.26 only gets A1ft

					This part had an answer of exactly 5.175, which should be left as such, but the answer 5.18 was accepted. Inevitably some answers were based on *suvat* expressions, more commonly in (ii) where integration was needed than in (iii) which used differentiation.	
		iii	$a = d(0.6t^2 + 3)/dt$	M1*	Differentiates v	MR $(0.6t^3 + 3)$ gives t = 1.82(57..)
		iii	$6 = 2 \times 0.6t$	D*M1	Plus attempt to solve $a(t) = 6$	
		iii	$v(5) = 18$ ms^{-1}	A1	**Examiner's Comments** Inevitably some answers were based on *suvat* expressions, more commonly in (ii) where integration was needed than in (iii) which used differentiation.	v(1.8257..) = 6.65 (3 sf)
			Total	8		
5		i	$v = \int 4 + 12t \, dt$	M1*	Integrates acceleration	Must see one term correct.
		i	$v = 4t + 12t^2/2 \, (+ c)$	A1	Award without (+ c)	
		i	$(t = 0,\ v = 2)$ $c = 2$ and $v(3) = 4 \times 3 + 12 \times 3^2/2 \, (+ 2)$	D*M1	Evaluates constant	
		i	$v = 68$ m s^{-1}	A1	**Examiner's Comments** The variable acceleration and hence the need to use differentiation and integration in this question was well understood with very few cases where the use of *suvat* equations was thought appropriate. The main cause of lost marks was failure to evaluate the constant of integration. Candidates should be aware that the constant of integration is not always zero.	

		ii	∫ $4t + 6t^2$ (+2)dt	M1*	Integrates velocity	
		ii	$x = 4t^2/2 + 6t^3/3 + 2t (+ d)$	A1ft	accept omission of d for all subsequent marks	ft on incorrect (non-zero) c from (i)
		ii	$x(3) = 4 \times 3^2/2 + 6 \times 3^3/3 (+ 3 \times 2)$	D*M1		
		ii	$x = 78$ m	A1	**Examiner's Comments** The variable acceleration and hence the need to use differentiation and integration in this question was well understood with very few cases where the use of suvat equations was thought appropriate. Failure to evaluate the constant of integration affected the accuracy of work in (ii). Candidates should be aware that the constant of integration is not always zero.	
		iii	$k = 132$	B1ft	ft cv(78) + 54 **Examiner's Comments** This part was not always done correctly; answers were often incorrect because of errors made when solving a simple linear equation or because the displacement of Q was taken to be zero.	
		iv	$v = \mathrm{d}(k - 2t^3)/\mathrm{d}t$	M1*	Differentiates displacement	
		iv	$v = -2 \times 3t^2$	A1	Award even if k wrong earlier	
		iv	$v(3) = -6 \times 3^2$ (=– 54)	D*M1	Substitutes $t = 3$	
		iv	$68m - 54m = 2mv$	M1	Conservation of momentum, must have $2m$, cv(68)	No marks if g included, even if apparently cancelled
		iv	$v = 7$ m s^{-1}	A1	**Examiner's Comments**	

					The differentiation was usually correct, although some lost the minus sign. The final step, requiring the use of conservation of momentum, was sometimes omitted. The most common error at the final stage was to use a mass of m instead of 2m for the after momentum.	
			Total	14		
6		a	Uniform deceleration from 12 m s^{-1} to –4 ms^{-1} Changes direction after 6 seconds	E1 (AO2.4) E1 (AO2.4) [2]	One relevant comment A second relevant comment	
		b	–2 m s^{-2}	B1 (AO1.1) [1]		
		c	Area of triangle for $0 \le t \le 6$ is $\frac{1}{2} \times 6 \times 12$ Area of trapezium for $6 \le t \le 16$ is $\frac{1}{2}(10+6) \times 4$ Displacement = 36 – 32 = 4 m	M1 (AO3.4) M1 (AO1.1) A1 (AO1.1) [3]		
		d	$36a + 6b + 12 = 0$ and $256a + 16b + 12 = 0$	M1 (AO1.1a)	Substitute (6, 0) and (16, 0) and	

