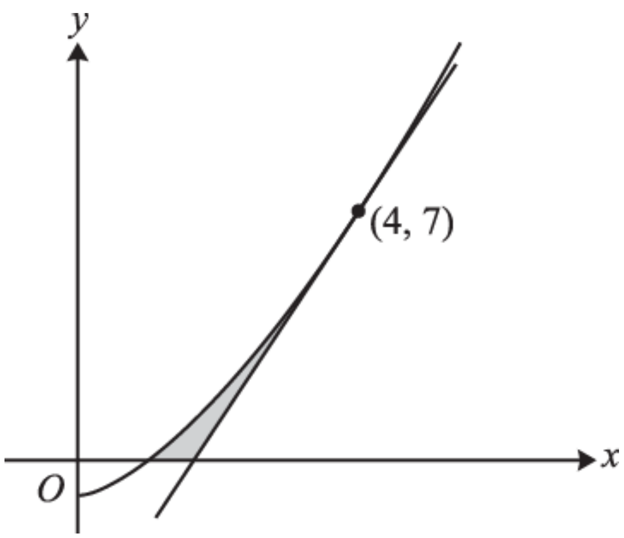
1.



The diagram shows the curve $y = x^{\frac{3}{2}} - 1$, which crosses the x-axis at (1, 0), and the tangent to the curve at the point (4, 7).

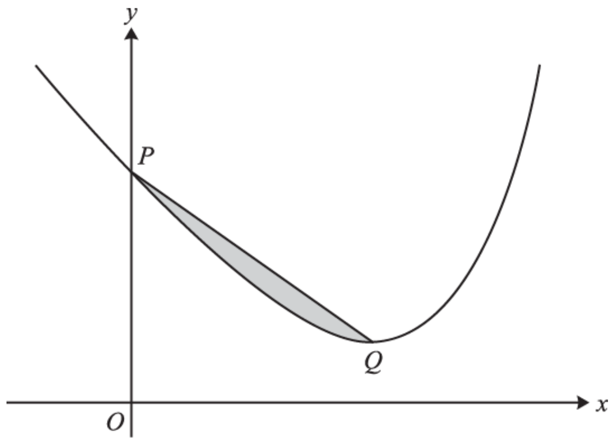
i. Show that $\int_1^4 (x^{\frac{3}{2}} - 1)\,dx = 9\frac{2}{5}$.

[4]

ii. Hence find the exact area of the shaded region enclosed by the curve, the tangent and the x-axis.

[5]

2.



The diagram shows the curve

$$y = e^{2x} - 18x + 15.$$

The curve crosses the y-axis at P and the minimum point is Q. The shaded region is bounded by the curve and the line PQ.

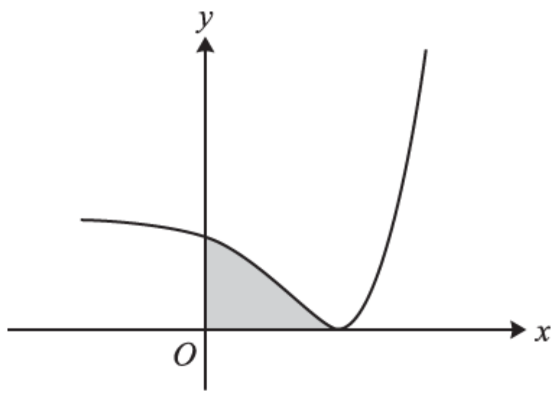
i. Show that the x-coordinate of Q is ln3.

[3]

ii. Find the exact area of the shaded region.

[8]

3.



The diagram shows the curve $y = e^{3x} - 6e^{2x} + 32$.

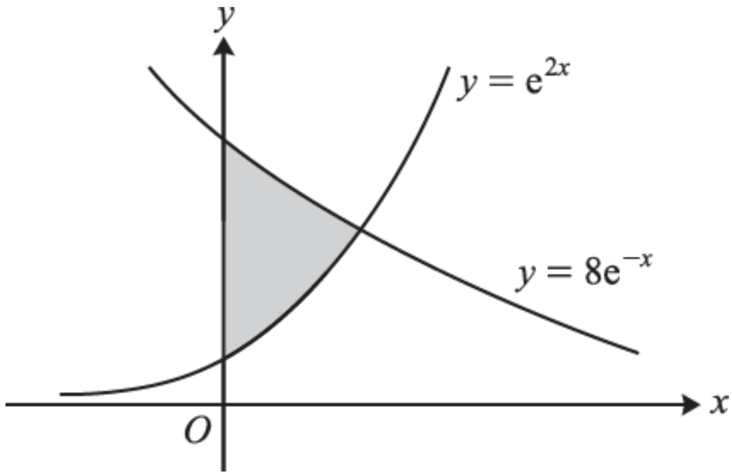
i. Find the exact x-coordinate of the minimum point and verify that the y-coordinate of the minimum point is 0.

