1. Three horizontal forces, acting at a single point, have magnitudes 12 N, 14 N and 5 N and act along bearings 000°, 090° and 270° respectively. Find the magnitude and bearing of their resultant.

[5]

2. Two forces of magnitudes 8 N and 12 N act at a point O.

i. Given that the two forces are perpendicular to each other, find

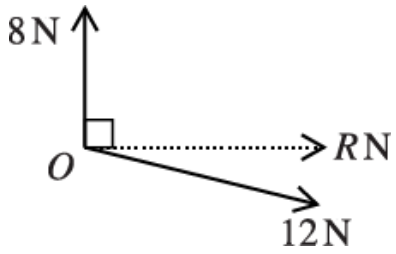
a. the angle between the resultant and the 12 N force,

[2]

b. the magnitude of the resultant.

[2]

ii. It is given instead that the resultant of the two forces has magnitude R N and acts in a direction perpendicular to the 8 N force (see diagram).



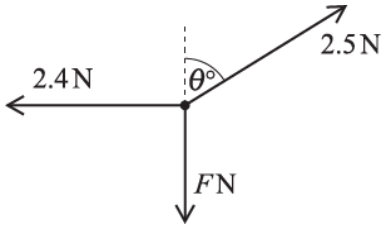
a. Calculate the angle between the resultant and the 12 N force.

[3]

b. Find R.

[2]

3.



A particle rests on a smooth horizontal surface. Three horizontal forces of magnitudes 2.5 N, FN and 2.4 N act on the particle on bearings $\theta°$, 180° and 270° respectively (see diagram). The particle is in equilibrium.

i. Find θ and F.

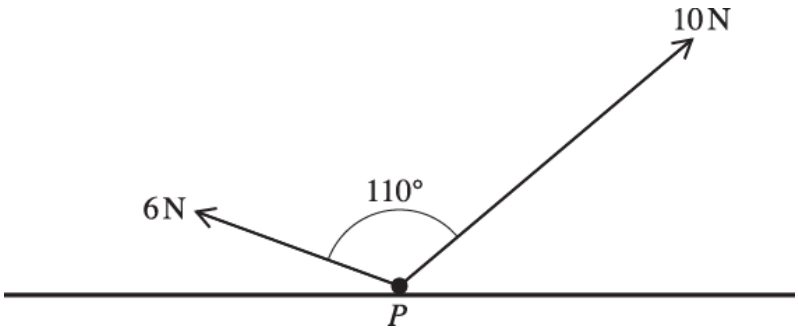
[4]

The 2.4 N force suddenly ceases to act on the particle, which has mass 0.2 kg.

ii. Find the magnitude and direction of the acceleration of the particle.

[3]

4.



Two forces of magnitudes 6 N and 10 N separated by an angle of 110° act on a particle P, which rests on a horizontal surface (see diagram).

i. Find the magnitude of the resultant of the 6 N and 10 N forces, and the angle between the resultant and the 10 N force.

[6]

The two forces act in the same vertical plane. The particle P has weight 20 N and rests in equilibrium on the surface. Given that the surface is smooth, find

ii. the magnitude of the force exerted on P by the surface,

[1]

iii. the angle between the surface and the 10 N force.

[2]

5. Two forces each of magnitude 4 N have a resultant of magnitude 6 N.

i. Calculate the angle between the two 4 N forces.

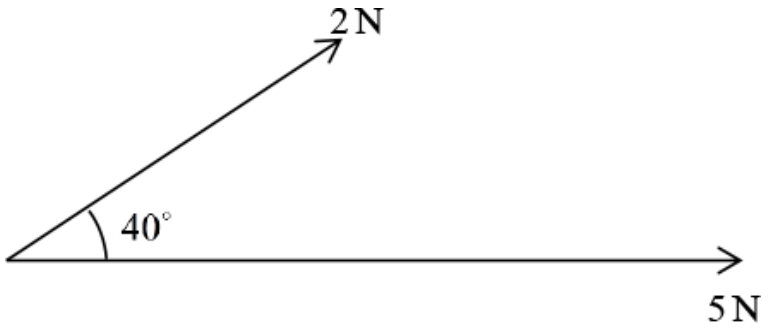
[4]

The two given forces of magnitude 4 N act on a particle of mass m kg which remains at rest on a smooth horizontal surface. The surface exerts a force of magnitude 3 N on the particle.

ii. Find m, and give the acute angle between the surface and one of the 4 N forces.

