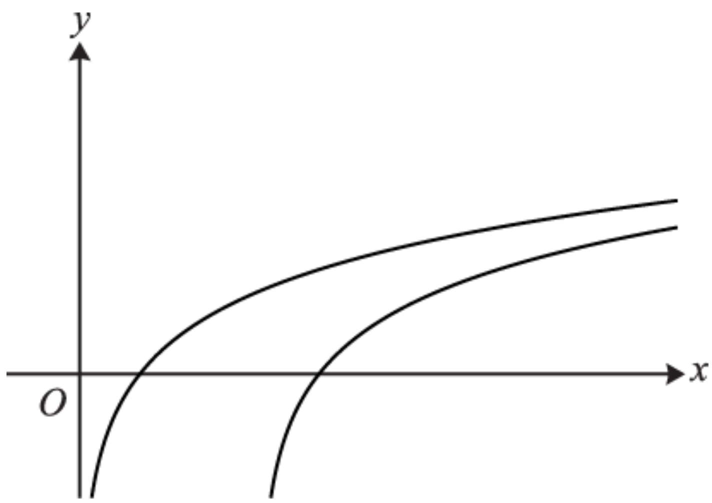
1.



The diagram shows the curves $y = \log_2 x$ and $y = \log_2 (x - 3)$.

i. Describe the geometrical transformation that transforms the curve $y = \log_2 x$ to the curve $y = \log_2 (x - 3)$.

[2]

ii. The curve $y = \log_2 x$ passes through the point (a, 3). State the value of a.

[1]

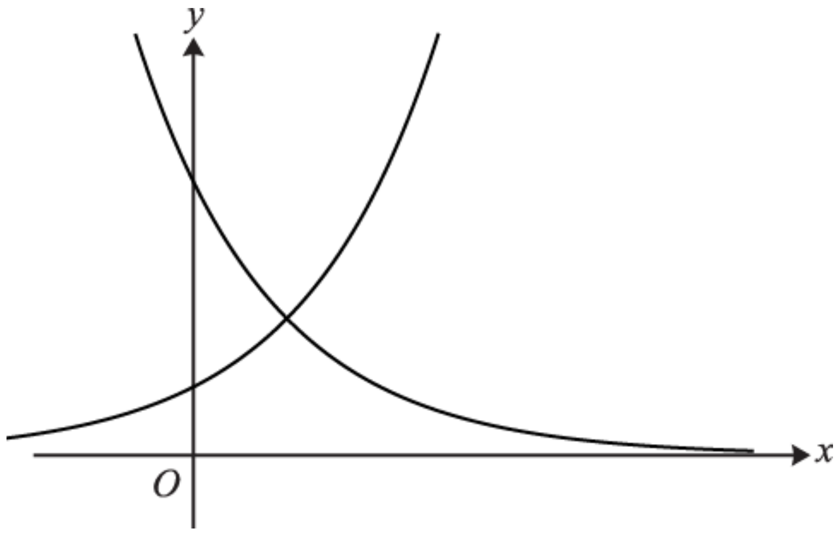
iii. The curve $y = \log_2 (x - 3)$ passes through the point (b, 1.8). Find the value of b, giving your answer correct to 3 significant figures.

[2]

iv. The point P lies on $y = \log_2 x$ and has an x-coordinate of c. The point Q lies on $y = \log_2 (x - 3)$ and also has an x-coordinate of c. Given that the distance PQ is 4 units find the exact value of c.

[4]

2.



The diagram shows the curves $y = a^x$ and $y = 4b^x$.

i.

a. State the coordinates of the point of intersection of $y = a^x$ with the y-axis.

[1]

b. State the coordinates of the point of intersection of $y = 4b^x$ with the y-axis.

[1]

c. State a possible value for a and a possible value for b.

[2]

ii. It is now given that $ab = 2$. Show that the x-coordinate of the point of intersection of $y = a^x$ and $y = 4b^x$ can be written as

$$x = \frac{2}{2\log_2 a - 1}.$$

[5]

3.

Solve the equation $2^{4x-1} = 3^{5-2x}$, giving your answer in the form $x = \frac{\log_{10} a}{\log_{10} b}$.

[6]

4. a. Use logarithms to solve the equation

$$2^{n-3} = 18000,$$

giving your answer correct to 3 significant figures.

[4]

b. Solve the simultaneous equations

$$\log_2 x + \log_2 y = 8, \qquad \log_2\left(\frac{x^2}{y}\right) = 7.$$

[5]

5. i. Express $2\log_3 x - \log_3(x + 4)$ as a single logarithm.

[2]

ii. Hence solve the equation $2\log_3 x - \log_3(x + 4) = 2$.

[4]

6. a. The mass, M grams, of a substance at time t years is given by

$$M = 58e^{-0.33t}.$$

Find the rate at which the mass is decreasing at the instant when $t = 4$. Give your answer correct to 2 significant figures.

[3]

b. The mass of a second substance is increasing exponentially. The initial mass is 42.0 grams and, 6 years later, the mass is 51.8 grams. Find the mass at a time 24 years after the initial value.

[4]

7. The mass of a substance is decreasing exponentially. Its mass is m grams at time t years. The following table shows certain values of t and m.

t	0	5	10	25
m	200	160		

i. Find the values missing from the table.

[2]

ii. Determine the value of t, correct to the nearest integer, for which the mass is 50 grams.

[4]

8. The number of members of a social networking site is modelled by $m = 150e^{2t}$, where m is the number of members and t is time in weeks after the launch of the site.

(a) State what this model implies about the relationship between m and the rate of change of m. [2]

(b) What is the significance of the integer 150 in the model? [1]

(c) Find the week in which the model predicts that the number of members first exceeds 60 000. [3]

(d) The social networking site only expects to attract 60 000 members. Suggest how the model could be refined to take account of this. [1]