[4]

ii. Find the exact area of the region (shaded in the diagram) enclosed by the curve and the axes.

[4]

4.



The diagram shows the curves $y = e^{2x}$ and $y = 8e^{-x}$. The shaded region is bounded by the curves and the y-axis. Without using a calculator,

i. solve an appropriate equation to show that the curves intersect at a point for which $x = \ln 2$,

[2]

ii. find the area of the shaded region, giving your answer in simplified form.

[5]

5.

Show that $\int_0^{\frac{1}{4}\pi} \frac{1-2\sin^2 x}{1+2\sin x\cos x}\,dx = \frac{1}{2}\ln 2$.

[5]

6.

i. Use the quotient rule to show that the derivative of $\frac{\cos x}{\sin x}$ is $\frac{-1}{\sin^2 x}$.

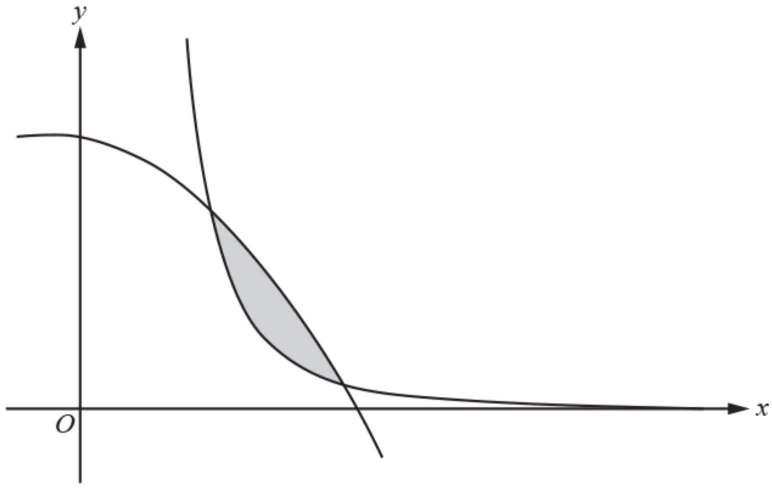
[2]

ii. Show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{\sqrt{1+\cos 2x}}{\sin x \sin 2x}\,dx = \frac{1}{2}(\sqrt{6}-\sqrt{2})$.

[6]

7. Use integration to find the exact value of $\int_{\frac{1}{16}\pi}^{\frac{1}{8}\pi}(9-6\cos^2 4x)\,\mathrm{d}x$. [5]

8.



The diagram shows parts of the curves $y = 11 - x - 2x^2$ and $y = \frac{8}{x^3}$. The curves intersect at (1, 8) and (2, 1).
Use integration to find the exact area of the shaded region enclosed between the two curves. [7]

9. A curve is defined by the parametric equations $x = \frac{2t}{1+t}$ and $y = \frac{t^2}{1+t}$, $t \neq -1$.

(a) (i) Show that the curve passes through the origin. [1]

(ii) Find the y-coordinate when $x = 1$. [1]

(b) Show that the area enclosed by the curve, the x-axis and the line $x = 1$ is given by