			$a=\frac{1}{8}$ and $b=-\frac{11}{4}$	A1 (AO1.1) [2]	attempt to solve resulting simultaneous equations BC	
	e		$s=\int\left(\frac{1}{8}t^2-\frac{11}{4}t+12\right)\mathrm{d}t=\frac{1}{24}t^3-\frac{11}{8}t^2+12t \quad (+c)$ $t=0,\ s=0 \Rightarrow c=0$ Distance travelled is $2\left(\frac{1}{24}(6)^3-\frac{11}{8}(6)^2+12(6)\right)-\left(\frac{1}{24}(16)^3-\frac{11}{8}(16)^2+12(16)\right)$ = 52.3 m (3 sf)	M1 (AO2.1) A1 (AO1.1) A1 (AO3.4) M1 (AO3.4) A1 (AO2.2a) [5]	Attempt integration of v (increase powers; but not just multiplication by t) Correct integration (with or without $+c$) Dependent on M mark but not previous A Dealing with motion both before and after $t=6$	

		f	$\frac{1}{4}t-\frac{11}{4}=-2$ $t = 3$ s	M1 (AO3.4) A1 (AO2.2a) [2]	Differentiate their v and equate to their value from part **(b)**	
			Total	15		
7		i	Differentiates to find accn $dv/dt = 3 \times 0.8t^2 - 2 \times 4t + 5.6$ Solve $2.4t^2 - 8t + 5.6 = 0$ $t = 1, 2.33(33..)$ (Accept7/3) $v = 2.4$ m s^{-1}, 1.45 m s^{-1}	M1* A1 D*M1 A1 A1 [5]	3 term QE and evidence of method of solution if answer incorrect. *OR* t=1 and v=2.4 m s^{-1} A1 *OR* t=7/3 and v=1.45 m s^{-1} A1	As there are two values needed for each A1 mark, accept values correct to 2 sig fig.
		ii	$x = \int 0.8t^3 - 4t^2 + 5.6t\,dt$ $x = 0.8t^4/4 - 4t^3/3 + 5.6t^2/2\ (+c)$ $x = 0.2t^4 - 4t^3/3 + 2.8t^2$	M1 A1 A1	$x = 0.2t^4 - 1.33t^3 + 2.8t^2$	Simplified coefficients and c

			$x(2.3333) - x(1) = (0.2 \times 2.3333^4 - 4 \times 2.3333^3/3 + 2.8 \times 2.3333^2) - (0.2 \times 1^4 - 4 \times 1^3/3 + 2.8 \times 1^2)$	M1*	Evaluates x at two times found from a = 0 / discarded	
				D*M1	Subtraction of values (4.23–1.67) / These are the answers in (i)	
			Distance = 2.57 m	A1		
				[6]	**Examiner's Comments** This question was well answered by most candidates, who used calculus accurately, and used their answers in (i) to complete part (ii) successfully. The commonest mistakes arose from calculator error, and in (ii) substituting $t = 2.33 - 1 = 1.33$ into the formula for *x(t)*	
			Total	**11**		
8		i	A: $v = \int 0.18t\,dt$	M1*	Integration indicated by change in coefficient and	
		i	$v = 0.18/2\ t^2$ (+c)	A1	increase in power	
		i	$9 = 0.09t^2$	D*M1		
		i	$t = 10$	A1	**Examiner's Comments** Frequently this part of the question was answered correctly, with candidates integrating the acceleration of *A*.	
		ii	B: $v = d(Ut + 0.08t^3)/dt$	M1*	Differentiation indicated by change in coefficient and	
		ii	$v = U + 0.24t^2$ $9 = U + 0.24 \times 5^2$	D*M1	reduction in power	

		ii	$U = 3$ $SB(5) = 3 \times 5 + 0.08 \times 5^3$	A1	There are instances of solutions in which $SB(5) = 25$ is used to show that $U = 3$, and then demonstrate that $SB(5) = 25$. Such work can gain no marks. $u = 3$ without any supporting work. M0A0. **Examiner's Comments** This part of the question caused a large number of circular solutions to be presented (which gained no marks). Candidates could (and did) deduce that $U = 3$ from the position of B when $t = 5$. The substitution of $U = 3$ and $t = 5$ into the distance formula of B was then held to verify the value of 25 m. That said, the correct solution based on differentiation of the distance formula of B was frequently seen.	
		ii	$SB(5) = 25$ m AG	A1		
		iii	A: $x = \int 0.09t^2 dt$ $x = 0.09t^3 /3$	M1*	Integration of $v(A)$	
		iii	$x(16) = 0.03 \times 16^3$	D*M1		
		iii	$x = 122.88$ (may be implied by later work)	A1	Accept 123	
		iii	$122.88 = 25 + 10 \times 9 + (9 + v)(x1) /2$	M1		
		iii	$v = 6.76$ m s^{-1} *OR*	A1		
		iii	$122.88 - 25 - 10 \times 9 = 9 \times 1 +/- a \times 1^2 /2$ Deceleration = 2.24 m s^{-2} $v = 9 - 2.24 \times 1$	M1	$s = ut +/- at^2 /2$	
		iii	$v = 6.76$ m s^{-1}	A1	**Examiner's Comments** There were a lot of very good answers to this demand. Candidates	