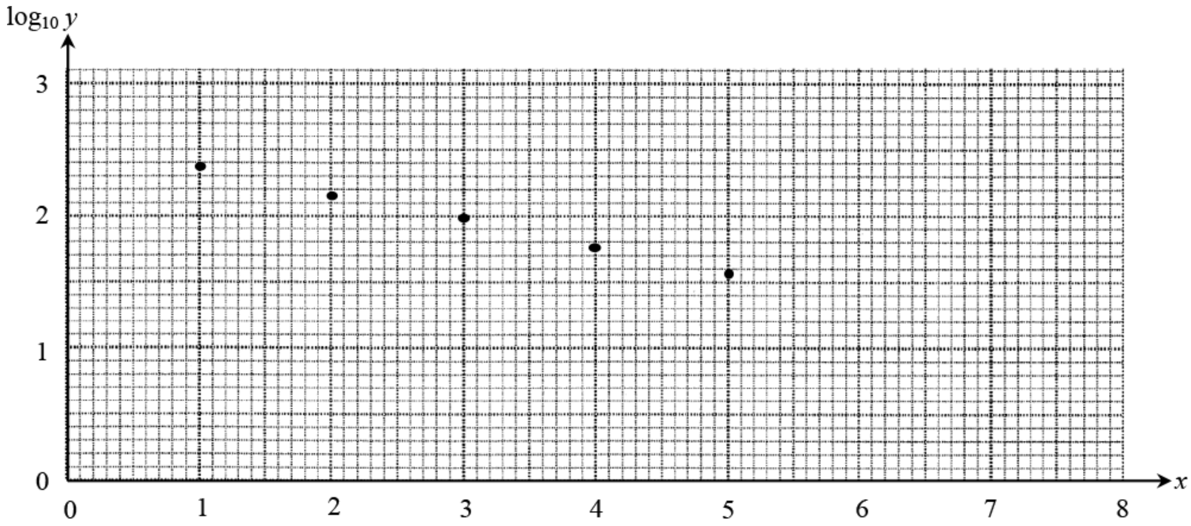
9. A doctors' surgery starts a campaign to reduce missed appointments. The number of missed appointments for each of the first five weeks after the start of the campaign is shown below.

Number of weeks after the start (x)	1	2	3	4	5
Number of missed appointments (y)	235	149	99	59	38

This data could be modelled by an equation of the form $y = pq^x$ where p and q are constants.

(a) Show that this relationship may be expressed in the form $\log_{10} y = mx + c$, expressing m and c in terms of p and/or q. [2]

The diagram below shows $\log_{10} y$ plotted against x, for the given data.



(b) Estimate the values of p and q. [3]

(c) Use the model to predict when the number of missed appointments will fall below 20.
Explain why this answer may not be reliable. [2]

10. **In this question you must show detailed reasoning.**

Use logarithms to solve the equation

$$3^{2x+1} = 4^{100},$$

giving your answer correct to 3 significant figures. [4]

11. Sanjeep invests £250 at 4% compound interest per annum. Interest is added at the end of each complete year.

(a) What is Sanjeep's investment worth after 5 years? [2]

(b) After how long will Sanjeep's investment be worth £500? [2]

(c) State briefly a limitation of the model used in part (b) [1]

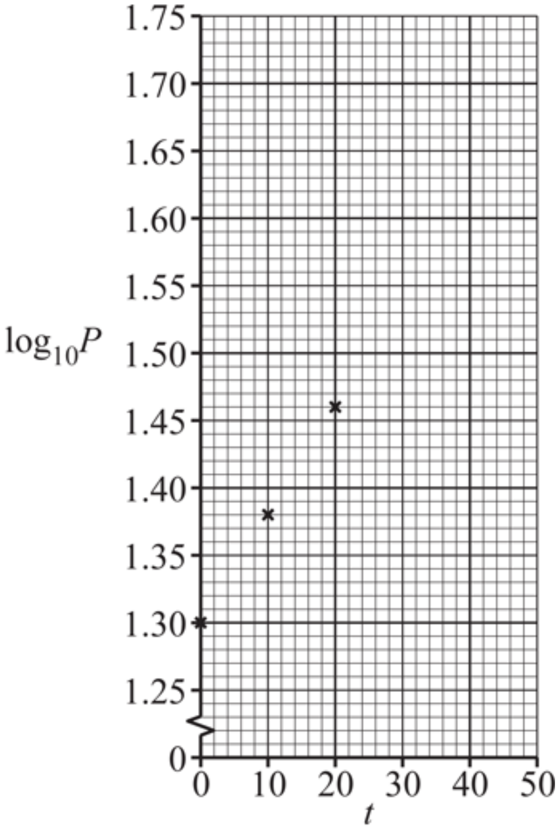
12. The population of fish, P, in a lake is recorded at 10 day intervals. The table below shows the data collected, where t is the number of days since the population was first recorded.

t	0	10	20	30	40	50
P	20	24	29	34	42	50

It is proposed the population can be modelled by the equation $P = ab^t$, where a and b are constants.

(a) Complete the table of values below. Plot the final three values of $\log_{10}P$ against t on the axes provided. [1]

t	0	10	20	30	40	50
$\log_{10}P$	1.30	1.38	1.46			



(b) By drawing an appropriate straight line on your graph, find the values of a and b. [3]

(c) Use the model to predict the population of fish when $t = 200$. [1]

(d) Explain why this prediction may not be reliable. [1]

13. (a) Show that the equation $\log_2 (y + 1) - 1 = 2\log_2 x$ can be written in the form $y = ax^2 + b$, where a and b are integers. [4]

(b) Hence solve the simultaneous equations

$$\log_2 (y + 1) - 1 = 2\log_2 x, \qquad \log_2 (y - 10x + 14) = 0.$$ [4]

14. A sequence of three transformations maps the curve $y = \ln x$ to the curve $y = e^{3x} - 5$. Give details of these transformations.

[4]

15. A pan of water is heated until it reaches 100°C. Once the water reaches 100°C, the heat is switched off and the temperature T°C of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where t denotes the time, in minutes, after the heat is switched off and a and k are positive constants.

(a) Write down the value of a. [1]

(b) Explain what the value of 25 represents in the equation $T = 25 + ae^{-kt}$. [1]

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15 °C per minute.

(c) Calculate the value of k. [3]

(d) Find the time taken for the temperature of the water to drop from 100 °C to 45 °C. [3]

(e) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than 100°C. Suggest how the equation for the temperature as the water cools would be modified by this.

[1]

16. In a science experiment a substance is decaying exponentially. Its mass, M grams, at time t minutes is given by $M = 300e^{-0.05t}$.

(a) Find the time taken for the mass to decrease to half of its original value. [3]

A second substance is also decaying exponentially. Initially its mass was 400 grams and, after 10 minutes, its mass was 320 grams.

(b) Find the time at which both substances are decaying at the same rate. [8]

17. A student was asked to solve the equation $2(\log_3 x)^2 - 3\log_3 x - 2 = 0$. The student's attempt is written out below.

$$2(\log_3 x)^2 - 3\log_3 x - 2 = 0$$
$$4\log_3 x - 3\log_3 x - 2 = 0$$
$$\log_3 x - 2 = 0$$
$$\log_3 x = 2$$
$$x = 8$$

(a) Identify the two mistakes that the student has made. [2]

(b) Solve the equation $2(\log_3 x)^2 - 3\log_3 x - 2 = 0$, giving your answers in an exact form. [4]

18. An analyst believes that the sales of a particular electronic device are growing exponentially. In 2015 the sales were 3.1 million devices and the rate of increase in the annual sales is 0.8 million devices per year.

