1. Find $\int x \cos 3x \, dx$.

[4]

2. Find $\int x^8 \ln(3x) \, dx$.

[5]

3.

i. Use division to show that $\frac{t^3}{t+2} \equiv t^2 - 2t + 4 - \frac{8}{t+2}$.

[3]

ii. Find $\int_1^2 6t^2 \ln(t+2) \, dt$. Give your answer in the form $A + B \ln 3 + C \ln 4$.

[6]

4. Find the exact value of $\int_1^8 \frac{1}{\sqrt[3]{x}} \ln x \, dx,$ giving your answer in the form $A \ln 2 + B$, where A and B are constants to be found.

[5]

5. Find $\int (2x+1) \ln x \, dx$. [5]

6. Show that $\int_0^1 16x e^{4x} \, dx = 3e^4 + 1.$ [5]

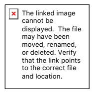
END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$u = x$ and $dv = \cos 3x$	M1	integration by parts as far as $f(x) \pm \int g(x)dx$	Check if labelled v,du
			$x \times \frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x\,dx$	A2	**A1** for $x \times k\sin 3x - \int k\sin 3x\,dx$; $k \neq \frac{1}{3}$ or 0 Not ….. $\frac{1}{3}\left(\frac{1}{3}\cos 3x\right)$ or ….. $--\frac{1}{9}\cos 3x$ **Examiner's Comments**	k may be negative
			$\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x\ [+\mathrm{c}]$ cao www ISW	A1	The vast majority recognised this question as one suitable for integration by parts, the main errors arising from the integrations of cos3x and sin3x. Provided the method of integrating by parts was fully understood, some credit was given to candidates who used a wrong sign or 3 instead of $\frac{1}{3}$ n the integrals. Candidates were expected to simplify $\frac{1}{3}(\frac{1}{3}$ cos 3x) and – – $\frac{1}{9}$cos 3x in their answers but, needless to say, they were not expected to multiply their result by 9 to make it look 'better'.	
			Total	4		
2			$u = \ln 3x$ and dv or $\frac{dv}{dx} = x^8$	M1	integ by parts as far as $f(x)+/- \int g(x)(dx)$	If difficult to assess, x^8 must be integrated, so look for term in x^9
			$\frac{d}{dx}(\ln 3x) = \frac{1}{x}$ or $\frac{3}{3x}$	B1	stated or clearly used	
			$\frac{x^9}{9}\ln 3x - \int \frac{x^9}{9}$ their $\frac{du}{dx}(dx)$ FT	√A1	i.e. correct understanding of 'by parts'…	..even if ln(3x) incorrectly differentiated

			Answer	Marks	Guidance	
			Indication that $\int kx^8 \mathrm{d}x$ is required	M1	i.e. before integrating, product of terms must be taken	The product may already have been indicated on the previous line
			$\frac{x^9}{9}\ln 3x - \frac{x^9}{81}$ or $\frac{1}{9}x^9\left(\ln 3x - \frac{1}{9}\right)$ ISW (+c) cao	A1	$\frac{1}{9}\frac{x^9}{9}$ to be simplif to $\frac{x^9}{81}$; $\frac{3x^9}{243}$ satis	
			If candidate manipulates ln(3*x*) first of all ln(3*x*) = ln 3 + ln *x*	B1		If, however, ln(3*x*) is said to be ln 3.ln *x*, then B0 followed by possible M1 A1
			u = ln *x* and d*v* = x^8	M1	In order to find $\int x^8 \ln x \,\mathrm{d}x$:	A1 in line with alternative solution on
			$\frac{x^9}{9}\ln x - \int \frac{x^9}{9}.\frac{1}{x}(\mathrm{d}x)$ or better	A1		
			$\frac{x^9}{9}\ln x - \frac{x^9}{81}$	A1		LHS, where the 'M' mark is for dealing with $\int x^8 \ln x \,\mathrm{d}x$ 'by parts' in the right order and the 2 @ A1 are for correct results.
			Their $\int x^8 \ln x \,\mathrm{d}x + \frac{x^9}{9}\ln 3$ (+ c) FT ISW	√A1		
					Examiner's Comments This was a relatively straightforward question but two specific errors occurred. The first could have been forecast: the differentiation of ln(3x) as $\frac{1}{3x}$; the other, perhaps not so predictable, involved the integration (at the second stage) of $\frac{x^9}{9}.\frac{1}{x}$. Here examiners were looking, first of all, to see if candidates were integrating an expression of the form kx^8. Even the correct simplification at that stage was of ten followed by the incorrect result of $\frac{1}{72}x^9$. However, it can be said that the technique of 'integration by parts' was generally known.	

			Total	5		
3		i	t^2 in quotient and $t^3 + 2t^2$ seen	B1	or $\dfrac{t(t^2-4)+4t}{(t+2)}$	or $\dfrac{(t+2)^3 - 6t^2 - 12t - 8}{(t+2)}$
		i	$-2t$ in quotient $-2t^2 - (-2t^2 - 4t) = 4t$ seen	B1	$\dfrac{t(t+2)(t-2)}{(t+2)} + \dfrac{4t}{t+2}$	$\dfrac{(t+2)^3}{(t+2)} - \dfrac{6((t+2)^2 - 4t - 4) + 12t + 8}{(t+2)}$ oe $(t+2)^2 - 6(t+2) + \dfrac{12t+16}{t+2}$ oe
		i	completion to obtain correct quotient and remainder identified www	B1	$t(t-2) + \dfrac{4(t+2)-8}{t+2}$	or B1 for $\dfrac{t^2(t+2) - 2t^2}{(t+2)}$ both steps needed for final B1
			alternatively			
		i	$\dfrac{t^3}{t+2} \equiv At^2 + Bt + C + \dfrac{D}{(t+2)}$	B1	or $t^3 \equiv (At^2 + Bt + C)(t+2) + D$	or B1 for $\dfrac{t^2(t+2) - 2t^2}{(t+2)}$
		i	equate coefficients to obtain correctly $A = 1$, $0 = 2A + B$ and $B = -2$ www	B1		B1 for $t^2 + \dfrac{-2t(t+2) + 4t}{(t+2)}$
		i	$0 = 2B + C$ and $0 = 2C + D$ obtained and solved correctly www	B1	<u>Examiner's Comments</u> Most candidates took the expected route and showed the required result successfully using long division, although a proportion who adopted this approach made sign errors and fudged the rest. A variety of other approaches were also seen. Candidates are reminded that in this type of question, a convincing argument is required – it appeared that some strong candidates lost marks because the answer a andppeared obvious to them.	B1 for $t^2 - 2t + \dfrac{4(t+2)-8}{(t+2)}$