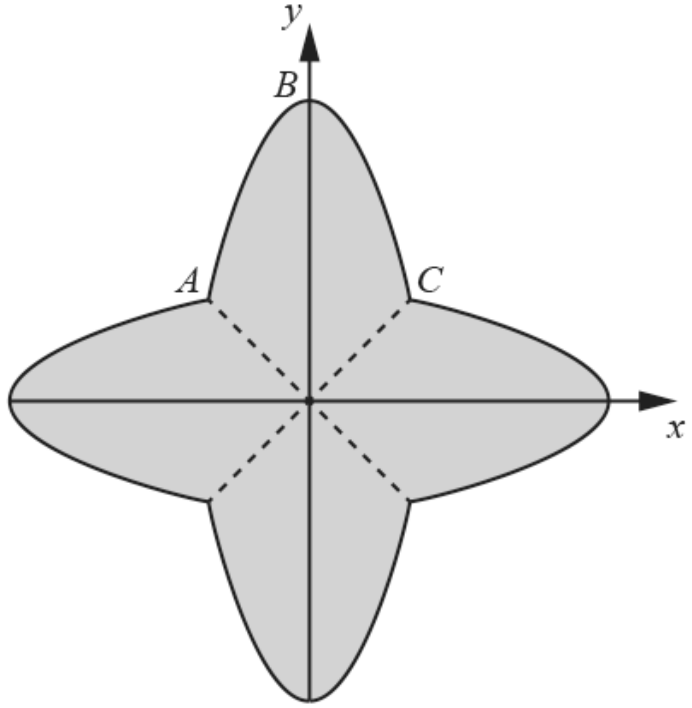
$$\int_0^1 \frac{2t^2}{(1+t)^3}\,\mathrm{d}t.$$ [5]

(c) **In this question you must show detailed reasoning.**

Hence use an appropriate substitution to find the exact area enclosed by the curve, the x-axis and the line $x = 1$. [6]

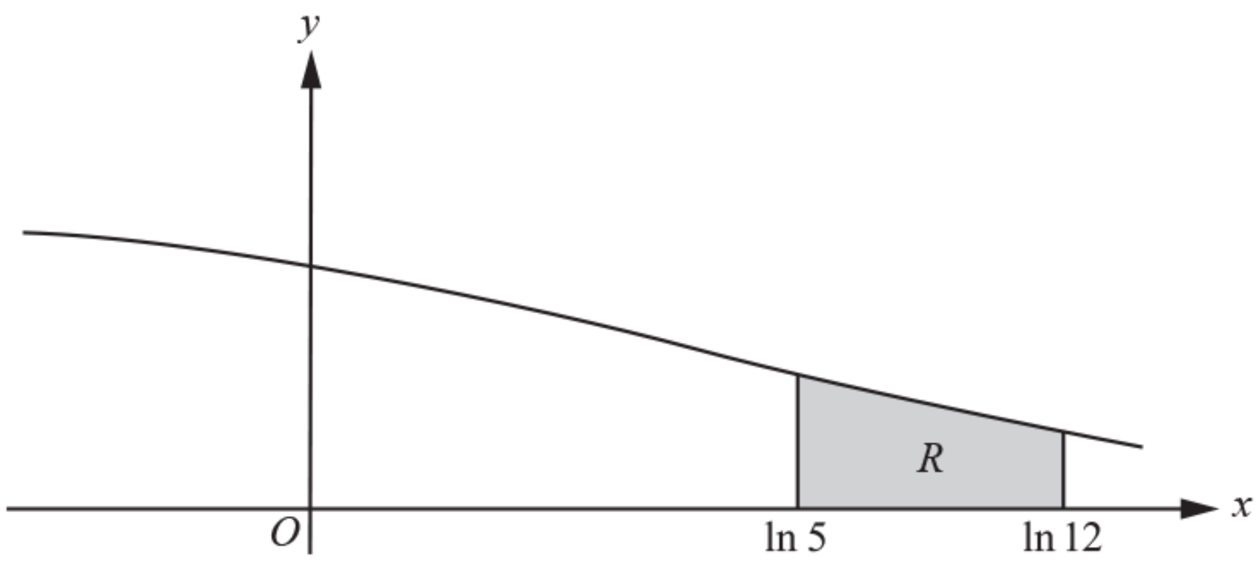
10. Find the area of the region enclosed by the curve $y = 5x - x^2$ and the line $y = 2x$. [5]

11. The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$.



Find the area of the shaded region. [7]

12.



The diagram shows the curve with parametric equations $x = \ln(t^2 - 4), \quad y = \frac{4}{t^2}$, where $t > 2$.

The shaded region R is enclosed by the curve, the x-axis and the lines $x = \ln 5$ and $x = \ln 12$.

