1. A curve has an equation which satisfies $\frac{dy}{dx} = kx(2x - 1)$ for all values of x. The point $P(2, 7)$ lies on the curve and the gradient of the curve at P is 9.

i. Find the value of the constant k.

[2]

ii. Find the equation of the curve.

[5]

2.

i. Find the binomial expansion of $\left(x^3 + \frac{2}{x^2}\right)^4$, simplifying the terms.

[5]

ii. Hence find $\int \left(x^3 + \frac{2}{x^2}\right)^4 dx$.

[4]

3. (a) It is given that $y = x^2 + 3x$.

(i) Find $\frac{dy}{dx}$ [2]

(ii) Find the values of x for which y is increasing. [2]

(b) Find $\int (3 - 4\sqrt{x})dx$. [5]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$2k \times 3 = 9$ $k = 1.5$	M1	Attempt to find k	Substitute $x = 2$ and $^{dy}/_{dx} = 9$ into given differential equation and attempt to find k Allow any exact equiv. including $^9/_6$
		i		A1	Obtain $k = 1.5$	Examiner's Comments Most candidates scored full marks on this question, with just a few using the y-coordinate rather than the gradient.
		ii	$y = x^3 - 0.75x^2 + c$ $7 = 8 - 3 + c$ hence $c = 2$ $y = x^3 - 0.75x^2 + 2$	M1	Expand bracket and attempt integration	M0 if bracket not expanded first M1 can still be gained for integrating an incorrect expansion as long as there are two terms For an 'integration attempt' there must be an increase in power by 1 for both terms
		ii		A1ft	Obtain at least one correct term (allow still in terms of k)	Follow through on their value of k (but not on an incorrect expansion at start of part (ii)) Can also get A1 if still in terms of k Allow unsimplified coefficients
		ii		A1	Obtain $x^3 - 0.75x^2$ (condone no + c)	Must now be numerical, and no f-t Allow unsimplified coefficients A0 if integral sign or dx still present, unless it later disappears
		ii		M1	Attempt to find c using (2, 7)	There must have been an attempt at integration, but can follow M0 eg if the bracket was not expanded first Need to get as far as actually attempting c M1 could be implied by eg 7 = 8 – 3 followed by an attempt to include a constant to balance the equation M0 if no + c seen or implied M0 if using $x = 7$, $y = 2$

		ii		A1	Obtain $y = x^3 - 0.75x^2 + 2$	Coefficients now need to be simplified (0.75 or $^3/_4$) Must be an equation ie $y = \ldots$, so A0 for 'f(x) = …' or 'equation = …' Allow aef, such as $4y = 4x^3 - 3x^2 + 8$
		ii				<u>Examiner's Comments</u> Most candidates also scored full marks on this part of the question, although some spoiled an otherwise correct solution by failing to write the final answer as an equation. Whilst the majority of candidates recognised the need to integrate and could attempt to do so, a surprising number then stopped at this point and made no attempt to evaluate c. There were a few candidates who, upon seeing the request to find an equation, immediately attempted to use $y = mx + c$ without first considering whether a linear function was involved. The majority of candidates appreciated the need to first expand the bracket, but it was disappointing that, at this level, some were unable to do so accurately.
			Total	7		
2		i	$(x^3)^4 + 4(x^3)^3(2x^{-2}) + 6(x^3)^2(2x^{-2})^2 + 4(x^3)(2x^{-2})^3 + (2x^{-2})^4$	M1*	Attempt expansion - products of powers of x^3 and $2x^{-2}$	Must attempt at least 4 terms Each term must be an attempt at a product, including binomial coeffs if used
		i	$= x^{12} + 8x^7 + 24x^2 + 32x^{-3} + 16x^{-8}$			Allow M1 if no longer $2x^{-2}$ due to index errors Allow M1 for no, or incorrect, binomial coeffs Powers of x^3 and $2x^{-2}$ must be intended to sum to 4 within each term (allow slips if intention correct) Allow M1 even if powers used incorrectly with $2x^{-2}$ ie only applied to x^{-2} and not to 2 as well Allow M1 for expansion of