		ii	integration by parts with $u = \ln(t+2)$ and $dv = 6t^2$ to obtain $f(t) \pm \int g(t)(dt)$	M1*	$f(t)$ must include t^3 and $g(t)$ must **not** include a logarithm	ignore spurious dx etc
		ii	$2t^3 \ln(t+2) - \int \frac{2t^3}{t+2}(\mathrm{d}t)$ cao	A1		alternatively, following $u = t + 2$
		ii	result from part (i) seen in integrand; must follow award of at least first M1	M1*	no integration required for this mark	$\int 2(u^2 - 6u + 12 - \frac{8}{u})\mathrm{d}u$ oe
		ii	$F[t] = 2t^3 \ln(t+2) \pm \frac{2t^3}{3} \pm 2t^2 \pm 8t \pm 16\ln(t+2)$	A1	$2t^3 \ln(t+2) - \frac{2t^3}{3} + 2t^2 - 8t + 16\ln(t+2)$	$\frac{2u^3}{3} - 6u^2 + 24u - 16\ln u$ and $2t^3 \ln(t+2)$
		ii	their F[2] – F[1]	M1dep*	at least one of their terms correctly integrated	NB limits following substitution are $u = 4$ and $u = 3$
		ii	$-6\frac{2}{3} - 18\ln 3 + 32\ln 4$ oe cao	A1		
		ii			<u>Examiner's Comments</u> Most candidates made some progress here. Integration by parts was generally used, and mostly successfully. Weak candidates failed to make the connection with part (i), but those who did make the connection generally went on to achieve at least the method marks. It was often in the manipulation following integration that marks were lost. A surprisingly common error was 	
			Total	9		
4			$Ax^{\frac{2}{3}} \ln x - \int Bx^{\frac{2}{3}} \times \frac{1}{x}\mathrm{d}x$ oe	M1*	A and B are non-zero constants;	

			Answer	Marks	AO / Guidance		Additional guidance
			$\frac{3}{2}x^{\frac{2}{3}}\ln x - \int \frac{3}{2}x^{\frac{2}{3}} \times \frac{1}{x}\,\mathrm{d}x$	A1	ignore + c		NB $\frac{3}{2}x^{\frac{2}{3}}\ln x - \int \frac{3}{2}x^{-\frac{1}{3}}\mathrm{d}x$ Allow both marks if dx omitted
			$\mathrm{F}[x] = \frac{3}{2}x^{\frac{2}{3}}\ln x - \frac{3/2}{2/3}x^{\frac{2}{3}}$	A1	ignore limits for first three marks		
			F[8] – F[1]	M1*dep	and also dependent on integration of their $\frac{3}{2}x^{-\frac{1}{3}}$		NB A0 for $6\ln 8 - \frac{27}{4}$
			$18\ln 2 - \frac{27}{4}$ cao	A1			**Examiner's Comments** Most candidates knew how to integrate by parts, but many made accuracy errors, particularly when dealing with the second integral. Some candidates worked carefully through the problem, but either didn't see the instruction to leave the answer in terms of ln2, or didn't know how to resolve ln8.
			Total	5			
5			$u = \ln x$, $\mathrm{d}v = 2x + 1$ $\mathrm{d}u = \frac{1}{x}$, $v = x^2 + x$	M1(AO1.1a) B1(AO1.2) A1(AO1.1)	Recognise integration by parts with correct u and dv State or imply that $\mathrm{d}u = \frac{1}{x}$		

			$I = (x^2 + x)\ln x - \int (x^2 + x)\frac{1}{x}\,dx$ $= (x^2 + x)\ln x - \int (x + 1)\,dx$ $= (x^2 + x)\ln x - (\frac{1}{2}x^2 + x) + c$	M1(AO1.1a) A1(AO1.1) [5]	Correct unsimplified expression Attempt to simplify and integrate Obtain fully correct integral	Including + c
			Total	5		
6			$\frac{1}{4}e^{4x}$ soi $[16]x \times \frac{1}{4}e^{4x} - \int [16] \times \frac{1}{4}e^{4x}\,dx$ oe $F[x] = [4xe^{4x} - e^{4x}]$ $F[1] - F[0]$ $= 3e^4 + 1$ NB AG	B1 M1* A1 M1dep* A1 [5]	from integration allow sign errors only allow bracket errors, but substitution of limits must be shown convincing intermediate step needed eg $4e^4 - e^4 - (0 - e^0)$	ignore limits at this stage **NB** double negative may be implied by plus sign no recovery from bracket errors for this mark

					Examiner's Comments This was question was done very well, with many candidates achieving full marks. A few candidates integrated x when applying integration by parts, and more often than not the correct result mysteriously appeared from wrong working.	
			Total	5		