(a) Show that the area of R is given by

$$\int_a^b \frac{8}{t(t^2 - 4)} \, dt,$$

where a and b are constants to be determined. [4]

(b) **In this question you must show detailed reasoning.**

Hence find the exact area of R, giving your answer in the form $\ln k$, where k is a constant to be determined. [8]

(c) Find a cartesian equation of the curve in the form $y = f(x)$. [3]

END OF QUESTION paper

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$\int_1^4 \left(x^{\frac{3}{2}} - 1\right) \mathrm{d}x = \left[\frac{2}{5}x^{\frac{5}{2}} - x\right]_1^4$	M1	Attempt integration	Increase in power by 1 for at least one term – allow the –1 to disappear
		i		A1	Obtain fully correct integral	Coeff could be unsimplified eg $^{1}/_{2.5}$ Could have + *c* present
		i	= (12.8 – 4) – (0.4 – 1)	M1	Attempt correct use of limits	Must be explicitly attempting F(4) ‰ F(1), either by clear substitution of 4 and 1 or by showing at least (8.8) – (–0.6) Allow M1 if + *c* still present in both F(4) and F(1), but M0 if their *c* is now numerical Allow use in any function other than the original
		i	= $9^{2}/_{5}$**AG**	A1	Obtain $9^{2}/_{5}$ **Examiner's Comments** This was very well answered with most candidates gaining full marks. As the answer was given candidates were expected to show sufficient detail in their method, including the use of limits. Some candidates were clearly aided by the answer being given and were able to go back and amend incorrect working, though they must ensure that this is done consistently throughout the entire solution.	**AG**, so check method carefully Allow $^{47}/_{5}$ or 9.4
		ii	$m = {}^{3}/_{2} \times \sqrt{4} = 3$	M1*	Attempt to find gradient at (4, 7) using differentiation	Must be reasonable attempt at differentiation ie decrease the power by 1 Need to actually evaluate derivative at *x* = 4

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$y = 3x - 5$	M1d*	Attempt to find point of intersection of tangent with x-axis or attempt to find base of triangle	Could attempt equation of tangent and use $y = 0$ Could use equiv method with gradient eg $3 = {}^{7}/_{4-x}$ Could just find base of triangle using gradient eg $3 = {}^{7}/_{b}$
		ii	tangent crosses x-axis at $({}^{5}/_{3}, 0)$	A1	Obtain $x = {}^{5}/_{3}$ as pt of intersection or obtain ${}^{7}/_{3}$ as base of triangle	Allow decimal equiv, such as 1.7, 1.67 or even 1.6 www Allow M1M1A1 for $x = {}^{5}/_{3}$ with no method shown
		ii	area of triangle = $\frac{1}{2} \times (4 - {}^{5}/_{3}) \times 7 = 8{}^{1}/_{6}$	M1d**	Attempt complete method to find shaded area	Dependent on both previous M marks Find area of triangle and subtract from $9{}^{2}/_{5}$ Must have 1 < their x < 4, and area of triangle < $9{}^{2}/_{5}$ If using $\int(3x - 5)\mathrm{d}x$ then limits must be 4 and their x M1 for area of trapezium – area between curve and y -axis

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	shaded area = $9^2/_5 - 8^1/_6 = 1^7/_{30}$	A1	Obtain $1^7/_{30}$, or exact equiv **Examiner's Comments** This proved to be one of the most challenging questions on the paper. Whilst 25% of the candidates were able to provide accurate and concise solutions, 50% were unable to score any credit at all. Most candidates did seem to recognise that they needed to find the equation of the tangent, but many did not realise that differentiation was required to find the gradient. Of those who did find the correct equation, a number then just integrated this equation between 1 and 4, rather than appreciating that a different lower limit was required and that this would be given by the point of intersection of the tangent with the *x*-axis. Some of the more successful candidates drew a sketch graph and gave consideration to the area that was to be subtracted from the answer to part (i), but too many launched straight into calculations with seemingly no clear strategy.	A0 for decimal answer (1.23), unless clearly a recurring decimal (but not eg 1.2333…)
			Total	9		

