1. i. The first three terms of an arithmetic progression are $2x$, $x + 4$ and $2x - 7$ respectively. Find the value of x.

[3]

ii. The first three terms of another sequence are also $2x$, $x + 4$ and $2x - 7$ respectively.

a. Verify that when $x = 8$ the terms form a geometric progression and find the sum to infinity in this case.

[4]

b. Find the other possible value of x that also gives a geometric progression.

[4]

2. Sarah is carrying out a series of experiments which involve using increasing amounts of a chemical. In the first experiment she uses 6 g of the chemical and in the second experiment she uses 7.8 g of the chemical.

i. Given that the amounts of the chemical used form an arithmetic progression, find the total amount of chemical used in the first 30 experiments.

[3]

ii. Instead it is given that the amounts of the chemical used form a geometric progression. Sarah has a total of 1800 g of the chemical available. Show that N, the greatest number of experiments possible, satisfies the inequality

$$1.3^N \leqslant 91,$$

and use logarithms to calculate the value of N.

[6]

3. a. The first term of a geometric progression is 50 and the common ratio is 0.8. Use logarithms to find the smallest value of k such that the value of the kth term is less than 0.15.

[4]

b. In a different geometric progression, the second term is –3 and the sum to infinity is 4. Show that there is only one possible value of the common ratio and hence find the first term.

[8]

4. A geometric progression has first term 3 and second term – 6.

i. State the value of the common ratio.

[1]

ii. Find the value of the eleventh term.

[2]

iii. Find the sum of the first twenty terms.

[2]

5. An arithmetic progression $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 5$ and $u_{n+1} = u_n + 1.5$ for $n \geqslant 1$.

i. Given that $u_k = 140$, find the value of k.

[3]

A geometric progression $w_1, w_2, w_3, \ldots$ is defined by $w_n = 120 \times (0.9)^{n-1}$ for $n \geqslant 1$.

ii. Find the sum of the first 16 terms of this geometric progression, giving your answer correct to 3 significant figures.

[2]

iii. Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{N} u_n > \sum_{n=1}^{\infty} w_n$.

[6]

6. Business A made a £5000 profit during its first year.
In each subsequent year, the profit increased by £1500 so that the profit was £6500 during the second year, £8000 during the third year and so on.

Business B made a £5000 profit during its first year.
In each subsequent year, the profit was 90% of the previous year's profit.

(a) Find an expression for the total profit made by business A during the first n years. Give your answer in its simplest form. [2]

(b) Find an expression for the total profit made by business B during the first n years. Give your answer in its simplest form. [3]

(c) Find how many years it will take for the total profit of business A to reach £385 000. [3]

(d) Comment on the profits made by each business in the long term. [2]

7. **In this question you must show detailed reasoning.**
It is given that the geometric series

$$1+\frac{5}{3x-4}+\left(\frac{5}{3x-4}\right)^2+\left(\frac{5}{3x-4}\right)^3+\ldots$$

is convergent.

(a) Find the set of possible values of x, giving your answer in set notation. [5]

(b) Given that the sum to infinity of the series is $\frac{2}{3}$, find the value of x. [3]

8(i). The seventh term of a geometric progression is equal to twice the fifth term. The sum of the first seven terms is 254 and the terms are all positive. Find the first term, showing that it can be written in the form $p+q\sqrt{r}$ where p, q and r are integers. [6]

(ii). The seventh term of a geometric progression is equal to twice the fifth term. The sum of the first seven terms is 254 and the terms are all positive. Find the first term, showing that it can be written in the form $p+q\sqrt{r}$ where p, q and r are integers. [6]

9. The first term of a geometric progression is 12 and the second term is 9.

(a) Find the fifth term. [3]

The sum of the first N terms is denoted by S_N and the sum to infinity is denoted by S_∞. It is given that the difference between S_∞ and S_N is at most 0.0096.

(b) Show that $\left(\frac{3}{4}\right)^N \leqslant 0.0002$ [3]

(c) Use logarithms to find the smallest possible value of N. [2]

10. **In this question you must show detailed reasoning.**

The nth term of a geometric progression is denoted by g_n and the nth term of an arithmetic progression is denoted by a_n. It is given that $g_1 = a_1 = 1+\sqrt{5}$, $g_3 = a_2$ and $g_4 + a_3 = 0$.

