1. Solve the equations

i. $3^n = 1$,

[1]

ii. $t^{-3} = 64$,

[2]

iii. $\left(8p^6\right)^{\frac{1}{3}} = 8$.

[3]

2. Express each of the following in the form $a\sqrt{5}$, where a is an integer.

i. $4\sqrt{15} \times \sqrt{3}$

[2]

ii. $\dfrac{20}{\sqrt{5}}$

[1]

iii. $5^{\frac{3}{2}}$

[1]

3. Express each of the following in the form 5^k.

i. 25^4

[1]

ii. $\dfrac{1}{\sqrt[4]{5}}$

[2]

iii. $(5\sqrt{5})^3$

[2]

4. Express $\frac{8}{\sqrt{3}-1}$ in the form $a\sqrt{3}+b$, where a and b are integers.

[3]

5. Express the following in the form 2^p.

i. $(2^5 \div 2^7)^3$

[2]

ii. $5\times 4^{\frac{2}{3}}+3\times 16^{\frac{1}{3}}$

[3]

6. Express $\frac{3+\sqrt{20}}{3+\sqrt{5}}$ in the form $a+b\sqrt{5}$.

[4]

7. Simplify fully.

(a) $\sqrt{a^3}\times\sqrt{16a}$. [2]

(b) $(4b^6)^{\frac{5}{2}}$ [2]

8. Simplify

(a) $\frac{(3x)^3\times 2x^{-1}}{9x^2}$, [2]

(b) $(49x^{-4})^{-\frac{1}{2}}$. [2]

9. In this question you must show detailed reasoning.

Express each of the following in the form $a+b\sqrt{2}$, where a and b are integers.

(a) $\sqrt{3}(\sqrt{12}+\sqrt{54})$ [3]

(b) $\dfrac{6}{2+\sqrt{2}}$ [3]

10. Express $\frac{2+\sqrt{7}}{\sqrt{7}-2}$ in the form $a+b\sqrt{7}$, where a and b are rational numbers. [3]

11. In this question you must show detailed reasoning.

(a) Express $3^{\frac{7}{2}}$ in the form $a\sqrt{b}$, where a is an integer and b is a prime number. [2]

(b) Express $\frac{\sqrt{2}}{1-\sqrt{2}}$ in the form $c+d\sqrt{e}$, where c and d are integers and e is a prime number. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$n = 0$	B1	Allow 3^0 Examiner's Comments Only a tiny number of candidates failed to secure the mark for this simple recall of index notation; $\frac{1}{3}$ and 1 were occasionally seen.	
		ii	$\frac{1}{t^3} = 64 \text{ (or } 4^3\text{)}$	M1	or $t^3 = \frac{1}{64}$ or $64t^3 = 1$ or $\left(\frac{1}{t}\right)^3 = 64$ 4^{-1} is **A0** $t = \pm \frac{1}{4}$ is **A0**	Allow embedded
		ii	$t = \frac{1}{4}$	A1	Examiner's Comments Most candidates knew how to deal with the negative index and rewrote the equation as $\frac{1}{t^3} = 64$ or equivalent. Thereafter, however, a significant number could not proceed further, with –4 being a common wrong answer.	4^{-1} **www** alone implies **M1 A0**
		iii	$2p^2 = 8$	M1	or $8p^6 = 8^3$. Allow $2p^{\frac{6}{3}} = 8$ for **M1**	If not 512, evidence of 8 × 8 × 8 needed.
		iii	$p = 2$	A1	**www**	**SC** Spotted **B1** for 2, **B1** for –2, **B1** for justifying exactly 2 solutions
		iii	or $p = -2$	A1	**www**	**SC** $8p^2 = 8$, $p = \pm 1$ **B1**

					Examiner's Comments Although a large majority of candidates realised the need to find a cube root, many applied this only to the p^6 term and not to the 8. Those that were successful often omitted the negative solution thus surrendering the final mark.	
			Total	6		
2		i	$4\sqrt{45}$	M1	or $4\sqrt{5}\sqrt{3}\times\sqrt{3}$ (not just $4\sqrt{5\times3}\times\sqrt{3}$) or $\sqrt{720}$ or $\sqrt{240}\times\sqrt{3}$ or better Correctly simplified answer Examiner's Comments Most candidates were successful with this easy starter, but a significant minority found it quite challenging. Most earned at least a method mark for correct surd manipulation of some kind, but the accuracy was more of a problem, with some arithmetic errors and also conceptual ones such as $4\times3\sqrt{5}=7\sqrt{5}$	For method mark, makes a correct start to manipulate the expression i.e. at least combines two parts correctly or splits one part correctly
		i	$=12\sqrt{5}$	A1		
		ii	$\frac{20\sqrt{5}}{5}=4\sqrt{5}$	B1	**cao**, do not allow unsimplified, do not allow if clearly from wrong working Examiner's Comments Around 85% of candidates were successful in rationalising the denominator. Where no credit was earned, this was usually due to a lack of understanding rather than arithmetical error with a significant minority appearing not to know how to rationalise the given expression. Simply rewriting it as $20(\sqrt{5})^{-1}$ was quite common.	
		iii	$5\sqrt{5}$	B1	**cao www**, do not allow unsimplified, do not allow if clearly from wrong working	