Question			Answer/Indicative content	Marks	Part marks and guidance	
2		i	Differentiate to obtain $2e^{2x} - 18$	B1		
		i	Equate first derivative to zero and use legitimate method to reach equation without e involved	M1		
		i	Confirm $x = \ln 3$	A1	AG; necessary detail needed (in particular, for solutions concluding $x = \frac{1}{2}\ln 9 = \ln 3$ or equiv award A0) **Examiner's Comments** The majority of candidates had no difficulty in realising that equating the first derivative to zero was needed. With the answer ln 3 given in the question, solutions were expected to be sufficiently detailed. Many candidates failed to earn the final mark because their solutions went immediately from $x = \frac{1}{2}\ln 9$ or $x = \frac{\ln 9}{2}$ to x = ln3.	
		ii	Attempt integration	*M1	confirmed by at least one correct term	
		ii	Obtain $\frac{1}{2}e^{2x} - 9x^2 + 15x$	A1	or equiv	
		ii	Apply limits 0 and ln 3 to obtain exact unsimplified expression	M1	dep *M	
		ii	Obtain $4 - 9(\ln 3)^2 + 15 \ln 3$	A1	or exact (maybe unsimplified) equiv perhaps still involving e	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	Attempt area of trapezium or equiv, retaining exactness throughout	M1	using $\frac{1}{2}\ln 3\times(y_1+y_2)$ where y_1 is 15 or 16 and y_2 is attempt at y-coordinate of Q; if using alternative approach involving rectangle and triangle, complete attempt needs to be seen for M1; another alternative approach involves equation of PQ ($y=\frac{8-18\ln 3}{\ln 3}x+16$)with integration: M1 for attempting equation and integration, A1 for correct answer	
		ii	Obtain $\frac{1}{2}\ln 3\times(16+24-18\ln 3)$	A1	or equiv perhaps still including e	
		ii	Subtract areas the right way round, retaining exactness	M1	dep on award of all three M marks	
		ii	Obtain 5 ln 3 – 4	A1	or similarly simplified exact equiv **Examiner's Comments** Most candidates were able to make some progress with this question but only 14% of candidates succeeded in recording full marks. The integration was usually carried out accurately but a few used incorrect limits such as ln 3 and 16. It was quite common for the limit 0 to be ignored when evaluating the area under the curve. A considerable problem for many involved the term $9(\ln 3)^2$. This was often carelessly written as $9\ln 3^2$ and then 'simplified' to 9ln 9 or 18ln	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					3. Perhaps it was this awkward term that prompted many to resort to decimal approximations. A variety of approaches for finding the shaded area were seen. One that was wrong was to treat the equation of *PQ* as the tangent to the curve at *P*. The most successful approach was to find the area of the trapezium between *PQ* and the *x*-axis. A similar approach involved the areas of triangle and rectangle but there was more scope for sign errors for those adopting this. An alternative approach, successful occasionally, involved finding the equation of *PQ* and integrating this; again, for some, an awkward-looking gradient for the line was the motive for moving to decimal approximations. Candidates who earned all eight marks usually provided clear and succinct solutions, where the steps were briefly described and where appropriate simplifications were carried out as the solution progressed.	
			Total	11		

Question			Answer/Indicative content	Marks	Part marks and guidance	
3		i	State first derivative is $3e^{3x} - 12e^{2x}$	B1	Or equiv	
		i	Equate first derivative to zero and attempt solution of equation of form $k_1e^{3x} - k_2e^{2x} = 0$	M1	At least as far as $e^x = c$; M0 for false method such as $\ln(3e^{3x}) - \ln(12e^{2x}) = 0$	
		i	Obtain ln 4 or exact equiv and no other	A1	Obtained by legitimate method	
		i	Substitute $x = \ln 4$ or $e^x = 4$ to confirm $y = 0$	A1	AG; using exact working with all detail present: needs sight of $4^3 - 6 \times 4^2 + 32$ or similar equiv	
		ii	Integrate to obtain $k_3e^{3x} + k_4e^{2x} + 32x$	M1	For non-zero constants	
		ii	Obtain $\frac{1}{3}e^{3x} - 3e^{2x} + 32x$ or equiv	A1		
		ii	Apply limits correctly to expression of form $k_3e^{3x} + k_4e^{2x} + 32x$	M1	Using limits 0 and their answer from part (i)	
		ii	Simplify to obtain $32 \ln 4 - 24$ or $64 \ln 2 - 24$	A1	Or suitably simplified equiv **Examiner's Comments** For a question involving routine techniques, it was disappointing that only 37% of the candidates recorded all eight marks. Almost all candidates differentiated correctly in part (i) but then many struggled to find the x-coordinate of the minimum point. The equation $3e^{3x} - 12e^{2x} = 0$ prompted some to a next incorrect step of $\ln(3e^{3x}) - \ln(12e^{2x}) = 0$; others followed $\ln e^{3x} = \ln 4e^{2x}$ with $3x = 2x\ln 4$. Those with an approach involving factorisation such as $3e^{2x}(e^x - 4) = 0$ often included extra	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					incorrect roots such as 0 or $\frac{1}{2}$. Confirmation that the minimum point lies on the *x*-axis required a little more detail than the mere statement $e^{3\ln4} - 6e^{2\ln4} + 32 = 0$ and, as a result, some candidates did not earn the final mark of part (i). Some candidates also found the second derivative but no confirmation that the stationary point is indeed a minimum was needed. There was more success with part (ii). Integration was handled efficiently and the area was produced in a suitably simplified form. There were occasional sign errors and some answers were not exact. Surprisingly, there were a few cases where $\int \pi y^2 \, dx$ was attempted.	
			Total	8		