Given also that the geometric progression is convergent, show that its sum to infinity is $4+2\sqrt{5}$. [12]

11. The table shows information about three geometric series. The three geometric series have different common ratios.

	First term	Common ratio	Number of terms	Last term
Series 1	1	2	n_1	1024

Series 2	1	r_2	n_2	1024
Series 3	1	r_3	n_3	1024

(a) Find n_1. [2]

(b) Given that r_2 is an integer less than 10, find the value of r_2 and the value of n_2. [2]

(c) Given that r_3 is **not** an integer, find a possible value for the sum of all the terms in Series 3. [4]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$(x + 4) - 2x = (2x - 7) - (x + 4)$	M1	Attempt to eliminate d to obtain equation in x only	Equate two expressions for d, both in terms of x Could use $a + (n - 1)d$ twice, and then eliminate d Could use $u_1 + u_2 + u_3 = S_3$ or $u_2 = ½ (u_1 + u_3)$
		i	**OR**			
		i	$2x + d = x + 4 \quad 2x + 2d = 2x - 7$	A1	Obtain correct equation in just x	Allow unsimplified equation A0 if brackets missing unless implied by subsequent working or final answer
		i	$2x = 15$ $x = 7.5$	A1	Obtain $x = 7.5$	Any equivalent form Allow from no working or T&I
		i				**Alt method:** B1 – state, or imply, $2x + 2d = 2x - 7$, to obtain $d = -3.5$ M1 – attempt to find x from second equation in x and d A1 – obtain $x = 7.5$
		i				<u>Examiner's Comments</u> Many candidates were successful in this part of the question, with the most popular approach being to first find d = -3.5 and then use a second equation to find x. This was usually successful, although sign errors proved a pitfall for some. However, a number of candidates made no further progress beyond finding d, often because they did not consider a third equation. The other common method was to find two expressions for

						d by considering the difference of consecutive terms which could then be equated and solved. This was an elegant and concise method, but a lack of brackets resulted in errors being made. Other, more creative, solutions were also seen including adding the sum of the three terms and equating this to an expression for S_3.
		ii	terms are 16, 12, 9 $^{12}/_{16} = 0.75$, $^{9}/_{12} = 0.75$	B1	List 3 terms	Ignore any additional terms given Must show two values of 0.75, or unsimplified fractions Must state, or imply, that ratio has been checked twice, using both pairs of consecutive terms No need to show actual division of terms to justify 0.75, so allow eg arrows from one term to the next with 'x0.75' **SR B2** if 16, 12, 9 never stated explicitly in a list but are soi in a convincing method for $r = 0.75$ twice
		ii	common ratio of 0.75 so GP	B1	Convincing explanation of why it is a GP	
		ii	$S_\infty = {}^{16}/_{1-0.75} = 64$	M1	Attempt use of $^{a}/_{1-r}$	Must be correct formula Could be implied by method Allow if used with their incorrect a and / or r Allow if using $a = 8$, even if 16 given correctly in list
		ii		A1	Obtain 64	A0 if given as 'approximately 64'
		ii				<u>**Examiner's Comments**</u> Virtually all of the candidates gained the first mark for stating the three relevant terms, and most also gained the final two marks for finding the sum to infinity, though a few used $\frac{4}{3}$ as their ratio. It was the second mark that proved to be the most challenging. Candidates had been asked to verify

						that the terms did form a geometric progression, and were expected to provide a convincing proof that considered the ratio between two pairs of terms, or an equivalent justification. Whilst some candidates did provide this explanation, far too many assumed that it was a geometric progression and simply found the ratio from a single pair of terms.
		iii	$^{(2x-7)}/_{(x+4)} = {}^{(x+4)}/_{2x}$ $4x^2 - 14x = x^2 + 8x + 16$	M1*	Attempt to eliminate r to obtain equation in x only	Equate two expressions for r, both in terms of x Could use ar^{n-1} twice, and then eliminate r from simultaneous eqns
		iii	OR			
			$2xr = x + 4$ $2xr^2 = 2x - 7$			Allow $6x^2 - 44x - 32 = 0$ Allow $3x^3 - 22x^2 - 16x = 0$, or a multiple of this
		iii	$3x^2 - 22x - 16 = 0$ $(3x + 2)(x - 8) = 0$ $x = {}^{-2}/_3$, $x = 8$	A1	Obtain $3x^2 - 22x - 16 = 0$	Allow any equivalent form, as long as no brackets and like terms have been combined Condone no = 0, as long as implied by subsequent work
		iii		M1d*	Attempt to solve quadratic	Dependent on first M1 for valid method to eliminate r See guidance sheet for acceptable methods
		iii		A1	Obtain $x = {}^{-2}/_3$	Allow recurring decimal, but not rounded or truncated Condone $x = 8$ also given Allow from no working or T&I **<u>Examiner's Comments</u>**
		iii				This proved to be a challenging question for many candidates. Whilst most were able to make some attempt at it, it was often not enough to gain even the first mark. The most efficient solution was to equate two algebraic expressions for the ratio,