(a) Find a model to represent the annual sales, defining any variables used. [5]

(b) In 2017 the sales were 5.2 million devices. Determine whether this is consistent with the model in part (a).

[2]

(c) The analyst uses the model in part (a) to predict the sales for 2025. Comment on the reliability of this prediction.

[1]

19. **In this question you must show detailed reasoning.**

Solve the simultaneous equations

$e^x - 2e^y = 3$
$e^{2x} - 4e^{2y} = 33.$

Give your answer in an exact form. [5]

20. Use logarithms to solve the equation $2^{3x-1} = 3^{x+4}$, giving your answer correct to 3 significant figures. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Translation of 3 units in positive *x*-direction	B1	State translation	Must be 'translation' and not 'move', slide', 'shift' etc
		i		B1	State or imply 3 units in positive *x*-direction	Independent of first B1 Allow vector notation, but not a coordinate ie (3, 0) Worded descriptions must give clear intention of direction, so B0 for just '*x*-direction' or 'parallel to *x*-axis' unless + 3 also stated (as ' + ' implies the direction) For the direction, allow 'in the positive *x*-direction', 'parallel to the positive *x*-axis' or 'to the right' Do not allow 'in the positive *x*-axis' or 'along the positive *x*-axis' even if combined with correct statement eg 'right' Allow '3' or '3 units' but not '3 places', '3 squares', 'sf 3'… Ignore irrelevant statements (eg intercepts on axes), but penalise contradictions **B0 B0** if second transformation also given <u>Examiner's Comments</u> The majority of candidates could identify the relevant transformation, but many then lost marks through a lack of precision when describing it. Examiners expected to see the word translation used, rather than more

						colloquial descriptions such as move or shift. Equally, the description of the translation had to indicate three units in the positive x-direction, with no ambiguity. The most successful candidates made effective use of vector notation.
		ii	$a = 8$	B1	State 8	Allow x not a Allow implied value eg (8, 3) or $\log_2 8 = 3$ **Examiner's Comments** The vast majority of candidates were able to state the correct value, with 3^2 and $\log_2 3$ being the most common errors.
		iii	$b - 3 = 2^{1.8}$	B1	State or imply $b - 3 = 2^{1.8}$	Allow x not b More accurate answer is 6.482202253… Answer only can gain B2 as long as accurate **Examiner's Comments**
		iii	$b = 6.48$	B1	Obtain 6.48, or better	Most candidates were also able to find the required value in this part as well, though it was not quite so well done. Candidates seemed familiar with the method to remove the logarithm, though in some cases this was spoiled by first attempting to split $\log_2 (x - 3)$ into two terms. The other common error was to use 1.8^2 rather than $2^{1.8}$.
		iv	$\log_2 c - \log_2(c - 3) = 4$ $\log_2 {}^{c}/_{c-3} = 4$ ${}^{c}/_{c-3} = 2^4$	M1	Equate difference in y-coordinates to ± 4	Allow in terms of x not c Allow any equiv eg $\log_2 c = \log_2(c - 3) + 4$ Brackets must be seen, or implied by later working

		iv	$c = 16c - 48$ $c = {}^{48}/_{15} = {}^{16}/_{5}$			Allow if subtraction is the other way around, but M0 if two log terms are summed Allow as part of an attempt at Pythagoras' theorem eg $\sqrt{\{(c - c)^2 + (\log_2 c - \log_2(c - 3))^2\}} = 4$ Could be implied if $\log_2$ dealt with at the same time Must be used on difference not sum if using the two algebraic terms ie $\pm (\log_2 c - \log_2(c - 3))$ Starting with $\log_2 c = \log_2 (c - 3)$, rearranging to equal 0 and then using a log law could get M1 Allow if 4 is attempted as $\log_2 k$ ($k \neq 4$) and then combined with at least one of the other two terms (possibly using $\log a + \log b$) Allow if attempted with their now incorrect 4 Allow if they started with a constant other than ± 4 ie attempting to rewrite k as $\log_2 2^k$ and then combining with at least one of the algebraic logs gets M1
		iv		M1	Use $\log a - \log b = \log {}^{a}/_{b}$	
		iv		A1	Obtain ${}^{c}/_{c-3} = 2^4$	Any correct equation, in a form not involving logs Allow 3.2, or unsimplified fraction **SR B2** for answer only or T&I Examiner's Comments
		iv		A1	Obtain ${}^{16}/_{5}$ oe	This final part of the question proved to be somewhat more challenging. Most candidates could gain the first mark for attempting a relevant equation, although some simply equated the two *y*-coordinates. A second method mark was then available for correctly combining two logarithm terms, and a reasonable number gained this mark, including some who had not gained the first mark. Successful candidates were then able

						to complete the question to gain full marks. Some candidates failed to get more than the first two marks as they subtracted the functions in the incorrect order when equating the difference to 4. A few of the more astute candidates considered both possible differences and then justified which to select as their final answer.
			Total	9		
2		i	(0, 1)	B1	State (0, 1)	Allow no brackets B1 for $x = 0$, $y = 1$ – must have $x = 0$ stated explicitly B0 for $y = a^0 = 1$ (as $x = 0$ is implicit)
		i	(0, 4)	B1	State (0, 4)	Allow no brackets B1 for $x = 0$, $y = 4$ – must have $x = 0$ stated explicitly B0 for $y = 4b^0 = 4$ (as $x = 0$ is implicit)
		i	State a possible value for a	B1	Must satisfy $a > 1$	Must be a single value Could be irrational eg e^{-1} Must be fully correct so B0 for eg 'any positive number such as 3'
		i	State a possible value for b	B1	Must satisfy $0 < b < 1$ **Examiner's Comments** Most candidates were able to give the coordinates of the two required points of intersection. Many of the unsuccessful candidates had the correct idea but just gave the y-value rather than the required coordinates. The final part was not so well done. Most candidates were able to give an appropriate value for a, but many were less successful on b, with a negative value being the most common incorrect answer.	Must be a single value Could be irrational eg e^{-1} Must be fully correct **SR** allow B1 if both a and b given correctly as a range of values

