1. In this question the unit vectors **i** and **j** are in the directions east and north respectively.

Distance is measured in metres and time in seconds.

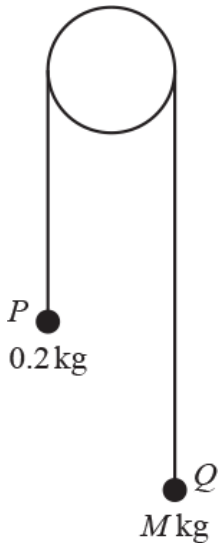
A ship of mass 100 000 kg is being towed by two tug boats. The cables attaching each tug to the ship are horizontal. One tug produces a force of (350**i** + 400**j**) N and the other tug produces a force of (250**i** – 400**j**) N. The total resistance to motion is 200 N. At the instant when the tugs begin to tow the ship, it is moving east at a speed of 1.5 m s^{-1}.

(a) Explain why the ship continues to move directly east. [2]

(b) Find the acceleration of the ship. [2]

(c) Find the time which the ship takes to move 400 m while it is being towed. Find its speed after moving that distance. [6]

2.



Particles P and Q, of masses 0.2 kg and M kg respectively, where $M > 0.2$, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley (see diagram). The system is in motion with the string taut and with each of the particles moving vertically.

The tension in the string is 2.1 N.

(a) Show that the acceleration of P is 0.7 m s^{-2}. [2]

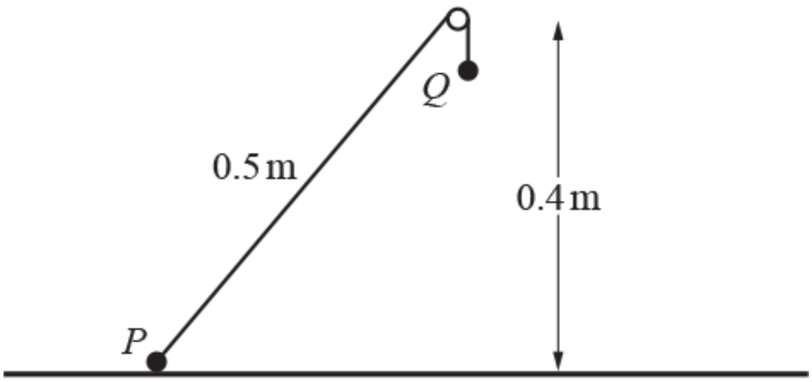
(b) Find the value of M. [2]

(c) At one instant P has speed 0.3 m s^{-1} upwards. Find its speed 1.5 seconds later, assuming that it has not yet reached the pulley. [1]

3. In this question the horizontal unit vectors **i** and **j** are in the directions east and north respectively.
A toy car of mass 0.5 kg is moving so that its acceleration vector **a** ms^{-2} at time t seconds is given by
$\mathbf{a} = 6t\,\mathbf{i} + (2 - 3t^2)\mathbf{j}$. When $t = 2$ the horizontal force acting on the car is **F** N.
Find

- the magnitude of **F**,
- the bearing of **F**. [5]

4.



A particle P of mass 0.4 kg is attached to one end of a light inextensible string. The string passes over a small smooth fixed pulley, and a particle Q of mass 0.1 kg is attached to the other end of the string. P rests in limiting equilibrium on a horizontal surface which is 0.4 m below the pulley, with the string taut and in the same vertical plane as P, Q and the pulley. P is 0.5 m from the pulley (see diagram).

(i) Calculate the coefficient of friction and the magnitude of the contact force exerted on P by the surface. [7]

Q is now moved to the position on the surface below the pulley such that the portion of the string attached to Q is vertical. P hangs freely below the pulley and the portion of the string attached to P is vertical. Both particles are at rest when Q is released.

(ii) Find the acceleration of the particles and the tension in the string while P is descending. [5]

P strikes the surface and remains at rest. Q comes to instantaneous rest immediately before reaching the pulley.