[3]

6. Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.



(a) Calculate the magnitude of the resultant force on the particle. [3]

(b) Calculate the angle between this resultant force and the force of magnitude 5 N. [1]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$X = 14 - 5$	B1	Or 5 – 14	
			$R^2 = (14 - 5)^2 + 12^2$	M1	Pythagoras, R as hypotenuse, 3 squared terms	
			$R = 15$ N	A1		
			$\tan\theta = (14 - 5)/12$	M1	Any correct trig, angle between 12 and R targetted. Accept 37, 037 **Examiner's Comments** Most frequently candidates started the paper by gaining full marks, though 3 out of 5 was a common total among candidates who did not target the "bearing". Some scripts contained a confusion between bearings and polar angles. A small minority of candidates began by finding the resultant of the 5 N and 12 N forces, and then combining it with the 14 N force. This gives a diagram which should contain an obtuse angle and calculation which incorporates the ambiguous case of the sine rule. This was not done.	
			$\theta = 36.9°$	A1		
			Total	**5**		
2		i	***(a)*** $\tan\theta = 8/12$	M1	Must be correct angle.	
		i	$\theta = 33.7°$	A1		
		i	*OR* correct trig using ans (**i**)(**b**)			
		i	$\sin\theta = 8/\text{cv}(14.4)$ or $\cos\theta = 12/\text{cv}(14.4)$	M1	Must be correct angle	
		i	$\theta = 33.7°$	A1	A1 needs 2/2 in (i)(b). $\cos\theta = 12/14.4$ gives $\theta = 33.6$ A1	
		i	***(b)*** $R^2 = 8^2 + 12^2$	M1	Pythagoras, 3 squared terms, R as hypotenuse	
		i	$R = 14.4$ N	A1	Accept $4\sqrt{13}$ not $\sqrt{208}$	
		ii	***(a)*** $12\text{CorS}\theta = \pm 8$	M1	Either angle.	

		ii	12sin $\theta = 8$	A1		
		ii	$\theta = 41.8°$	A1		
		ii	*OR* correct trig using **(ii)(b)**	Enter text here.		
		ii	12CorS $\theta =$ cv(8.94), cv(8.94)tan $\theta = 8$, or 8tan $\theta =$ cv(8.94)	M1		
		ii	12cos $\theta = 8.94$ or 8.94tan $\theta = 8$	A1		
		ii	$\theta = 41.8°$	A1		
		ii	***(b)*** $R = 12\cos 41.8$	M1		
		ii	$R = 8.94$ N	A1		

4		i	x = +/– (10 – 6cos70), y = 6sin70 *OR* +/– (10cos70 – 6), 10sin70 *OR* correct resolving in 2 perpendicular directions	B1,B1	10cos55 + 6cos55 (= 9.177) B1 +/– (10sin55 – 6sin55) (= +/– 3.2766) B1	Uses cosine rule M1 $R^2 = 6^2 + 10^2 – 2 \times 6 \times 10$ cos B1 Uses angle of 70 B1
		i	R^2 = { +/– (10 – 6cos70)}^{2} + (6sin70)^2 = { +/– (10cos70 – 6)}^{2} + (10sin70)^2	M1	R^2 = (10cos55 + 6cos55)^2 + (10sin55 – 6sin55)^2	
		i	R = 9.74 N	A1	R = 9.74 N	R = 9.74 N A1
		i	Tanα = (6cos20)/(10 – 6sin20)	M1		Sinα/6 = sin70/9.744 M1
		i	α = 35.4°	A1	www **Examiner's Comments** This proved to be a difficult question for many candidates. For (i) there were two methods of solution, using trigonometry or by resolving; whichever approach was used the candidates who were most successful had usually draw a good diagram. The most common error made by those using trigonometry was to form a triangle by simply joining the ends of the forces instead of drawing the appropriate triangle of forces. In this case 110 instead of the correct value of 70 was used in the cosine and sine rules and a maximum of half-marks were available. Those who chose to resolve were most successful, particularly in finding the angle, if they resolved parallel and perpendicular to the 10N force. Any reference to horizontal and vertical was ignored if the candidate worked with an incorrect angle between one of the forces and the true horizontal, credit being given for correct work relative to their base directions. Using Pythagoras and basic trigonometric ratios with 10 and 6 gained no credit.	α = 35.4° A1
		ii	Force = (20 – 9.74) = 10.3 N	B1ft	Difference of weight and Resultant ft 20 – cv(9.74) **Examiner's Comments** For (ii) and (iii) candidates needed to realise that the resultant was vertical. In both parts credit could be gained from correct use of the answers	