		ii	$\log_2 a^x = \log_2(4b^x)$	M1	Equate a^x and $4b^x$ and introduce logarithms at some stage	Could either use the two given equations, or b could have already been eliminated so using two eqns in a only Must take logs of each side soi so M0 for $4\log_2(b^x)$ Allow just log, with no base specified, or $\log_2$ Allow logs to any base, or no base, as long as consistent Or correct use of log $^a/_b$ = log a – log b
		ii	$\log_2 a^x = \log_2 4 + \log_2 b^x$	M1	Use log ab = log a + log b correctly	Used on a correct expression eg $\log_2(4b^x)$ or $\log_2 4(^2/_a)^x$ Equation could either have both a and b or just a Must be used on an expression associated with $a^x = 4b^x$, either before or after substitution, so M0 for $\log_2(ab) = 1$ hence $\log_2 a + \log_2 b = 1$ Could be an equiv method with indices before using logs eg $a^{2x} = 4 \times 2^x$ hence $a^{2x} = 2^{2+x}$
		ii	$x\log_2 a = \log_2 4 + x\log_2 b$	M1	Use log a^b = b log a correctly at least once	Allow if used on an expression that is possibly incorrect Allow M1 for $x\log_2 a = x\log_2 4b$ as one use is correct Equation could either have both a and b or just a
		ii	$x\log_2 a = \log_2 4 + x\log_2(^2/_a)$	B1	Use $b = {}^2/_a$ to produce a correct equation in a and x only	Can be gained at any stage, including before use of logs If logs involved then allow for no, or incorrect, base as long as equation is fully correct – ie if log $2^k = k$ used then base must be 2 throughout equation Could be an equiv method eg $(a \times a)^x = 4(a \times b)^x$ hence $a^{2x} = 4 \times 2^x$ Must be eliminating b, so $(^2/_b)^x = 4b^x$ is B0 unless the equation is later changed to being in terms of a

		ii	$x\log_2 a = 2 + x\log_2 2 - x\log_2 a$ $x(2\log_2 a - 1) = 2$ $x = \dfrac{2}{2\log_2 a - 1}$ **AG**	A1	Obtain given relationship with no wrong working **Examiner's Comments** As in previous series, candidates seem to have a basic understanding of the rules of logarithms but struggle to apply them consistently and accurately throughout a convincing proof. Most candidates gained a mark for equating and introducing logarithms, and a second mark for using the power rule correctly at least once. The most common error by far was for $\log 4b^x$ to become $x\log 4b$, which meant that no further progress could be made. Some of the more successful solutions eliminated *b* as the first step and simplified the resulting equation before introducing logarithms. Candidates must appreciate that a proof needs to be convincing throughout, which here included consistent use of bases, brackets being used correctly and sufficient detail being provided for each step. It was also noticeable that a number of candidates made multiple attempts at this question; they must recognise that it is the last attempt that will be marked unless they indicate otherwise by deleting unwanted attempts.	Proof must be fully correct with enough detail to be convincing Must use $\log_2$ throughout proof for A1 – allow 1 slip Using numerical values for *a* and *b* will gain no credit Working with equation(s) involving *y* is M0 unless *y* is subsequently eliminated
			Total	9		
3			$(4x - 1)\log_{10}2 = (5 - 2x)\log_{10}3$	M1*	Introduce logs throughout and drop power(s)	Allow no base or base other than 10 as long as consistent, including $\log_3$ on LHS or $\log_2$ on RHS Drop single power if $\log_3$ or $\log_2$ or both powers if any other base
				A1	Obtain $(4x - 1)\log_{10}2 = (5 - 2x)\log_{10}3$	Brackets must be seen, or implied by later working Allow no base, or base other than 10 if consistent Any correct linear equation ie $4x - 1 = (5 - 2x)\log_2 3$ or $(4x - 1)\log_3 2 = 5 - 2x$
			$x(4\log_{10}2 + 2\log_{10}3) = \log_{10}2 + 5\log_{10}3$	M1*	Attempt to make *x* the subject	Expand bracket(s) and collect like terms - as far as their $4x\log_{10}2 + 2x\log_{10}3 = \log_{10}2 + 5\log_{10}3$ Expressions could include $\log_2 3$ or $\log_3 2$ Must be working exactly, so M0 if log(s) now decimal equivs

				A1	Obtain a correct equation in which x only appears once	LHS could be $x(4\log_{10}2 + 2\log_{10}3)$, $x\log_{10}144$ or even $\log_{10}144^x$ Expressions could include $\log_2 3$ or $\log_3 2$ RHS may be two terms or single term
			$x\log_{10}144 = \log_{10}486$	M1d*	Attempt correct processes to combine logs	Use $b\log a = \log a^b$, then $\log a + \log b = \log ab$ correctly on at least one side of equation (or $\log a - \log b$) Dependent on previous M1 but not the A1 so $\log_{10}486$ will get this M1 irrespective of the LHS
			$x = \dfrac{\log_{10}486}{\log_{10}144}$	A1	Obtain correct final expression	Base 10 required in final answer - allow A1 if no base earlier, or if base 10 omitted at times, but A0 if different base seen previously (unless legitimate working to change base seen) Do not isw subsequent incorrect log work eg $x = {}^{\log 27}/_{\log 8}$
			Alternative solution			
				M1	Use index laws to split both terms	Either into fractions, or into products involving negative indices ie $2^{4x} \times 2^{-1}$
			$2^{4x} \div 2 = 3^5 \div 3^{2x}$ $2^{4x} \times 3^{2x} = 3^5 \times 2$ $16^x \times 9^x = 243 \times 2$	A1		Combine like terms on each side
			$144^x = 486$		Obtain $2^{4x} \times 3^{2x} = 3^5 \times 2$ oe	
			$\log_{10} 144^x = \log_{10} 486$	M1		Use at least once correctly
			$x\log_{10} 144 = \log_{10} 486$ $x = \dfrac{\log_{10}486}{\log_{10}144}$		Use $a^{bx} = (a^b)x$	
				A1		Any correct equation in which x appears only once - logs may have been introduced prior to this

				M1 A1	Obtain $144^x = 486$ Introduce logs on both sides and drop power Obtain correct final answer **Examiner's Comments** Most candidates were able to gain the first two marks for taking logarithms of both sides and using the power rule, though a number of candidates failed to use brackets. This lack of precision was penalised unless subsequent working clearly showed the correct intention. In order to make further progress candidates had to then expand the brackets and gather like terms which, only the better candidates realised the need to do. Even fewer managed the next step of making x the subject of the equation although some did manage to get a method mark for correctly combining two relevant logarithms. Recent examination sessions have shown candidates becoming more proficient in using logarithms to solve equations when a decimal answer is required, but it appears that algebraic manipulation of logarithms is still a challenge for many. Nevertheless, a pleasing number of fully correct solutions were still seen.	Allow no base, or base other than 10 if consistent Do not isw subsequent incorrect log work
			Total	6		
4			$2\log_2 x - \log_2 y = 7$	M1	Correct use of one log law – on a correct equation	Either on first eqn to get $\log_2(xy) = 8$, or on second eqn to get at least $\log_2 x^2 - \log_2 y = 7$ Allow for one correct use, even if error made with other equation Must be used on a correct equation so M0 if an error has already occurred eg $\log(x^2/y) = 2\log(xy) = 2(\log x + \log y)$ is M0