					found the correct strategy for finding the final velocity of *B* and executed it precisely.	
			Total	13		
9		a	(i) $18=\left(\frac{8+u}{2}\right)(9)$ u = –4 therefore the speed of *P* is 4 (ms^{-1})	M1 (AO 3.4) A1 (AO 1.1) [2]	Use of $s=\left(\frac{u+v}{2}\right)t$ AG Examiner's Comments To gain full credit in this part examiners expected to see u = –4 in the working as well as 4 appearing. Whilst $s = ½(u + v)t$ was widely used, sign fudging was seen. Explanations clearly distinguishing between velocity and speed were unusual.	
		a	(ii) eg $8 = –4 + 9a$ $a=\frac{4}{3}$ (m s^{-2})	M1 (AO 3.4) A1 (AO 1.1) [2]	Use of $v = u + at$ with their u or $s = vt - ½at^2$ or $v^2 = u^2 + 2as$ with their u **or** $s = ut + ½at^2$ with their u Accept 1.33 or better Examiner's Comments	

					Once again the necessary methods were widely used, with $v = u + at$ the equation used most. This was often done with $u = 4$, candidates not realising the importance of the minus sign. Those who used the equation $s = vt - \frac{1}{2}at^2$ avoided this consideration here.	
	b	$0 = -4 + \frac{4}{3}t$ $t = 3$ $-s_{\text{max}} = -4t + \frac{1}{2}\left(\frac{4}{3}\right)t^2$ $S_{\text{max}} = 6 < 10$ so P is never at B OR $-10 = -4t + \frac{1}{2}\left(\frac{4}{3}\right)t^2$	M1 (AO 3.1b) A1 (AO 1.1) M1 (AO 3.4) A1 (AO 2.2a) [4] M1 A1 M1 A1	Use of $v = u + at$ with $v = 0$ and their a and u Use of $s = ut + \frac{1}{2}at^2$ with their a, u & t Compare with 10 or suitable comment Use of $s = ut + \frac{1}{2}at^2$ with their u and a and suitable s Consider $b^2 - 4ac$ or attempt to solve three term		

			e.g. det = −24 therefore not possible OR $0 = (\pm 4)^2 + 2\left(\frac{4}{3}\right)s$ or $v^2 = (\pm 4)^2 + 2\left(\frac{4}{3}\right)(-10)$ $s = -6$ or $v^2 = -\frac{32}{3}$ Suitable conclusion	 M2 A1 A1	quadratic in t Or 36 – 60 < 0 therefore not possible Use of $v^2 = u^2 + 2as$ with their a and u and either $v = 0$ or $s = \pm 10$ Dependent on previous A mark **Examiner's Comments** This part proved to be a challenge and, although there are various ways of solving the problem, candidates did not always make their intentions easy to follow. Were they considering the motion from O or B? Some even appeared to be considering A. Those trying $s = ut + \frac{1}{2}at^2$ once again had sign difficulties, with $u = 4$ and/or $s = 10$ used quite widely.	
		c	$x = at^3 + bt^2 + ct$ $\dot{x} = 3at^2 + 2bt + c$	M1 (AO 1.1)	Attempt to differentiate once Two terms differentiated correctly	