						and then rearrange them to get a quadratic which could then be solved. Some candidates were able to provide a concise and elegant solution in this way. Some candidates did embark on this method, but then attempted to first simplify their fractions which invariably went wrong. Others started with the generic equations for the *n*th term of a geometric progression so that when they eliminated *r* their equation involved the square or square root of a rational expression.
			Total	11		
2		i	$S_{30} = {}^{30}/_{2}\ (2 \times 6 + 29 \times 1.8)$	M1	Use $d = 1.8$ in AP formula	Could be attempting S_{30} or u_{30} Formula must be recognisable, though not necessarily fully correct, so allow M1 for eg 15(6 + 29 × 1.8), 15(12 + 14 × 1.8) or even 15(12 + 19 × 1.8) Must have $d = 1.8$ (not 1.3), $n = 30$ and $a = 6$
		i		A1	Correct unsimplified S_{30}	Formula must now be fully correct Allow for any unsimplified correct expression If using $½n(a + l)$ then l must be correct when substituted
		i	= 963	A1	Obtain 963 **Examiner's Comments** The vast majority of candidates were able to gain full marks on this question. A few gained just one mark by finding the 30th term rather than the required sum of the first 30 terms.	Units not required
		ii	$r = {}^{7.8}/_{6} = 1.3$	M1	Use $r = 1.3$ in GP formula	Could be attempting S_N, u_N or even S_∞ Formula must be recognisable, though not necessarily fully correct Must have $r = 1.3$ (not 1.8) and $a = 6$

		ii	$\frac{6(1-1.3^N)}{1-1.3} \le 1800$	A1	Correct unsimplified S_N	Formula must now be fully correct Allow for any unsimplified correct expression
		ii	$1 - 1.3^N \ge -90$	M1	Link sum of GP to 1800 and attempt to rearrange to $1.3^N \le k$	Must have used correct formula for S_N of GP Allow =, ≥ or ≤ Allow slips when rearranging, including with indices, so rearranging to $7.8^N \le k$ could get M1
		ii	$1.3^N \le 91$ **AG**	A1	Obtain given inequality	Must have correct inequality signs throughout Correct working only, so A0 if 6×1.3^N becomes 7.8^N, even if subsequently corrected
		ii	$N \log 1.3 \le \log 91$	M1	Introduce logs throughout and attempt to solve equation / inequality	Must be using $1.3^N \le 91$, $1.3^N = 91$ or $1.3^N \ge 91$ This M1 (and then A1) is independent of previous marks Must get as far as attempting N M0 if no evidence of use of logarithms M0 if invalid use of logarithms in attempt to solve
		ii	$N \le 17.19$ hence $N = 17$	A1	Conclude $N = 17$ **Examiner's Comments** Most candidates were able to gain some credit on this question, but only a few scored full marks. The sum of N terms was usually quoted correctly and candidates could then make an attempt to rearrange it. A common error was the failure to reverse the direction of the inequality sign when multiplying or dividing by a negative number. Others started with an equality and then tried to justify the inequality sign at the end, which was not sufficient to gain the accuracy mark. Some candidates were unable to manipulate the indices, with $6 \times 1.3^N = 7.8^N$ being a fairly common error. When solving the given inequality, most candidates could use logarithms correctly to get a decimal answer, but did not then appreciate that the context of the question meant that N had to be an integer value. Some	Must come from solving $1.3^N \le 91$ or $1.3^N = 91$ (ie not incorrect inequality sign) Answer must be integer value Answer must be equality, so A0 for $N \le 17$ **SR** Candidates who use numerical value(s) for N can get M1 Use $r = 1.3$ in a recognisable GP formula (M0 if N is not an integer value) A1 Obtain a correct unsimplified S_N Candidates who solve $1.3^N \le 91$ and then use a value associated with their N (usually 17 and / or 18) in a GP formula will be eligible for the M1A1 for solving the inequality and also the M1A1 in the **SR** above