			$(\log_2 x + \log_2 y) + (2\log_2 x - \log_2 y) = 15$	M1	Attempt to eliminate one variable	To get an equation in just one variable, which may or may not still involve logs Must be a sound algebraic process with the two equations that they are using, though errors may have been made earlier with log / index laws
			$3\log_2 x = 15$	A1	Obtain correct equation in just one variable	Which may or may not still involve logs Depending on the method used, possible equations are $3\log_2 x = 15$, $\log_2 x^3 = 15$, $x^3 = 32768$ or $3\log_2 y = 9$, $\log_2 y^3 = 9$, $y^3 = 512$ The variable should only appear once so $\log_2 x^2 + \log_2 x = 15$ is A0 until the two log terms are correctly combined
			$x = 2^5$	M1	Correctly use 2^k as inverse of $\log_2$	At any stage – may even be the very first step to obtain $x^2/y = 128$ M0 for eg $\log_2 x + \log_2 y = 8$ becoming $x + y = 2^8$ as incorrect method to remove logs
			$x = 32,\ y = 8$	A1	Obtain $x = 32,\ y = 8$ **<u>Examiner's Comments</u>** This part of the question proved to be more challenging, and a variety of different approaches were seen. The most effective method tended to be to remove the logarithms as a first step and then solve the resulting simultaneous equations. The most common method however was to use equations that still involved logarithms. Candidates usually gained the first two method marks, for using a log law and eliminating a variable, with ease. However the resulting equation of $\log_2 x^2 + \log_2 x - 15 = 0$ was seen as a quadratic, and a solution attempt made based on this misunderstanding. Candidates who dropped the index on the first term, either at this stage or earlier in their solution, tended to then produce a fully correct solution. Candidates would be well advised to show their method clearly and not attempt to do more than one step at a time. It was quite common to see $\log_2 x + \log_2 y = 8$ become $xy = 3$ in the next line. Whilst this may suggest that a correct log law was used	Both values required, and no others Answer only, with no evidence of log or index work, is 0/5

					before the logs were incorrectly removed, with no clear evidence of this the method mark cannot be awarded.	
			Total	5		
5		i	$\log_3 x^2 - \log_3(x+4)$ $= \log_3 \frac{x^2}{x+4}$	B1*	Obtain $\log_3 x^2 - \log_3(x+4)$	Allow no base Could be implied if both log steps done together Allow equiv eg $2(\log_3 x - \log_3(x+4)^{0.5})$ CWO so B0 if eg $\frac{\log x^2}{\log(x+4)}$ seen in solution No ISW if subsequently incorrectly 'simplified' eg $\log_3(\frac{x}{4})$ Must now have correct base in final answer - condone if omitted earlier **Examiner's Comments** The majority of candidates were able to produce a fully correct solution to this part of the question. Of the remainder, most were aware of the power law but too often this was not used as the first step or the second term was incorrect at this stage so no fully correct expression was ever seen. Some candidates obtained the correct expression but then incorrectly cancelled within the logarithm, which was penalised. Another relatively common error was for the difference of the two logarithms to result in a fraction with a logarithm appearing in the denominator. Even if this subsequently was written as the required single term, the error in the method was still penalised.
		i		B1d*	Obtain $\log_3 \frac{x^2}{x+4}$ or equiv single term	
		ii	$\frac{x^2}{x+4} = 3^2$ $x^2 = 9(x+4)$	M1*	Attempt correct method to remove logs	Equation must be of format $\log_3 f(x) = 2$, with $f(x)$ being the result of a legitimate attempt to combine logs (but condone errors such as

			$x^2 - 9x - 36 = 0$ $(x - 12)(x + 3) = 0$ $x = 12$			incorrect simplification of fraction) Allow use of their (i) only if it satisfies the above criteria, so $x^2 - (x + 4) = 9$ is M0 whether or not in (i)
		ii		A1	Obtain any correct equation	Not involving logs
		ii		M1d*	Attempt complete method to solve for x	Solving a 3 term quadratic - see additional guidance Must attempt at least one value of x
		ii		A1	Obtain $x = 12$ as only solution	Must be from a correct solution of a correct quadratic, and A0 if other root (if given) is not $x = -3$ A0 if $x = -3$ still present Not necessary to consider $x = -3$, and then discard, but A0 if discarded for incorrect reason **NB** Despite not being 'hence' allow full credit for other valid attempts, such as combining $\log_3(x + 4)$ with $\log_3 9$ on right-hand side before removing logs, or starting with $\log_3 x - \frac{1}{2}\log_3(x + 4) = 1$ **SR** in (i) $\frac{\log x^2}{\log(x+4)}$ becoming $\log_3 \frac{x^2}{x+4}$ was penalised as an error in notation, but is eligible for full credit in (ii) **Examiner's Comments** Most candidates who had correctly combined logarithms in the first part of the question could then carry out the correct process to remove the logarithms in this part of the question and solve the ensuing equation with ease. Only the most astute candidates appreciated that −3 was not a

						valid solution to the given equation and thus needed discarding, which meant that three out of four was the modal mark. To gain any credit in this part of the question it was expected that there had been a valid attempt in part (i) to write the two logarithms as a single term.
			Total	6		
6			Either: State or imply formula $42e^{kt}$ or $42a^t$ Attempt to find k from $42e^{6k} = 51.8$ or a from $42a^6 = 51.8$ Obtain $k = 0.035$ or $a = 1.0356$ Substitute 24 to obtain value between 97.1 and 97.3 inclusive Or: Use ratio $\frac{51.8}{42}$ in calculation Attempt calculation of form $42 \times r^n$ Obtain $42 \times (\frac{51.8}{42})^4$ or $51.8 \times (\frac{51.8}{42})^3$ Obtain value between 97.1 and 97.3 inclusive	B1 M1 A1 A1 B1 M1 A1 A1	$42e^{-kt}$, $42e^{-kx}$, etc. also acceptable using sound process involving logarithms at least as far as $6k = \ldots$ or $a = \ldots$ or greater accuracy 0.03495… or exact equiv $\frac{1}{6}\ln\frac{37}{30}$ allow greater accuracy than 3 s.f. allow greater accuracy than 3 s.f. **Examiner's Comments** Part (b) presented more problems and some candidates made the incorrect assumption that the mass would increase by 9.8 grams in each period of 6 years. Others made no progress because of an	