					obtained in (i). These parts were done particularly badly, if at all, with 20 N being the most common wrong answer in (ii).	
		iii	Tanθ = +/– (10 – 6cos70) /6sin70	M1	Uses resultant is vertical	Angle = 90 – cv(35.4) M1
		iii	*OR* Tanϕ = +/– (6sin70) /(10 – 6cos70)			Angle = 54.6° A1
					Examiner's Comments	
		iii	Angle = 54.6°	A1	For (ii) and (iii) candidates needed to realise that the resultant was vertical. In both parts credit could be gained from correct use of the answers obtained in (i). These parts were done particularly badly, with (iii) frequently not attempted.	
			Total	**9**		
5		i		M1	Resolve / Resultant	
		i	$4\cos\theta + 4\cos\theta = 6$	A1		
		i	$\cos\theta = 6/8$	M1		
		i	Angle (= $2\theta = 2\cos^{-1}0.75$) = 82.8° *OR*	A1		
		i	$6^2 = 4^2 + 4^2 - 2 \times 4 \times 4\cos\alpha$	M1	Cosine rule for triangle of forces	
		i	α = 97.2°	A1	Cosine rule must give obtuse angle	
		i	Angle = 180 – 97.2	M1		
		i	Angle = 82.8° *OR*	A1	Do not accept 82.8° from incorrect working *OR*	
		i	$6^2 = (4\sin\theta)^2 + (4 + 4\cos\theta)^2$	M1	$6^2 = (4\cos\theta)^2 + (4 + 4\sin\theta)^2$	
		i	$36 = 16 + 32\cos\theta + 16$	A1	$36 = 16 + 32\sin\theta + 16$ hence θ = 7.2°	
		i	$\cos\theta$ =4/32	M1	θ = 90 – 7.2 θ = 82.8°.	
					Examiner's Comments	
		i	θ = 82.8°	A1	The simplest solution (resolving parallel to the bisector of the required angle) was seldom seen. Resolving parallel and perpendicular to one of the 4 N forces was more common.	

					Both approaches were generally successful. However, using the cosine rule on an incorrect diagram which reflected subtraction – not addition – was by far the most frequent approach, and led to candidates usually obtaining 2 marks out of 4.	
		ii	$mg = 6 + 3$ *OR* $mg = 4\cos(\text{Ans(i)}/2) + 4\cos((\text{Ans(i)}/2) + 3$	M1	Must have signs correct	
		ii	$m = 0.918$	A1	Ft(90– cv(angle in (i)) /2) **Examiner's Comments** Candidates needed to appreciate that the 6 N resultant would be perpendicular to the smooth surface on which the particle rested. This would be demonstrated by having the angle in (ii) equal to the complement of half the angle in (i), for which a mark was awarded. The value of *m* could be determined using the given 6 N resultant. So candidates who resolved 4 N forces (using an erroneous answer for the angle in (i)) gained a method mark, but not an accuracy mark. Some candidates lost a mark through giving the mass as 0.92 kg, rather than 0.918 kg.	
		ii	Angle = 48.6°	B1√		
			Total	**7**		
6		a	Attempt resolution of forces Horizontal component = 5 + 2cos 40 (= 6.5321) Vertical component = 2sin 40 (=1.2856) $\sqrt{6.5321^2 + 1.2856^2} = 6.66\text{N}$	**M1(AO 1.1a)** **A1(AO 1.1)** **A1(AO 1.1)** **[3]**	Allow sin / cos confusion Allow for either the horizontal or vertical component correct Use correct method for magnitude OR M1 Form triangle of forces A1 Use cosine rule with 140° A1	

					Obtain 6.66 N	
		b	$\tan^{-1}\left(\frac{2\sin 40}{5+2\cos 40}\right)=11.1^\circ$	B1FT(AO 1.1) [1]	FT their components from part (a)	
			Total	4		