					candidates simply solved the given inequality and made no attempt to show where it had come from. Others solved the inequality and then tested their solution in the sum formula to justify it, without appreciating that they had not fully answered the question.	
			Total	9		
3			$ar = -3,\ \frac{a}{1-r} = 4$	B1	State $ar = -3$	Any correct statement, including $a \times r^{(2-1)} = -3$ etc soi
				B1	State $\frac{a}{1-r} = 4$	Any correct statement, not involving r^∞ (unless it becomes 0) soi
			$-\frac{3}{r} = 4(1-r)$	M1*	Attempt to eliminate either a or r	Using valid algebra so M0 for eg $a = -3 - r$ Must be using ar^k and $\pm a/(\pm 1 \pm r)$ Award as soon as equation in one variable is seen
			$4r^2 - 4r - 3\ (= 0)$ / $a^2 - 4a - 12\ (= 0)$	A1	Obtain correct simplified quadratic	Any correct quadratic not involving fractions or brackets ie $4r^2 = 4r + 3$ gets A1
			$(2r - 3)(2r + 1) = 0$ / $(a - 6)(a + 2) = 0$	M1d*	Attempt to solve 3 term quadratic	See Appendix 1 for acceptable methods
			$r = -\frac{1}{2}$	M1**	Identify $r = -\frac{1}{2}$ as only ratio with a minimally acceptable reason	M0 if no, or incorrect, reason given Must have correct quadratic, correct factorisation and correct roots (if stated) If $r = -\frac{1}{2}$ is not explicitly identified then allow M1 when they use only this value to find a (or later eliminate the other value) Could accept $r = -\frac{1}{2}$ as $\|r\| < 1$ or reject $r = \frac{3}{2}$ as > 1 Could reject $a = -2$ as S_∞ is positive Could refer to convergent / divergent series

			$a = 6$	A1	Obtain $a = 6$ only Convincing reason for $r = -\frac{1}{2}$ as the only possible ratio **Examiner's Comments** Candidates were able to make a good start to this question, but only the most able could make progress beyond the first five marks. The majority could attempt the two relevant equations and then eliminate one of the variables, usually *a*. Substituting the equation for the sum to infinity into the equation for the second term usually resulted in the correct quadratic, whereas the fraction involved in doing the substitution the other way around caused problems for some. Nevertheless, many candidates did obtain the correct quadratic which they could then attempt to solve. Candidates then had to select the correct common ratio and also provide some reasoning for this choice. No credit was available for picking $r = -0.5$ with no, or an incorrect, reason. To gain full marks, the reasoning for the selection of $r = -0.5$ had to be convincing and fully complete. It was not sufficient to reject $r = 1.5$ without also explaining why the other was being accepted.	If solving quadratic in *a*, then both values of *a* may be seen initially - A1 can only be awarded when $a = 6$ is given as only solution
			for sum to infinity $-1 < r < 1$	A1d**		Must refer to $\lvert r \rvert < 1$ or $-1 < r < 1$ oe in words A0 if additional incorrect statement No credit for answer only unless both *r* first found
			Total	**8**		
4		i	$r = -2$	B1	State -2 **Examiner's Comments** This question was a straightforward start to the paper,	Not $^{-6}/_{3}$ as final answer No need to see $r = \ldots$, and also condone other variables

					and nearly all of the candidates were able to state the correct value for the common ratio. The most common incorrect answer was r = – 9, indicating a confusion between the definitions of arithmetic and geometric progressions.	
		ii	$3 \times (-2)^{10} = 3072$	M1	Attempt u_{11}	Must be using correct formula, with $a = 3$ and $r = -2$, or their r from (i) Allow M1 for 3×-2^{10} Using $r = 2$ is M0, unless this was their value in (i) Allow M1 for listing terms as far as u_{11}
		ii		A1	Obtain 3072	CWO Allow A1 BOD for $3 \times -2^{10} = 3072$ If listing terms, then need to indicate that 3072 is the required value
		ii			<u>Examiner's Comments</u> Most candidates knew how to find the eleventh term of the GP, but many were unable to evaluate correctly the expression as it included a negative number. The most successful candidates included brackets in their expression, and then used these in their evaluation. Some candidates included brackets but ignored them in the evaluation, and too many candidates wrote the expression as 3×-2^{10} and duly evaluated this as −3072. At this level, candidates should both be able to use their calculator proficiently and should also consider whether their answer is sensible; they should be aware that a negative number to an even power should give a positive answer.	
		iii	$\frac{3(1-(-2)^{20})}{1-(-2)} = -1048575$	M1	Attempt S_{20}	Must be using correct formula, with $a = 3$ and $r = -2$, or their r from (i) Allow M1 for correct formula, but with no brackets around the −2 Allow M1 for attempting to sum first 20 terms