					assumption that the formula from part (a) was still relevant. The usual method adopted was to set up a formula of the form $42e^{kt}$ and proceed to establish the value of *k*. A lack of accuracy in the working marred some solutions. Some candidates displayed a clear understanding of exponential growth, knowing that the mass increases by the same proportion over equal time intervals, and were able to find the answer immediately from the calculation $42.0 \times (\frac{51.8}{42.0})^4$.	
			Total	4		
7		i	Obtain 128 for value corresponding to 10	B1	Allow any value rounding to 128	
		i	Obtain 65.5 for value corresponding to 25	B1	Allow any value rounding to 65 or 66; whether obtained using powers of 0.8 or by use of formula	
		ii	Attempt to find formula for *m* of form $200e^{kt}$ or $200 \times r^{\lambda t}$	M1	Whether attempted in part (i) or (ii)	If formula attempted in part (i), marks earned must be recorded in part (ii)
		ii	Obtain $200e^{(0.21n0.8)t}$ or $200e^{-0.0446t}$ or $200 \times 0.8^{0.2t}$ or 200×0.956^{t}	A1	Or equiv	
		ii	Show correct process for solving equation of form $200e^{kt} = 50$ or $200r^{\lambda t} = 50$	M1		
		ii	Obtain 31	A1	Or greater accuracy rounding to 31; ignore any units given; second M1 is implied by correct answer **Examiner's Comments** It was pleasing to see this question on exponential decay handled competently by the majority of candidates; all 6 marks were earned by 78% of the candidates. A minority adopted an approach for part (i) based on powers of 0.8 and this worked well in most cases, just a few multiplying 200 by an incorrect power of 0.8. Most candidates though, perhaps having looked ahead to what was required in part (ii), decided that it was appropriate to establish a formula for *m* in terms of *t*. They then used this to find the two values in part (i) and to answer part (ii). Usually there was no difficulty in finding the formula although there was	Special case: no formula anywhere and answer 31 (or greater accuracy) given, award B2 (i.e. 2/4 for part (ii))

					some lack of attention given to the signs involved. Some candidates were guilty of having values in the formula that were insufficiently accurate. Lack of care with signs did lead in some instances to a negative value of t in part (ii). Other candidates were careless with units, some concluding with 31 seconds in part (ii) and others with 31 grams. These errors with units were not penalised on this occasion.	
			Total	6		
8		a	The model is exponential so the rate of change of m is proportional to m In this case, the rate of change of m is $2m$	M1(AO1.1) E1(AO2.2a) [2]	Gradient of $e^{kx} = ke^{kx}$ In context	
		b	The initial membership	B1(AO1.1) [1]		
		c	$60000 = 150e^{2t}$ $\ln 400 = 2t$ $2.995 = t$ and hence 3	M1(AO3.4) A1(AO1.1) A1(AO1.1) [3]	Correct equation and use correct order of operations Obtain correct intermediate step Or $\ln 60000 = \ln 150 + 2t$ Obtain correct answer	
		d	E.g. When the graph reaches 60 000 the graph becomes constant.	B1(AO3.5c) [1]	Correct suggestion	
			Total	7		
9		a	$\log_{10} y = \log_{10} p + x \log_{10} q$	B1(AO2.1)		

			$m = \log_{10} q$, $c = \log_{10} p$	B1(AO2.4) [2]		
		b	E.g. $\log_{10} q = \dfrac{2.4-1.6}{1-5} = -0.2$ $q = 10^{-0.2} = 0.63$ $\log_{10} p = 2.5$ so $p = 380$	M1(AO3.3) A1(AO1.1) B1(AO1.1) [3]	Measure gradient from graph and identify it as log q | Accept q in [0.6, 0.7] Accept p in [320, 400]	
		c	$\log_{10} 20 = 1.3$ so week 7 E.g. Extrapolation is unjustified because it assumes that the assumptions made in the model will hold true in the long term	B1(AO3.4) E1(AO3.5b) [2]	One valid explanation	
			Total	7		
10			DR $\log 3^{2x+1} = \log 4^{100}$ $(2x+1)\log 3 = \log 4^{100}$ $2x + 1 = 126(.18…)$ $x = 62.6$	*M1(AO1.1a) A1(AO1.1) dep*M1(AO1.1) A1(AO1.1) [4]	Correctly introduce logs (can use any base, if consistent) Obtain linear equation in x, with logarithm(s) allow $2x + 1\log 3 = \log 4^{100}$ cao | OR **M1** $\log_3 3^{2x+1} = \log_3 4^{100}$ **A1** $2x + 1 = \log_3 4^{100}$	

			Total	4		
11		a	250×1.04^5 = £304.16	M1(AO1.1a) A1(AO1.1) [2]	Allow £304	
		b	$250 \times 1.04^x = 500$ $1.04^x = 2$ $x = \dfrac{\ln 2}{\ln 1.04}$ = 17.7 18 years	M1 (AO3.1a) M1(AO1.1) A1(AO3.2a) [3]		
		c	eg Assumes constant interest rate.	E1(AO3.5b) [1]	or, eg, Bank may collapse	Interest rate may change
			Total	6		
12		a	Points at (30, 1.53), (40, 1.62), (50, 1.70)	B1(AO1.1) [1]	Plot $\log_{10}P$ against t	Allow one error
		b	$\log_{10}a = 1.30$ so $a = 20$ $\log_{10}b = 0.008$ $b = 1.02$	B1(AO3.3) M1(AO3.4) A1(AO1.1) [3]	Correct value for a State or imply that gradient is $\log_{10}b$ Obtain $b = 1.02$ (awrt)	Could just be stated Method must show use of graph not substitution into given model

		c	Answer in range 700 to 1050	B1ft(AO3.4) [1]	ft their a and b	
		d	Accept any sensible explanation	B1(AO3.5b) [1]	Eg extrapolation unreliable Eg the model is continuous, not discrete | Eg Model may no longer be valid eg insufficient food to support larger population	
			Total	6		
13		i	$\log_2(y + 1) - \log_2 2 = \log 2x^2$ $\log_2(\frac{y+1}{2}) = \log_2 x^2$ $y + 1 = 2x^2$ $y = 2x^2 - 1$ ie $a = 2$, $b = -1$	B1 M1 A1	$2\log_2 x = \log_2 x^2$ — Used correctly at any point, even if equation is no longer fully correct. Allow no base Correctly combine at least two log terms — Could be the 2 log terms in the given equation, or could involve $\log_2 2$. The terms being combined must be correct, even if an error has occurred elsewhere in the equation. M0 for incorrect method eg $\frac{\log(y+1)}{\log 2}$ even if it then becomes $\log(\frac{y+1}{2})$	