					Examiner's Comments This was generally less successful than parts (i) and (ii), with just under three-quarters of candidates earning the mark. Many of those who did not give the answer in the required form did at least understand the notation as $\left(\sqrt{5}\right)^3$ was often seen, but then simplified to $3\sqrt{5}$.	
			Total	4		
3		i	5^8	B1	cao Examiner's Comments Almost all candidates secured this easy mark, but the error of $(5^2)^4 = 5^6$ was quite common.	
		ii	$5^{-\frac{1}{4}}$	M1	Fourth root $\equiv \frac{1}{4}$ soi cao www	
		ii		A1	Examiner's Comments Again, most candidates were able to gain both marks dealing with both the fractional and negative elements of the index.	
		iii	$5^{\frac{9}{2}}$	M1	$(5^{\frac{3}{2}})^3$ or $5^3 \times 5^{\frac{3}{2}}$ or other correct product of two simplified powers of 5 oe cao www	
		iii		A1	Examiner's Comments This part of the question proved rather more demanding with a minority of	

					candidates securing both marks. Those who recognised that $\sqrt{5} = 5^{\frac{1}{2}}$ were usually able to go on and complete the question successfully; those who tried to multiply out were less successful.	
			Total	**5**		
4			$\frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $\frac{8\sqrt{3}+8}{3-1}$ $4\sqrt{3}+4$	M1 A1 A1	Multiply top and bottom by $\sqrt{3}+1$ or $-\sqrt{3}-1$ evidence of multiplying out needed Either numerator or denominator correct Final answer **cao** **Examiner's Comments** Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all three marks. Errors were sometimes seen both in evaluating the numerator and the denominator, and occasionally in performing the final division	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{3}+b$ **A1** Either *a* or *b* correct **A1** Both correct
			Total	**3**		
5		i	$(2^{-2})^3$ or $2^{15} \div 2^{21}$	B1	Valid attempt to simplify	Correct use of either index law
		i	2^{-6}	B1	Correct answer. Accept $p = -6$.	$\left(\frac{1}{2}\right)^6$ oe is B1

						Examiner's Comments This simple index question was very well done, with around 90% securing both marks.
		ii	$5\times\left(2^2\right)^{\frac{2}{3}}+3\times\left(2^4\right)^{\frac{1}{3}}$	M1	Attempts to express both terms or a combined term as a power of 2	e.g. **Both** $4 = 2^2$ **and** $16 = 2^4$ **soi**
		ii	$=5\times2^{\frac{4}{3}}+3\times2^{\frac{4}{3}}\ or\ 10\times2^{\frac{1}{3}}+6\times2^{\frac{1}{3}}$	B1	Correctly obtains $2^{\frac{4}{3}}$ or $2^{\frac{1}{3}}$ for **either** term	If **M0** **Examiner's Comments** Although there were a significant number of excellent solutions, this question proved much more demanding than expected with less than a third of candidates securing all three marks. Many reached $5\times2^{\frac{4}{3}}+3\times2^{\frac{4}{3}}$ but then went no further, or even "simplified" this to $10^{\frac{4}{3}}+6^{\frac{4}{3}}$. Many of those who did obtain $8\times2^{\frac{4}{3}}$ appeared not to realise 8 was a power or 2. Some of those who did then made errors adding the powers, either through incorrect addition or multiplying so that $5\times2^3\times2^{\frac{4}{3}}=2^4$
		ii	$=8\times2^{\frac{4}{3}}$			
		ii	$=2^{\frac{13}{3}}$	A1	Correct final answer	
			Total	**5**		

6			$\frac{3+\sqrt{20}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}$	M1	Attempt to rationalise the denominator – must attempt to multiply	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a\sqrt{5}+b$
				B1	$\sqrt{20}=2\sqrt{5}$ soi	
			$\frac{-1+3\sqrt{5}}{9-5}$	A1	Either numerator or denominator correct and simplified to no more than two terms	**B1** $\sqrt{20}=2\sqrt{5}$ soi **A1** Either *a* or *b* correct **A1** Both correct
			$-\frac{1}{4}+\frac{3}{4}\sqrt{5}$	A1	Fully correct and fully simplified. Allow $\frac{-1+3\sqrt{5}}{4}$ order reversed etc. **Do not ISW** if then multiplied by 4 etc.	**Examiner's Comments** Most candidates recognised the need to rationalise the denominator and did so efficiently and accurately, with many candidates securing all four marks. The conversion from $\sqrt{20}$ to $2\sqrt{5}$ was usually well done; most errors that occurred were seen when expanding and simplifying the numerator. Some candidates obtained the correct answer but then, seemingly unsatisfied with the fractional values of *a* and *b* found, multiplied by 4; this lost the final mark.
			Total	**4**		
7		a	$\sqrt{16a^4}$ or $4\sqrt{a^4}$ or $a\sqrt{a}\times 4\sqrt{a}$ $=4a^2$	M1(AO1.1) A1(AO1.1) **[2]**	Any correct first step	