		iii		A1	Obtain −1048575	Allow M1 for $\frac{3(1+2^{20})}{1+2}$ as long as correct general formula is also seen Could also come from manually summing terms NB $\frac{3(1--2^{20})}{1--2}$ gives 1048577
		iii			**Examiner's Comments** Once again, most candidates were able to quote a correct expression for the sum of the first twenty terms, but were unable to correctly evaluate this. A few candidates gave their answer to three significant figures, not appreciating that the instruction on the front of the question paper refers to non-exact numerical answers.	
			Total	5		
5		i	$u_k = 5 + 1.5(k - 1)$	M1*	Attempt *n*th term of an AP, using $a = 5$ and $d = 1.5$	Must be using correct formula, so M0 for 5 + 1.5*k* Allow if in terms of *n* not *k* Could attempt an *n*th term definition, giving 1.5*k* + 3.5
		i	$5 + 1.5(k - 1) = 140$ $k = 91$	M1d*	Equate to 140 and attempt to solve for *k*	Must be valid solution attempt, and go as far as an attempt at *k* Allow equiv informal methods
		i		A1	Obtain 91	Answer only gains full credit **Examiner's Comments**

						This proved to be a straightforward question for many candidates, and the majority gained full credit. Most candidates used the formula for the nth term of an arithmetic progression and another effective method was to generate an nth term expression for the sequence. Informal methods were rarely correct, and the other common error was to use the nth term as 5 + 1.5n or even n + 1.5.
		ii	$S_{16} = \frac{120(1-0.9^{16})}{1-0.9}$ = 978	M1	Attempt to find the sum of 16 terms of GP, with a = 120, r = 0.9	Must be using correct formula
		ii		A1	Obtain 978, or better	If > 3sf, allow answer rounding to 977.6 with no errors seen Answer only, or listing and summing 16 terms, gains full credit **Examiner's Comments** The majority of candidates were equally successful here, with solutions being mostly fully correct. Despite being told that it was a geometric progression, many candidates did not recognise w_n as being of the form a × r^{n-1} and instead generated the first few terms of the sequence to find the values of the first term and the common ratio, not always correctly.
		iii	$\frac{1}{2}N(10 + (N-1) \text{ x } 1.5) > \frac{120}{1-0.9}$	B1	Correct sum to infinity stated	Could be 1200 or unsimplified expression
		iii	$N(1.5N + 8.5) > 2400$ $3N^2 + 17N - 4800 > 0$ $N = 38$	B1	Correct S_N stated	Any correct expression, including unsimplified
		iii		M1*	Link S_N of AP to S_∞ of GP and attempt to rearrange	Must be recognisable attempt at S_N of AP and S_∞ of GP, though not necessarily fully correct

						Allow any (in)equality sign, including < Must rearrange to a three term quadratic, not involving brackets
		iii		A1	Obtain correct 3 term quadratic	aef - not necessary to have all algebraic terms on the same side of the (in)equation Allow any (in)equality sign
		iii		M1d*	Attempt to solve quadratic	See additional guidance for acceptable methods May never consider the negative root M1 could be implied by sight of 37.3, as long as from correct quadratic
		iii		A1	Obtain $N = 38$ (must be equality)	A0 for $N \geq 38$ or equiv in words eg '*N* is at least 38' Allow A1 if 38 follows =, > or ≥ being used but A0 if 38 follows < or ≤ being used A0 if second value of *N* given in final answer Must be from an algebraic method - at least as far as obtaining the correct quadratic **Examiner's Comments** The majority of candidates could identify that the sum to infinity was required, and correctly state this. There was then some uncertainty as to what was required on the left-hand side, with both the sum of the geometric progression and the *n*th term of the arithmetic progression being common errors. However many candidates could make a reasonable attempt at both of the summations, but there were a surprising number of errors when attempting to simplify their inequality. The most common errors included only multiplying one side by 2 in an attempt to remove the fraction or incorrect expansion of brackets. Candidates then had to solve the quadratic with

						both completing the square and use of the quadratic formula being seen, though the latter was by far the most common. A few candidates clearly anticipated that the quadratic would factorise and gave up when they realised that this was not the case. Some candidates, with an incorrect quadratic equation, simply wrote down two solutions with no method shown. In these circumstances, Examiners cannot speculate as to what method may have been used and no credit can be awarded. To gain full credit in this question, candidates had to appreciate that *N* had to be a positive integer and hence discard their negative root and round up their positive root. Some candidates spoilt an otherwise correct solution by failing to do so.
			Total	11		
6		a	Identify AP with $a = 5000$ and $d = 1500$ $\frac{n}{2}(2(5000)+(n-1)1500)$ $= n(750n+4250)$	M1(AO3.1b) A1(AO1.1) [2]	Identification recognised by an attempt at the sum formula or *n*th term formula for an AP Or $750n^2 + 4250n$	
		b	$\frac{5000(1-(0.9)^n)}{1-0.9}$	M1(AO3.1b) A1(AO3.1b) A1(AO1.1)	Identification recognised by an attempt at the sum formula with *n*, *n* – 1 or *n* + 1 or with a positive sign in numerator Obtain correct unsimplified	