				A1 [4]	<table><tr><td>Correct equation with at least two terms combined</td><td>Equation of form $\log_2 f(x, y) = k$ or $\log_2 f(y) = \log_2 g(x)$ Condone no base on the logs</td></tr><tr><td>Obtain $y = 2x^2 - 1$</td><td>Correct equation required, but no need for explicit statement of $a = 2$, $b = -1$</td></tr></table> **Examiner's Comments** This part of the question proved to be more challenging, with candidates clearly being familiar with the rules of logarithms but not always able to apply the relevant rule correctly. Most candidates gained the first mark for rewriting $2\log_2 x$ as $\log_2 x^2$, but only the more able candidates could make further progress. The most common error was to remove the logarithms term by term, and others explicitly 'expanded' the logarithm before achieving the same result. The most common method was to combine the two log terms before removing the logs, but a number of candidates rewrote 1 as $\log_2 2$ to produce a single term on the left-hand side. Most candidates who correctly combined at least two terms then went on to produce a fully correct solution. A more creative approach was to use an index, base 2, as the inverse of one of the log terms and then use rules of indices to simplify to the required equation.	
		ii	$y - 10x + 14 = 1$ $2x^2 - 1 - 10x + 14 = 1$	B1FT	<table><tr><td>Correct equation - www</td><td>State correct equation - aef not involving logs Allow FT on an incorrect equation from **(a)** if the</td></tr></table>	

			$2x^2 - 10x + 12 = 0 \Rightarrow x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2$, $x = 3$ $y = 7$, $y = 17$	M1* M1d* A1 [4]		substitution occurs before the log is removed ie B1FT is awarded for their $(ax^2 + b) - 10x + 14 = 1$
					Attempt to eliminate a variable	Using their $y - 10x + 14 = 1$ with their answer from **(a)**, which must be of the form $y = ax^2 + b$ oe, to obtain an equation in a single variable not involving logs M1 can still be awarded if the method to remove logs is not correct
					Attempt to solve 3 term quadratic	See additional guidance for valid methods
					Obtain both correct x,y pairs	Clear indication of which values are paired together could be implied by eg $y = 2 \times 2^2 - 1 = 7$ A0 if $y = 2x^2 - 1$ was obtained

					<table><tr><td></td><td>fortuitously in part (a)</td></tr></table> Examiner's Comments Most candidates appreciated the need to substitute their equation from part (a) into the new equation, with some substituting into the given equation and others removing the logarithm before doing so. Whichever method was employed, only the most able candidates were able to correctly remove the logarithm. The most common error was for the right-hand side to remain as 0, and some candidates never even removed the logarithm before solving the quadratic.	
			Total	8		
14			Reflection, stretch and translation (reflection) in the line $y = x$ (stretch) scale factor $\frac{1}{3}$ parallel to the x-axis (translation) $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	B1(AO2.5) B1(AO1.1) B1(AO1.1) B1(AO1.1) [4]	<table><tr><td>All three correct</td><td>Do not accept any other wording</td></tr><tr><td>Accept 'in the x-direction' accept 'factor' or 'SF' for 'scale factor'</td><td>Do not accept 'in/on/across/up the x-axis' or '$\frac{1}{3}$ units'</td></tr><tr><td>Accept '5 units in the negative y-direction' or '-5 units parallel to the y-axis'</td><td>Do not accept 'in/on/across/up the y-axis'</td></tr></table>	

					Order of transformations must be correct for all 4 marks to be awarded	
			Total	4		
15		a	(a =)75	B1 (AO 3.3) [1]	**Examiner's Comments** The correct value of a was frequent, but so too was a = 100.	
		b	25 is the value that T approaches after a long time So therefore it is the ambient temperature	B1 (AO 2.2a) [1]	oe e.g. room temperature, minimum, lowest, etc. — Not e.g. initial, etc. **Examiner's Comments** All the options on the mark scheme appeared. Some candidates did not realise that an explanation in the context of the model was needed and tried to give a geometrical interpretation.	
		c	$-ake^{-kt}$	B1 (AO 3.1a)	Correct rate of change of T	

			$-ak = -15$ $k = \frac{1}{5}$	M1 (AO 3.4) A1ft (AO 1.1) [3]	Substitute $t = 0$ into their rate of change and equate with + / –15 oe FT their $\frac{15}{a}$ **Examiner's Comments** This was not well understood, with very few candidates using the fact that the gradient of e^{kx}.is equal to ke^{kx}. It was very common to see attempts to solve 85 = 25 + ae^{-kt}, with the value of a from (a) and t = 1.	
		d	$45 = 25 + 75e^{-\frac{1}{5}t} \Rightarrow 75e^{-\frac{1}{5}t} = 20$ (eg) $-\frac{1}{5}t = \ln\left(\frac{4}{15}\right) \Rightarrow t = ...$ After 6.6 mins	M1 (AO 1.1) M1 (AO 1.1) A1 (AO 3.2a) [3]	Substitute $T = 45$ and subtract 25 from both sides — Their a and k Take logs correctly and attempt to solve for t Cao (no FT on this mark) with units — 6.6087792– **Examiner's Comments** Inevitably those who did not obtain a value of a and/or k were unable to make progress in this part. Those who had incorrect values seemed to understand the method. A few set T = 55. Because of the difficulties encountered in (c) very few correct answers were seen. Note that here	

					units were expected to be mentioned (cf AO3.2a) and candidates need to be aware that these are important, particularly in modelling questions.	
		e	Decrease the value of a	B1 (AO 3.5c) [1]	Ignore mention of changes to k and/or 25 Examiner's Comments A fair number of candidates made no response to this part, but where suggestions were seen the idea was understood.	
			Total	9		
16		a	When $t = 0$, $M = 300$ $300e^{-0.05t} = 150$ $e^{-0.05t} = 0.5$ $-0.05t = \ln 0.5$ $t = 13.9$ (minutes)	B1 (AO 2.2a) M1 (AO 3.1a) A1 (AO 1.1) [3]	Identify that the initial mass is 300g — Could be implied by eg $e^{-0.05t} = 0.5$ Equate to 150 and attempt to solve — Correct order of operations as far as attempting t If using logs on $300e^{-0.05t} = 150$ then the LHS must be dealt with correctly Obtain 13.86, or better — Allow 14 minutes www Or 13 minutes and 52 seconds	

					Examiner's Comments This question was very well answered with candidates identifying the initial mass, and then setting up and solving a relevant equation. Most candidates worked exactly throughout to provide a sufficiently accurate final answer.	
		b	$M_2 = 400e^{kt}$ $320 = 400e^{10k}$ $k = 0.1\ln 0.8$ $M_2 = 400e^{-0.0223t}$ Substance 1: $\frac{dM_1}{dt} = -15e^{-0.05t}$	B1 (AO 2.2a) M1 (AO 1.1a) A1 (AO 1.1) M1 (AO 3.1a)	<table><tr><td>State or imply $400e^{kt}$</td><td>Could be implied by stating general form of Ae^{kt} with $A = 400$ Any unknowns permitted</td></tr><tr><td>Attempt to find k</td><td>Substitute $M = 320$, $t = 10$ and attempt k Must be using valid method</td></tr><tr><td>Obtain correct expression for mass of second substance</td><td>Allow exact or decimal k (2sf or better) Must be seen or used as a complete term, not just implied by stated values of A and k</td></tr><tr><td>Attempt differentiation at least once</td><td>To obtain $ae^{-0.05t}$ or $be^{-0.0223t}$, where a and b are non-zero constants not 300 and 400</td></tr></table>	

