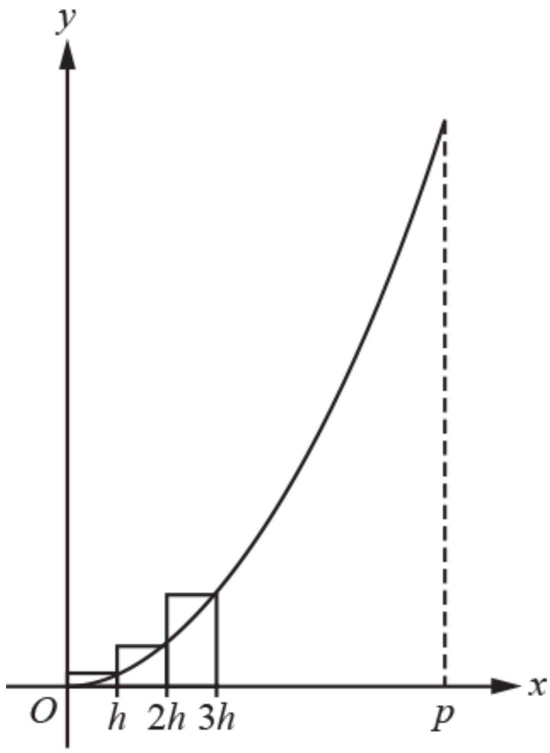
1. The diagram shows part of the curve $y = x^2$ for $0 \leq x \leq p$, where p is a constant.



The area A of the region enclosed by the curve, the x–axis and the line $x = p$ is given approximately by the sum S of the areas of n rectangles, each of width h, where h is small and $nh = p$. The first three such rectangles are shown in the diagram.

(a) Find an expression for S in terms of n and h. [2]

(b) Use the identity $\sum_{r=1}^{n} r^2 \equiv \frac{1}{6}n(n+1)(2n+1)$ to show that $S = \frac{1}{6}p(p+h)(2p+h)$. [3]

(c) Show how to use this result to find A in terms of p. [2]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance
1		a	Heights are h^2, $(2h)^2$, $(3h)^2$ etc $S = h\times h^2 + h\times(2h)^2 + h\times(3h)^2 + \ldots\ldots + h\times(nh)^2$	B1 (AO1.1a) B1 (AO1.1) [2]	soi or $h^3(1^2 + 2^2 + 3^2 + \ldots. + n^2)$ or $h^3 = \sum_{r=1}^{n} r^2$
		b	$S = h^3 \sum_{r=1}^{n} r^2$ $= \frac{h^3}{6} n(n+1)(2n+1)$ $= \frac{1}{6} nh(nh+h)(2nh+h)$ $= \frac{1}{6} p(p+h)(2p+h)$ AG	M1 (AO3.1a) A1 (AO2.1) A1 (AO1.1) [3]	oe
		c	$A = \lim_{h\to 0} S = \frac{1}{6} p \times p \times 2p$ $= \frac{p^3}{3}$	M1 (AO2.5) A1 (AO2.2a) [2]	Correctly expressed limit statement Answer without working: M0A0
			Total	7	