			Obtain $50000(1 - (0.9)^n)$	[3]	sum Or $50000 - 50000(0.9)^n$	
		c	Obtain $750n^2 + 4250n - 385000 = 0$ $n = 20$ or $n = -\frac{77}{3}$ State 20 years	M1(AO3.1b) A1(AO1.1) A1(AO3.4) [3]	Equate to 385 000 and solve a 3 term quadratic = 0 BC both required Allow different methods for solving the quadratic **OR** **M1** For writing down and summing the total profit for at least the first four years (may be implied BC) **A1** For finding that the total is equal to 385 000 for $n = 20$ **A1** state 20 years	
		d	Firm A's profits continue to grow Firm B's profits eventually plateau at £50 000 as $(0.9)n$ tends to 0 with large enough n	E1(AO3.4) E1(AO3.2a) [2]	Some mention is required about the effect of $(0.9)^n$	
			Total	10		

7		a	DR State $-1 < \frac{5}{3x-4}$ and/or $\frac{5}{3x-4} < 1$ Multiply by $(3x-4)^2$ and attempt to simplify Obtain either $9x^2 - 9x - 4 > 0$ or $3x^2 - 13x + 12 > 0$ Obtain critical values $\frac{4}{3}, -\frac{1}{3}$ or $\frac{4}{3}, 3$ $\{x : x < -\frac{1}{3}\} \cup \{x : x > 3\}$ Alternative method State $\left\|\frac{5}{3x-4}\right\| < 1$ Rewrite in the form $\| 3x - 4 \| > 5$ Obtain either $3x - 4 > 5$ or $3x - 4 < -5$ Obtain both critical values 3 and $-\frac{1}{3}$ $\{x : x < -\frac{1}{3}\} \cup \{x : x > 3\}$	B1(AO1.2) M1(AO1.1a) A1(AO1.1) A1(AO1.1) A1(AO2.5) B1 M1 A1 A1 A1 [5]	BC oe, eg $3x^2 - 8x - 3 > 0$	
		b	DR $S_\infty = \frac{1}{1 - \frac{5}{3x-4}}$	B1(AO1.1) M1(AO1.1)	Correct use of sum to infinity	

			$\frac{3x-4}{3x-9}=\frac{2}{3}\Rightarrow x=\ldots$ $x=-2$	A1(AO1.1) [3]	formula Equate to $\frac{2}{3}$ and attempt to solve for x	
			Total	8		
8	i		$r^2=2$ hence $r=\sqrt{2}$ $\frac{a(1-\sqrt{2}^7)}{1-\sqrt{2}}=254$ $a=\frac{254(1-\sqrt{2})}{1-8\sqrt{2}}$ $a=\frac{254(1-\sqrt{2})(1+8\sqrt{2})}{(1-8\sqrt{2})(1+8\sqrt{2})}$ $a=\frac{254(-15+7\sqrt{2})}{-127}$	B1 M1 A1	State $r=\sqrt{2}$ www Attempt $S_7=254$ Rearrange to obtain correct numerical expression for	B0 if from $ar^7=2ar^5$ (but then allow all of the remaining marks) Allow decimal value (1.41) Allow B1 for $r=\pm\sqrt{2}$ Must be correct formula, using their numerical r, which could be exact or a decimal value Must also equate to 254 Must be in an exact form,

			$a = 30 - 14\sqrt{2}$	B1 M1 A1 [6]	*a* aef Use $(\sqrt{2})^7 = 8\sqrt{2}$ soi Attempt to rationalise denominator Obtain correct value in surd form	but could involve $(\sqrt{2})^7$ or $\sqrt{128}$ rather than $8\sqrt{2}$ Ignore second value for *a* from using $r = -\sqrt{2}$ Equation may no longer be fully correct Must be using $r = \sqrt{2}$ only Must be explicit evidence of rationalizing Could use $(1 + (\sqrt{2})^7)$ or $(1 + \sqrt{128})$ Allow M1 if denominator now incorrect, as long as of form $\pm(1 - k\sqrt{2})$ or equiv M0 if rationalising $1 - \sqrt{2}$ only (ie before making *a* the subject)	