		b	$32b^{15}$	B2(AO1.1) (AO1.1) [2]	**B1** for 32 and **B1** for b^{15}	
			Total	**4**		
8		a	$\frac{(3x)^3 \times 2x^{-1}}{9x^2} = \frac{27x^3 \times 2x^{-1}}{9x^2}$ $= \frac{54x^2}{9x^2} = 6$	B1 (AO1.1) B1 (AO1.1) [2]	Correctly expands $(3x)^3$ as $27x^3$	
		b	$(49x^{-4})^{-\frac{1}{2}} = \frac{1}{7}x^2$	B1 (AO1.1) B1 (AO1.1) [2]	For $\frac{1}{7}$, independent of power of x For x^2	
			Total	**4**		
9		a	DR $\sqrt{36} + \sqrt{162}$ oe	M1(AO1.1a)	Attempt to expand	

			$\sqrt{6^2}+\sqrt{9^2\times 2}$ oe $=6+9\sqrt{2}$	A1(AO1.1) A1(AO1.1) [3]	bracket Obtain 6 Obtain $9\sqrt{2}$	 Must show sufficient method	
	b	DR $\frac{6(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}=\frac{12-6\sqrt{2}}{2}=6-3\sqrt{2}$	M1(AO1.1a) A1(AO1.1) A1(AO1.1) [3]	Multiply numerator and denominator by $2-\sqrt{2}$ Either numerator or denominator correct Fully correct expression	 Must be simplified Must show sufficient method		
		Total	6				
10		$\frac{2+\sqrt{7}}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$ $\frac{11+4\sqrt{7}}{7-4}$	M1 A1	Attempt to rationalise the denominator – must attempt to multiply. (May use $-\sqrt{7}-2$) Either numerator or denominator correct and simplified to no more than two terms	Alternative: **M1** Correct method to solve simultaneous equations formed from equating expression to $a+b\sqrt{7}$		

			$\frac{11}{3}+\frac{4\sqrt{7}}{3}$	A1 [3]	Fully correct and simplified. Allow $\frac{11+4\sqrt{7}}{3}$, terms in any order **Do not ISW** if then incorrect	**A1** Either *a* or *b* correct **A1** Both correct Do not allow $\frac{-11-4\sqrt{7}}{-3}$ for last A1.	
			Examiner's Comments Almost all candidates recognised the need to rationalise the denominator and many did so efficiently and accurately, securing all three marks. Arithmetical errors were sometimes seen both in evaluating the numerator and the denominator, more commonly in the latter e.g. by not squaring the 2 and ending up with a denominator of 5. A small but noticeable number obtained the correct answer but then proceeded to multiply by 3 to give integers for a and b; this error was not condoned. Likewise, a small number of those who chose to use $-\sqrt{7}-2$ as their multiplier, left their final answer with a negative denominator which was not deemed appropriate at AS level.				
			Total	**3**			
11	a		**DR** $(\sqrt{3})^7$ or $\sqrt{3^7}$ or $3^3\times\sqrt{3}$ or $3\sqrt{243}$	**M1** **(AO1.1a)**	or any **correct intermediate step**	**If this step is not seen, M0A0**	

			$27\sqrt{3}$	A1 (AO1.1) [2]	using √ or $3^3 \times 3^{\frac{1}{2}}$ or $a = 27$, $b = 3$		

Examiner's Comments

Many candidates answered this question correctly. A few made a correct first step, for example $\sqrt{3^7}$, but could not continue correctly. Some candidates gave the correct answer with no working or with incorrect working. These scored no marks.

		b	DR $\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$ $= \frac{\sqrt{2}+2}{1-2}$ or $\frac{\sqrt{2}+2}{-1}$ or $\frac{\sqrt{2}+2}{1+\sqrt{2}-\sqrt{2}-2}$ $= -2 - \sqrt{2}$ ISW	M1 (AO1.1a) A1 (AO1.1) A1 (AO1.1) [3]	A1 for correct num OR denom or $-2 + (-1\sqrt{2})$ or $c = -2$, $d = -1$ and $e = 2$	If this step is not seen, M0A0 Allow $-(2 + \sqrt{2})$	

Examiner's Comments

Many candidates answered this question correctly. A few made a correct

					first step, multiplying numerator and denominator by $1 + \sqrt{2}$, but made a subsequent error. Some candidates gave the correct answer with no working or with incorrect working. These scored no marks.	
			Total	5		