			Substance 2: $\frac{dM_2}{dt} = -8.93e^{-0.0223t}$ $-15e^{-0.05t} = -8.93e^{-0.0223t}$ $e^{0.0277t} = 1.681$ $0.0277t = 0.519$ time = 18.75 minutes	A1ft (AO 1.1) M1 (AO 3.1a) M1 (AO 1.1) A1 (AO 3.2a)	Both derivatives correct Equate derivatives and rearrange as far as $e^{f(t)} = c$ Attempt to solve equation of form $e^{f(t)} = c$ Obtain correct	respectively Following their equation for substance 2 Equation must be of the form $ae^{-0.05t} = be^{-0.0223t}$ Combining like terms to result in a two term equation – not necessarily on opposite sides If logs are introduced earlier then allow M1 only if the products are correctly split so eg ln(15) × (–0.05t) is M0 M0 if attempting to take a log of a term that is negative As far as attempting t Or equiv if logs have been taken earlier Units required Could be 18

				[8]	value for t Allow 18.7, 18.8 or 19 mins | minutes and 45 seconds Must have been working with 3sf or better throughout **Examiner's Comments** The vast majority of candidates were able to make some progress on this question, and a number of fully correct solutions were seen. Candidates appreciated the need to find an expression for the mass of the second substance, and were able to make a good attempt at finding the two parameters. As the substance was decaying, some candidates used an initial structure of $M = Ae^{-kt}$, but sign errors were relatively common when substituting back for k. A few candidates simply equated the two expressions for the mass, but most realised that it was the derivatives that should be equated and made a reasonable attempt to do so. Solving the ensuing equation was found to be challenging. Some attempted to rearrange first whereas others introduced logarithms straightaway. Sign errors were common, especially in solutions where candidates were working exactly as the coefficient of 0.1ln0.8 is not obviously negative. Some candidates spoilt an otherwise correct solution by not working to a sufficient degree of accuracy throughout their solution resulting in an incorrect final answer.	
			Total	11		
17		a	E.g. $\log_3 x^2 = 2\log_3 x$; the student has ignored the brackets and used the power rule incorrectly E.g. $x = 3^2$; the student has done 2^3	E1 (AO 2.3) E1 (AO 2.3) [2]	Error identified with explanation Error identified with explanation	

		b	$(2\log_3 x + 1)(\log_3 x - 2) = 0$ $\log_3 x = -0.5$, $\log_3 x = 2$ $x = 3^{-0.5}$ or $x = 3^2$ $x = \frac{1}{3}\sqrt{3}$ and $x = 9$	M1 (AO 3.1a) A1 (AO 1.1) M1 (AO 1.1a) A1 (AO 1.1) [4]	Attempt to solve quadratic in $\log_3 x$ Obtain two correct roots **BC** Attempt correct process to find x at least once Obtain both correct roots	soi Any equivalent exact form
			Total	6		
18		a	$S = Ae^{kt}$ $S = 3.1e^{kt}$ $\frac{dS}{dt} = 3.1ke^{kt}$	B1 (AO 3.3) B1 (AO 3.3) M1 (AO 3.3) M1 (AO 3.4)	State or imply appropriate exponential model Identify correct initial value Attempt differentiation	Other models are possible eg using t as number of years after a year other than 2015 OR $S = ab^t$ OR $a = 3.1$ May still be A and not 3.1 OR $\frac{dS}{dt} = 3.1(\ln b)b^t$

			$0.8 = 3.1\,k\mathrm{e}^{0}$ hence $k = 0.258$ $S = 3.1\mathrm{e}^{0.258t}$, where S is the annual sales in millions of devices and t is the number of years after 2015	A1 (AO 2.5) [5]	Substitute into derivative and attempt to find k Correct equation with variables clearly defined | OR $0.8 = 3.1(\ln b)$ so $b = 1.29$ OR $S = 3.1(1.29)t$	
		b	when $t = 2$, $S = 3.1\mathrm{e}^{0.516} = 5.19$ (millions) E.g. so observed value was 5.2 (millions) so model appears to be reliable	M1 (AO 3.4) E1 (AO 3.5a) [2]	Find value of S when $t = 2$ Comment on reliability of model | Using their model which must be of the form $A\mathrm{e}^{kt}$ or ab^{t}, with numerical parameters Must have correct 5.2 million, from correct model	
		c	E.g. unlikely to be a reliable prediction as market will become saturated so sales unlikely to increase at same rate	E1 (AO 3.5b)	Comment about trend unlikely to continue, or device becoming obsolete or extrapolation may not be reliable	
			Total	8		
19			DR $\mathrm{e}^{x} = 3 + 2\mathrm{e}^{y}$	M1(AO 3.1a)		

			$(3 + 2e^y)^2 - 4e^{2y} = 33$ $9 + 12e^y + 4e^{2y} - 4e^{2y} = 33$ $12e^y = 24$ $e^y = 2$ $y = \ln 2$ $e^x - 4 = 3$ $e^x = 7$ $x = \ln 7$	A1(AO 1.1) M1(AO 1.1a) A1(AO 1.1) A1(AO 2.1) [5]	Attempt to eliminate one variable Obtain correct equation in one variable – allow unsimplified Simplify and attempt to solve Obtain $y = \ln 2$ Obtain $x = \ln 7$, using either equation. | or $e^{2x} - 4(0.5e^x - 1.5)^2 = 33$ or $6e^x = 42$ etc	
			Total	5		
20			$2^{3x-1} = 3^{x+4} \Rightarrow 3x - 1 = \log_2(3^{x+4})$ $(3x - 1) = (x + 4)\log_2 3 \Rightarrow x = K$	M1 (AO 1.1a) M1 (AO 1.1) A1 (AO 1.1) [3]	Take logs of both sides – allow any (consistent) base including natural logs Bring both powers to	

			$x=\dfrac{4\log_2 3+1}{3-\log_2 3}=5.19$		<table><tr><td>the front and attempt to make *x* the subject</td><td>In base 10 $x=\dfrac{4\log 3+\log 2}{3\log 2-\log 3}=5.19$</td></tr></table>	
			Total	3		