						Allow any exact answer in form $p + q\sqrt{r}$ A0 if additional answer from using $r = -\sqrt{2}$ A0 if final answer results from subsequent attempt to simplify eg $a = 15 - 7\sqrt{2}$ (ie no ISW) Could use variables other than a and r If $a = 30 - 14\sqrt{2}$ obtained, but no evidence of dealing with $(\sqrt{2})^7$ or rationalising denominator then maximum of B1 M1 A1 ie 3 marks (as the given form has not been 'shown')	

					Examiner's Comments This proved to be the most challenging question on the paper, and only the most able candidates were able to provide fully correct and detailed solutions. Most candidates were able to set up the correct initial equation of $ar^7 = 2ar^5$, but many struggled to find a numerical solution to this equation. Despite it being given in the formula book, some students could not quote the correct sum formula with an incorrect index of *n* – 1 being the common error. However, many students could indeed find the correct ratio, substitute into the sum formula and rearrange to find an expressions for *a*. The more astute candidates were able to make further progress, by simplifying (√2)7 and/or rationalising the denominator, but fully correct solutions were in the minority.	
	ii		$r^2 = 2$ hence $r = \sqrt{2}$ $\frac{a(1-\sqrt{2}^7)}{1-\sqrt{2}} = 254$ $a = \frac{254(1-\sqrt{2})}{1-8\sqrt{2}}$	B1 M1 A1	State $r = \sqrt{2}$ www — B0 if from $ar^7 = 2ar^5$ (but then allow all of the remaining marks) Allow decimal value (1.41) Allow B1 for $r = \pm\sqrt{2}$ Attempt $S_7 = 254$ — Must be correct formula, using their numerical r, which could be exact or a	

			$a = \frac{254(1-\sqrt{2})(1+8\sqrt{2})}{(1-8\sqrt{2})(1+8\sqrt{2})}$ $a = \frac{254(-15+7\sqrt{2})}{-127}$ $a = 30 - 14\sqrt{2}$	B1 M1 A1 [6]	Rearrange to obtain correct numerical expression for *a* aef Use $(\sqrt{2})^7 = 8\sqrt{2}$ soi Attempt to rationalise denominator	decimal value Must also equate to 254 Must be in an exact form, but could involve $(\sqrt{2})^7$ or $\sqrt{128}$ rather than $8\sqrt{2}$ Ignore second value for *a* from using $r = -\sqrt{2}$ Equation may no longer be fully correct Must be using $r = \sqrt{2}$ only Must be explicit evidence of rationalizing Could use $(1 + (\sqrt{2})^7)$ or $(1 + \sqrt{128})$ Allow M1 if denominator now incorrect, as long as of form $\pm(1 - k\sqrt{2})$ or equiv

					Obtain correct value in surd form	M0 if rationalising 1 – √2 only (ie before making *a* the subject) Allow any exact answer in form $p + q\sqrt{r}$ A0 if additional answer from using $r = -\sqrt{2}$ A0 if final answer results from subsequent attempt to simplify eg $a = 15 - 7\sqrt{2}$ (ie no ISW) Could use variables other than *a* and *r* If $a = 30 - 14\sqrt{2}$ obtained, but no evidence of dealing with $(\sqrt{2})^7$ or rationalising denominator then

					<table><tr><td></td><td>maximum of B1 M1 A1 ie 3 marks (as the given form has not been 'shown')</td></tr></table> **Examiner's Comments** This proved to be the most challenging question on the paper, and only the most able candidates were able to provide fully correct and detailed solutions. Most candidates were able to set up the correct initial equation of $ar^7 = 2ar^5$, but many struggled to find a numerical solution to this equation. Despite it being given in the formula book, some students could not quote the correct sum formula with an incorrect index of *n* – 1 being the common error. However, many students could indeed find the correct ratio, substitute into the sum formula and rearrange to find an expressions for *a*. The more astute candidates were able to make further progress, by simplifying (√2)7 and/or rationalising the denominator, but fully correct solutions were in the minority.	
			Total	12		
9		a	$r = \frac{3}{4}$ $u_5 = 12r^4$ $u_5 = \frac{243}{64}$	B1(AO1.1a) M1(AO1.2) A1(AO1.1) [3]	<table><tr><td>Applying their *r* in the correct formula for u_5 with a = 12</td><td>Or repeated use of their *r*</td></tr></table>	