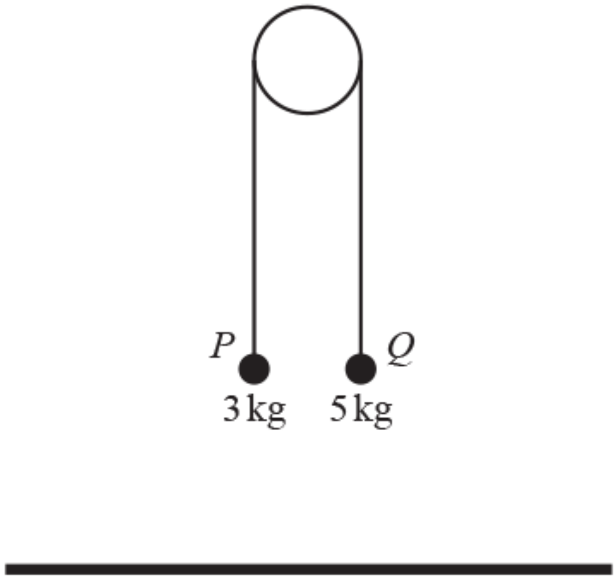
(iii) Find the length of the string. [5]

5. In this question the horizontal unit vectors **i** and **j** are in the directions east and north respectively.

A model ship of mass 2 kg is moving so that its acceleration vector **a** ms^{-2} at time t seconds is given by
a = $3(2t - 5)$**i** + 4**j**. When $t = T$, the magnitude of the horizontal force acting on the ship is 10 N.

Find the possible values of T. **[4]**

6. Particles P and Q, of masses 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and P and Q are above a horizontal plane (see diagram).



(a) Find the tension in the string immediately after the particles are released. **[4]**

After descending 2.5 m, Q strikes the plane and is immediately brought to rest. It is given that P does not reach the pulley in the subsequent motion.

(b) Find the distance travelled by P between the instant when Q strikes the plane and the instant when the string becomes taut again. **[4]**

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance	
1		a	Resultant force from the tug boats is positive so it is moving east There is zero resultant force in the **j** direction, so it is not moving north or south	E1(AO2.2a) E1(AO2.2a) [2]	(600**i**)	
		b	$350 + 250 - 200 = 100000a$ Obtain 0.004 m s^{-2}	M1(AO3.3) A1(AO1.1) [2]	Use $F = ma$. Allow sign errors and one missing force	
		c	$400 = 1.5t + \frac{1}{2}(0.004)t^2$ $0.002t^2 + 1.5t - 400 = 0$ Obtain 209 (seconds) $v^2 = 1.5^2 + 2(0.004)(400)$ Obtain 2.33 (m s^{-1})	M1(AO3.1b) A1(AO1.1) M1(AO3.4) A1(AO1.1) M1(AO3.4) A1(AO1.1) [6]	Use $s = ut + \frac{1}{2}at^2$ Obtain correct quadratic. Any equivalent form Use any method to solve their quadratic If negative root given (–958.63088) this must be clearly discarded Use $v^2 = u^2 + 2as$ with their a or $v = u + at$ with their a and t Accept better (2.3345235)	Including BC Accept better (208.630877) but not 208
			Total	10		
2		a	$T - 0.2g = 0.2a$ $a = \frac{2.1 - 0.2 \times 9.8}{0.2} = \frac{0.14}{0.2} = 0.7$	M1 (AO3.3) E1 (AO1.1) [2]	Attempt N2L for P AG	Must include sufficient working

						to justify the given answer
		b	$Mg - T = Ma$ $M = 0.231$ (3 sf)	M1 (AO3.3) A1 (AO1.1) [2]	Attempt N2L for Q	0.23076923…
		c	$v = 0.3 + 0.7 \times 1.5 = 1.35$ m s^{-1}	B1 (AO3.4) [1]	Use of $v = u + at$ with given information	
			Total	5		
3			a = 12**i** – 10**j** $F = 0.5\sqrt{(12)^2 + (-10)^2}$ $F = 7.81$ N $90 + \tan^{-1}\left(\frac{5}{6}\right)$ = 130°	B1 (AO1.1) M1 (AO3.3) A1FT (AO3.4) M1 (AO3.1a) A1 (AO1.1) [5]	Substitute $t = 2$ Use of **F = ma** and Pythagoras FT their **a** at $t = 2$ $90 + \tan^{-1}\left(\frac{y}{x}\right)$ to give a 3-figure bearing	
			Total	5		
4		i	$\sin\theta = 0.4/0.5$ or $\cos\theta = 0.3/0.5$ $T = 0.1g$ (=0.98) N $Fr = T\cos\theta$ (= 0.588) $R = 0.4g - T\sin\theta$ (= 3.136)	B1 B1 M1 M1	θ is angle between string and horizontal CorS. T, angle do not have to be numerical SorC. T, angle do	If two values of T are employed, award B1 for 0.1g associated with Q.