Question			Answer/Indicative content	Marks	Part marks and guidance	
4		i	Either State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		Verifying by substitution of ln2 in each equation earns B0B0
		i	Obtain $e^x = 2$ and hence $x = \ln 2$	B1	AG; necessary detail needed	
		i	Or 1 State $e^{2x} = 8e^{-x}$ and so $e^{3x} = 8$	B1		
		i	State $3x = 1n8$, $x = 1n8^{\frac{1}{3}}$ and hence $x = 1n2$	B1	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ does not earn the second B1
		i	Or 2 State $e^{2x} = 8e^{-x}$ and $2x = 1n8 - x$	B1		
		i	State $3x = \ln 8$, $x = \ln 8^{\frac{1}{3}}$ and hence $x = \ln 2$	B1	AG; necessary detail needed	Going immediately from $x = \frac{1}{3}$ 1n8 to $x =$ 1n2 does not earn the second B1
		ii	Integrate to obtain $K_1 e^{-x}$ and $k_2 e^{2x}$	M1	Any non-zero constants k_1 and k_2	
		ii	Obtain correct $-8e^{-x} - \frac{1}{2}e^{2x}$ or, if done separately, $-8e^{-x}$ and $\frac{1}{2}e^{2x}$	A1		
		ii	Apply limits 0 and ln2 correctly to their integral(s)	M1	Condone one sign slip; earned by sight of $-8e^{-\ln 2} - \frac{1}{2}e^{2\ln 2} + 8 + \frac{1}{2}$(or equivs if integrals treated separately)	M1 also implied by sight only of $-4 - 2 + \frac{1}{2}$ (or equivs …)
		ii	Obtain at least $-4 - 2 + 8 + \frac{1}{2}$(or equivs)	*A1		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	Obtain $\frac{5}{2}$ or equiv	A1	Final A1 dependent on *A1 **Examiner's Comments** In order for particular skills to be assessed, this question required candidates to answer both parts without the use of a calculator. This meant that sufficient detail had to be shown in solutions and many candidates lost some credit for not providing all the necessary steps. The candidate offering only $\int_0^{\ln 2} e^{2x}\,dx = \frac{3}{2}$as part of the solution in part (ii) did not earn many marks. Part (i) specified the use of an appropriate equation and therefore candidates showing that both curves had the same *y*-coordinate of 4 when $x = \ln 2$ did not earn the marks. With the answer given in the question, examiners needed to see a convincing solution of the equation $e^{2x} = 8e^{-x}$. For example, solutions going directly from $x = \frac{1}{3}\ln 8$ to $x = \ln 2$ did not earn the second mark. Other attempts showed basic errors in the application of logarithms to both sides of the equation. It was disappointing that only 52% of candidates earned both marks in part (i). Although 64% of the	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					candidates earned all the marks in part (ii), surprising problems were revealed by some of the solutions from other candidates. The trapezium rule was used in some cases for finding the area under one of the curves and incorrect limits were sometimes seen. There were also attempts to treat the region as one between the curves and the *y*-axis; this is a possible method although it involves integration techniques from Core Mathematics 4 and the attempts seldom succeeded. With the instruction to answer this part without the use of a calculator, candidates needed to show sufficient detail in their solutions. Most did so although some lost marks through a failure to show clearly how, for example, $\frac{1}{2}e^{21n\,2}$ becomes 2.	
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance	
5			$\int \frac{\cos 2x}{1+\sin 2x}\,(\mathrm{d}x)$	B1*	cos2x = 1 – 2sin^2x or (1 +)sin2x = (1 +) 2sin$x$cos$x$ seen	if B0B0M0A0, SC4 for F[x] = ½ln(1 + 2sin$x$cos$x$) or ½ln(1 + sin2x)
				B1*	numerator and denominator both correct in the integral soi	final mark may still be awarded
			F[x] = $k$ln(1 + sin2x) soi	M1dep*	or $k$ln(1 + u) or $k$ln(u) following their substitution www	
			k = ½	A1	correct k for their substitution	
			½ln(1 + sin($^\pi/_2$)) – ½ln(1 + 0) = ½ln2	A1 AG	correct use of limits www Examiner's Comments Most candidates recognised at least one of the double angle substitutions, and many went on to spot the correct form of the integral and score full marks. A surprising number achieved the correct integrand, however, and then failed to progress. Incorrect splitting of the fraction such as ∫(cos2x + cot2x) dx and substitutions which went astray were fairly common. A small number of candidates didn't bother with double angles, but differentiated the numerator and spotted the logarithmic form, often going on to achieve full marks.	minimum working: ½ln2 – ½ln1 or ½ln(1 + 1) oe
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance	
6		i	$\dfrac{\sin x \times -\sin x - \cos x \times \cos x}{\sin^2 x}$ may be implied by $\dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x}$	M1	or $-\sin x \times \dfrac{1}{\sin x}$ $+\cos x \times -(\sin x)^{-2} \times \cos x$ oe	allow sign errors only if **M0**, **SC1** for just $\dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \dfrac{-1}{\sin^2 x}$
		i	eg $= \dfrac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ and completion to $\dfrac{-1}{\sin^2 x}$ AG	A1	eg $= \dfrac{-\sin^2 x}{\sin^2 x} - \dfrac{\cos^2 x}{\sin^2 x}$ oe and completion to $\dfrac{-1}{\sin^2 x}$	need to see at least two correct, constructive steps and statement of given answer for **A1** NB $\sin^2 x + \cos^2 x = 1$ seen may be a constructive intermediate step
		i			<u>Examiner's Comments</u> This was very well-done, with most candidates achieving full marks. A few showed insufficient working and lost a mark, and a small minority either misquoted the Quotient Rule or the relevant trigonometric identity.	
		ii	$\cos 2x = 2\cos^2 x - 1$ substituted in numerator	M1	or alternative form of double angle formula plus Pythagoras leading to no term in $\sin^2 x$ in numerator	may be awarded if not seen as part of fraction
		ii	$\sin 2x = 2\sin x\cos x$ substituted in denominator	M1		
		ii	$\dfrac{\sqrt{2}\cos x}{2\sin^2 x \cos x}$	A1		NB $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{1}{\sqrt{2}\sin^2 x}\,dx$
		ii	$F[x] = \pm k\dfrac{\cos x}{\sin x}$	M1*	k must not be obtained from square rooting a negative number	NB $-\dfrac{\cos x}{\sqrt{2}\sin x}$