					<table><tr><td></td><td>3.796 875</td></tr></table>	
		b	$S_\infty = \dfrac{12}{1-\frac{3}{4}}$ or $S_N = \dfrac{12\left(1-\left(\frac{3}{4}\right)^N\right)}{1-\frac{3}{4}}$ $\dfrac{12}{1-\frac{3}{4}} - \dfrac{12\left(1-\left(\frac{3}{4}\right)^N\right)}{1-\frac{3}{4}} \le 0.0096$ $48 - 48\left(1-\left(\frac{3}{4}\right)^N\right) \le 0.0096 \Rightarrow \left(\frac{3}{4}\right)^N \le 0.0002$	B1ft(AO1.1) M1(AO2.1) A1(AO2.2a) [3]	<table><tr><td>Correctly applying formula for S_∞ or S_N with their value of r Attempt at $S_\infty - S_N$ compared with 0.0096 (dependent on previous B1) **AG** – completely correct working</td><td>Accept any inequality or equals for this mark</td></tr></table>	
		c	$N\log\left(\frac{3}{4}\right) \le \log(0.0002) \Rightarrow N \ge \ldots$ $N \ge 29.6\ldots \Rightarrow N = 30$	M1(AO1.1) A1(AO2.2a) [2]	<table><tr><td>Take logs and attempt to make N the subject (accept any inequality or equals</td><td>$N = \log_{\frac{3}{4}}(0.0002)$</td></tr></table>	

					for this mark)		
			Total	8		
10			DR $a + d = ar^2$ $a + 2d + ar^3 = 0$ $a + 2(ar^2 - a) + ar^3 = 0$ $r^3 + 2r^2 - 1 = 0$ f(−1) = −1 + 2 − 1 = 0 hence (r + 1) is a factor	B1 (AO 3.1a) B1 (AO 3.1a) M1 (AO 2.1) A1 (AO 2.1) B1 (AO 2.4) M1 (AO 3.1a)	Correct equation for $a_2 = g_3$ Correct equation for $a_3 + g_4 = 0$ Eliminate d	Allow a, a_1, g_1 or $1+\sqrt{5}$ and may be different in each term eg $a_1 + d = g_1r^2$ Allow a, a_1, g_1 or $1+\sqrt{5}$ and may be different in each term eg $a_1 + 2d + g_1r^3 = 0$ Could be a,	

			Answer	Marks	Guidance		
			$(r+1)(r^2+r-1)=0$	A1 (AO 1.1a)	Obtain correct cubic	a_1, g_1 or $1+\sqrt{5}$ but must now be consistent throughout (soi)	
			$r=-1,\ \dfrac{-1\pm\sqrt{5}}{2}$	B1 (AO 2.4)	Identify $(r+1)$ as a factor, with justification		
			GP is convergent so $-1<r<1$, so $r=\dfrac{-1+\sqrt{5}}{2}$		Attempt to find all 3 roots of cubic		
			$S_\infty=\dfrac{1+\sqrt{5}}{1-\frac{1}{2}(-1+\sqrt{5})}$	M1 (AO 1.1a)	For all three		
			$=\dfrac{2(1+\sqrt{5})}{2-(-1+\sqrt{5})}=\dfrac{2(1+\sqrt{5})}{3-\sqrt{5}}$	A1 (AO 2.1)			
				M1 (AO 3.1a)	Identify correct value of r, with reason		
			$=\dfrac{2(1+\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}=\dfrac{2(3+\sqrt{5}+3\sqrt{5}+5)}{9-5}$	A1 (AO 2.1)			
			$=\dfrac{2(8+4\sqrt{5})}{4}=4+2\sqrt{5}$ AG	[12]			
					Attempt sum to infinity,		

					using their r Simplify to correct expression Rationalise denominator Obtain given answer www | Need $-1 <$ their $r < 1$ Must also attempt expansion	
			Total	12		
11		a	$2^{n_1 - 1} = 1024$ $n_1 = 11$	M1 (AO 1.1) A1 (AO 1.1) [2]		
		b	$r_2 = 4$ $4^{n_2 - 1} = 1024$ $n_2 = 6$	B1 (AO 1.1) B1 (AO 2.2a) [2]		
		c	$r_3 = \sqrt{2}$	B1 (AO 1.1)	Other correct answers score similarly, eg $r_3 = \sqrt[4]{2}$	

			$(\sqrt{2})^{n_3-1} = 1024$ $n_3 = 21$ $S_{21} = 1 \times \frac{(\sqrt{2})^{21}-1}{\sqrt{2}-1}$ $= 2047 + 1023\sqrt{2}$ or 3490 (3 sf)	M1 (AO 3.1a) A1 (AO 2.2a) A1FT (AO 1.1) [4]	$((\sqrt[4]{2})^{n_3-1} = 1024$ $n_3 = 41$ $S_{21} = 1 \times \frac{(\sqrt[4]{2})^{41}-1}{\sqrt[4]{2}-1}$ 6430 (3 sf)	ft their r_3 and n_3
			Total	8		