						bracket in $x^4(1 + 2x^{-5})^4$ with $k = 3$ or 12 only, or $x^4(x^5 + 2)^4$ with $k = -2$ or -8 only, oe
		i		M1d*	Attempt to use correct binomial coeffs	At least 4 correct from 1, 4, 6, 4, 1 - allow missing or incorrect (but not if raised to a power) May be implied rather than explicit Must be numerical eg $^4C_1$ is not enough They must be part of a product within each term The coefficient must be used in an attempt at the relevant term ie $6(x^3)^3(2x^{-2})$ is M0 Allow M1 for correct coefficients when expanding the bracket in $x^4(1 + 2x^{-5})^4$ or $x^4(x^5 + 2)^4$ $x^{12} + 8x^7 + 12x^2 + 8x^{-3} + 2x^{-8}$ gets M1 M1 implied (even if no method seen) – will also get the first A1 as well
		i		A1	Obtain two correct simplified terms	Either linked by '+' or as part of a list Powers and coefficients must be simplified
		i		A1	Obtain a further two correct terms	Either linked by '+' or as part of a list Powers and coefficients must be simplified
		i		A1	Obtain a fully correct expansion **Examiner's Comments** Most candidates were able to write down a correct binomial expansion, including coefficients. The correct brackets were invariably seen in the initial statement, and around half of the candidates then used these effectively to produce a fully correct solution. However a significant minority simply ignored the brackets resulting in incorrect coefficients as each index was only applied to the x^{-2} and not the 2 as well. An equally common error was an ability to deal with the indices involved, which is basic GCSE and C1 work. Whilst the first and last terms were often correct, the multiple index laws required for the middle three terms caused problems for many with confusion over whether the add or multiply the relevant indices.	Terms must be linked by '+' and not just commas Powers and coefficients must be simplified A0 if subsequent attempt to simplify indices (eg x by x^8) **SR for reasonable expansion attempt:** M2 for attempt involving all 4 brackets resulting in a quartic with at most one term missing A1 for two correct, simplified, terms A1 for a further two correct, simplified, terms A1 for fully correct, simplified, expansion
		ii	$\frac{1}{13}x^{13} + x^8 + 8x^3 - 16x^{-2} - \frac{16}{7}x^{-7} + c$	M1*	Attempt integration	Increase in power by 1 for at least three terms (other terms could be incorrect)

						Can still gain M1 if their expansion does not have 5 terms Allow if the three terms include x^{-1} becoming $k\ln x$(but not x^0)
		ii		A1FT	Obtain at least 3 correct terms, following their (i)	Allow unsimplified coefficients
		ii		A1	Obtain fully correct expression	Coefficients must be fully simplified, inc x^8 not $1x^8$ isw subsequent errors eg $16x^{-2}$ then being written with 16 as well as x^2 in the denominator of a fraction
		ii		B1d*	+ c, and no dx or integral sign in answer **Examiner's Comments** Nearly all of the candidates were able to make a good attempt at integrating their expression from part (i) and the majority gained two marks for correctly integrating at least three of their terms. This allowed for slips with the negative indices, which proved more problematical for some. A third mark was available for including + c, but this was omitted from a number of solutions.	Ignore notation on LHS such as ∫ = …, y = …, $^{dy}/_{dx}$ = …
			Total	**9**		
3		a	(i) $2x + 3$	B1 (AO1.1) B1 (AO1.1) [2]	B1 for $2x$ or $2x^1$ B1 for + 3 or + $3x^0$ **Examiner's Comments** Almost all candidates answered this question correctly.	
		a	(ii) $2x + 3 > 0$ $x > -\frac{3}{2}$	M1 (AO1.1) A1f (AO2.2a) [2]	ft their (a)(i) Allow $x = -\frac{3}{2}$ min, stated or shown ft their **(a)(i)** so long as two terms **Examiner's Comments** A significant minority of candidates appeared not to understand	

					what is meant by “increasing”. Some did not appreciate that they could use their answer to part (a)(i), and started from scratch. Some of these found the minimum point, but could not proceed from this to the answer. Some common incorrect answers were x ≤ –1.5, x < –1.5 and x > 1.5. A few found the second derivative, but did not know how to proceed.	
		b	3x $-4x^{\frac{1}{2}}$ $-\frac{4x^{\frac{3}{2}}}{\frac{3}{2}}$ $-\frac{8}{3}x^{\frac{3}{2}}$ or equivalent $3x-\frac{8}{3}x^{\frac{3}{2}}+c$	B1 (AO1.1) M1 (AO1.1) M1 (AO1.2) A1 (AO1.1) B1f (AO2.5) [5]	M1 for $x^{\frac{1}{2}}$ seen before integration — May be implied by next line M1 for $x^{\frac{3}{2}}$ or equiv seen after integ or increase their fractional power by 1 ISW Their integral + c in final ans ISW eg “$y =$” or attempt find c B0 if include integral sign or dx. — Correct ans, no working: full mks Examiner’s Comments Many candidates answered this question correctly. A few candidates omitted "+ c". A few obtained $x^{\frac{3}{2}}$ correctly, but with an incorrect coefficient. Some thought that $4\sqrt{x}$ meant $4x^{-\frac{1}{2}}$ or $4x^{-2}$ or $x^{\frac{1}{4}}$. A few candidates "integrated" 3 to become $\frac{3^2}{2}$ or "integrated" $4\sqrt{x}$ to become $4\frac{(\sqrt{x})^2}{2}$. Some candidates integrated	

					correctly and then attempted to find the value of c.	
			Total	9		