			μ (= 0.588 /3.136) = 3/16 or 0.1875 $C^2 = 0.588^2 + 3.136^2$ $C = 3.19$ N	A1 M1 A1 [7]	not have to be numerical with 0.4g / R must be a difference of forces 0.187 or 0.188 Must have two non-zero numerical values **Examiner's Comments** Part (i) was often incomplete as candidates were unfamiliar with the term "contact force". The initial part was well answered by many, and candidates who could not find a relevant angle were still able to gain a majority of marks.
		ii	$0.4g - T = 0.4a$ $T - 0.1g = 0.1a$ $0.3g = 0.5a$ *OR* $0.4g - 0.1g = 0.4a + 0.1a$ $a = 5.88$ m s^{-2} $T = 1.568$ N = 1.57 N	M1 A1 M1 A1 A1 [5]	N2L for either particle, no components Both equations correct Solves two simultaneous equations Finding a correctly from the combined equation gets M1A1. Using a in an N2L equation for P or Q can get M1, and obtaining the correct value of T gets A1, hence 4 marks out of 5 **Examiner's Comments** In part (ii) many fully correct solutions were found, and the most common error was to omit an answer for the tension.
		iii	P descends = x m (= (2x0.4 - l) m) $v^2 = 2\text{x}5.88x$ (=11.76x) $0 = v^2 - 2g(0.4 - x)$ $x = 0.25$ String is 0.8-0.25 m long $l = 0.55$ m *OR* (P starts d m below pulley)	 M1 M1 A1 M1 A1 [5]	P and Q moving together / Eqn has two unknowns Q rising alone / Eqn has two unknowns

			$v^2 = 2\times5.88(0.4 - d)$ $v^2 = 2gd$ $d = 0.15$ String is 0.4+0.15 m long $l = 0.55$ m	M1 M1 A1 M1 A1	*P* and *Q* moving together — Eqn has two unknowns *Q* rising alone — Eqn has two unknowns **Examiner's Comments** Part (iii) was challenging for most, setting up and solving two simultaneous $v^2 = u^2 + 2as$ equations being unfamiliar.
			Total	17	
5			$F = \sqrt{36(2T-5)^2 + 64}$ $36(2T - 5)^2 = 36$ $2T - 5 = \pm1 \Rightarrow T = \ldots$ $T = 2$ and $T = 3$	M1* (AO 3.3) A1 (AO 1.1) Dep*M1 (AO 1.1) A1 (AO 2.2a) [4]	Correct use of **F** = m**a** and Pythagoras Correct equation(s) for both values of *T* e.g. $10 = 2\sqrt{9(2T-5)^2 + 16}$ Attempt to solve a quadratic leading to at least one value for *T* Allow *t* throughout **Examiner's Comments** This question proved to be challenging for the majority of candidates. The need to find the magnitude of the acceleration was not well understood. For those who did use it, algebraic and numerical errors sometimes marred the solution. $(\pm)10 = 2(3(2t - 5)\mathbf{i} + 4\mathbf{j})$ was common, even when the correct methods then followed, but in many cases equations like $10 = 12t - 30$ were used. A number of candidates ignored the mass and **F = a** pursued.
			Total	4	

6		a	$T - 3g = 3a$ $5g - T = 5a$ $5g - T = 5\left(\frac{T-3g}{3}\right) \Rightarrow T = ...$ $T = 36.75$ (N)	M1* (AO 3.3) A1 (AO 1.1) Dep*M1 (AO 1.1) A1 (AO 1.1) [4]	Attempt N2L for *P* and *Q* – three terms, mass required, condone sign errors / M0 for $a = 0$ or $\pm g$ Eliminate *a* and attempt to solve for *T* Accept $\frac{15}{4}$, 36.8 **Examiner's Comments** Whilst this standard situation was generally well understood, it was not uncommon to see attempts which assumed the acceleration after release was g or 0. Those who produced initial appropriate equations generally produced fully accurate solutions, although sign errors were sometimes seen.
		b	$a = 2.45$ ms^{-2} $v^2 = 0 + 2(2.45)(2.5)$ $0 = 12.25 + 2h(-g)$ $(2h =)$ 1.25(m)	B1 (AO 3.4) M1* (AO 3.3) Dep*M1 (AO 3.3) A1 (AO 1.1) [4]	$0.25g$ Use of $v^2 = u^2 + 2as$ for *P* with $u = 0$ / M0 for $a = 0$ or $\pm g$ Use of $v^2 = u^2 + 2as$ for *P* with $v = 0$ / $a = \pm g$ oe **Examiner's Comments** We expected to see an acceleration other than g or 0 used for the descent of *Q*. Some candidates used $v = 0$ for this phase rather than $u = 0$, presumably because '*Q* strikes the plane and is immediately brought to rest'. Those who now looked at the next phase of the motion sometimes did not realise that it was motion under gravity. Some good attempts omitted to double 0.625.
			Total	8	