			$\ddot{x} = 6at + 2b$ $c = -4$ and $b = \dfrac{2}{3}$ $18 = a(6)^3 + \dfrac{2}{3}(6)^2 - 4(6) \Rightarrow a = \dfrac{1}{12}$ Velocity of $Q = \left(\dfrac{1}{4}(6)^2 + \dfrac{4}{3}(6) - 4\right) = 13\,(\text{m s}^{-1})$	M1 (AO 2.1) A1ft (AO 1.1) A1ft (AO 1.1) A1 (AO 1.1) [5]	Attempt to differentiate again and substitute $t = 0$ into both equations and substitute their acceleration in their second derivative and their u in their first derivative — Two terms differentiated correctly following through from their first derivative $b = 0.5 \times$ their accel. and $c = \pm 4$ Allow $a = -\dfrac{5}{36}, -\dfrac{7}{108}, \dfrac{1}{108}$ which come from $u = 4$ — Allow $b = 0.665$ from accel. = 1.33 cao Examiner's Comments Few fully correct solutions were seen to this part. Many just earned one mark for differentiating the given displacement equation, seemingly not understanding that $t = 0$ needed to be used in the work to find b and c (not $t = 6$) and then $t = 6$ needed to obtain a. A number of candidates attempted to solve this part using the constant acceleration formulae.	
			Total	13		

10		a	$a = k + 0.06t$ $k + 0.06(20) = 1.3$ $k = 1.3 - 1.2 = 0.1$	B1(AO 1.1)E M1(AO 1.1)E A1(AO 1.1)E [3]	Use of $t = 20$ and $a = 1.3$ in their a **Examiner's Comments** Nearly all candidates correctly differentiated the expression for v and correctly obtained the value of k as 0.1.	
		b	$s = 0.05t^2 + 0.01t^3 (+c)$ $t = 0,\ s = 0 \Rightarrow c$ $t = 20,\ v = 14$ $s_1 = 0.05(20)^2 + 0.01(20)^3$ $25^2 = 14^2 + 2(1.3)s_2$	M1*(AO 3.1a)E A1ft(AO 1.1)E B1(AO 2.1)A B1ft(AO 1.1)E dep*M1(AO 3.4)C M1(AO 3.3)A	Attempt to integrate – all powers increased by 1 (but not just multiplying by t) $s = \frac{1}{2}kt^2 + 0.01t^3$ From a correct expression for s $12 + 20k$ Finding distance travelled after 20 s (for reference $s_1 = 100$) Use of $v^2 = u^2 + 2as$ with $v = 25$ | If $c = 0$ stated then must give a reason	

			Total distance = $s_1 + s_2 = 265$ m	A1(AO 2.2a)A [7]	and $a = 1.3$ and their u All previous marks must have been awarded **Examiner's Comments** This part was answered extremely well with many candidates correctly finding the total distance that car had travelled when its speed had reached 25 ms^{-1}. Many correctly realised that they had to use integration to find an expression for the displacement in terms of t which they could then use to find the distance travelled by the car in the first 20 seconds. However, many ignored the constant of integration that would arise from the corresponding indefinite integral; even though this constant was zero it is mathematically incorrect to simply ignore it (and for full marks candidates either had to consider this displacement expressed as a definite integral or explain why the constant was zero). Most candidates then used the SUVAT equations to work out the remaining distance travelled when the speed increased from 14 to 25 and correctly calculated the total distance as 265m.	
			Total	10		
11		a	$\frac{dv}{dt} = 8t^3 + 2kt$ $8(2)^3 + 2K(2) = 28$ $4K = 28 - 64 \Rightarrow k = -9$	B1 (AO 1.1) M1 (AOs 1.1) A1 (AO 2.2a) [3]	Correct expression for the acceleration Substitute $t = 2$ into their a and equate to 28 AG	

		b	$\frac{dv}{dt}=0 \Rightarrow 2t(4t^2-9)=0$ $t = 1.5$ (and $t = 0$) E.g. $\left.\frac{d^2v}{dt^2}\right\|_{t=1.5} = 24(1.5)^2 - 18 > 0$ so a minimum	M1 (AO 3.1b) A1 (AO 1.1) B1 (AO 2.1) [3]	Substituting the correct value of k and equating to zero AG Correctly finding the given value of t Showing that this value of t gives a minimum	Or complete argument from the shape of the curve, or from first derivatives
		c	$s = \frac{2}{5}t^5 - 3t^3 - 4t\ (+c)$ $-6.4125 = 0.4 - 3 - 4 + c \Rightarrow c =$ K $s = 0.4(1.5)^5 - 3(1.5)^3 - 4(1.5) + 0.1875$ $s = -12.9$ so distance of P from O is 12.9m	M1* (AO 1.1a) M1dep* (AO 2.1a) M1 (AO 1.1) A1 (AO 3.2a) [4]	Attempt to integrate v (all powers increased by 1) Attempt to find c Substitute 1.5 into their expression for s – dependent on both previous M marks	Constant not required for this first M mark $c = 0.1875$
			Total	10		