Question			Answer/Indicative content	Marks	Part marks and guidance	
		ii	$F\left[\frac{1}{4}\pi\right]-F\left[\frac{1}{6}\pi\right]$	M1dep*	eg $\frac{-\cos\pi/4}{\sqrt{2}\times\sin\pi/4}-\frac{-\cos\pi/6}{\sqrt{2}\times\sin\pi/6}$	eg $\frac{-1/\sqrt{2}}{\sqrt{2}\times 1/\sqrt{2}}-\frac{-\sqrt{3}/2}{\sqrt{2}\times 1/2}$ at least one correct intermediate step following substitution needed as well as statement of given result eg $-\frac{\sqrt{2}}{2}(1-\sqrt{3})$
		ii	$=\frac{1}{2}(\sqrt{6}-\sqrt{2})$ www **AG**	A1		
		ii			<u>Examiner's Comments</u> A surprisingly high proportion of candidates did not recognise that double angle formulae were needed here, and went round in circles trying to use integration by parts or achieve a logarithmic form. Some of those who did successfully use the correct identities to produce a multiple of the function in part (ii) didn't make the connection between the two parts and either ran out of steam or produced reams of incorrect work. That said, there were many examples of excellent work: well-presented, succinct solutions with sufficient detail to meet the show that demand.	
			Total	8		

Question			Answer/Indicative content	Marks	Part marks and guidance	
7			cos8x seen in integrand	M1		
			F[x] = Ax + Bsin8x oe	M1*	A and B are non-zero constants	
			$\mathrm{F}[x]=6x-\frac{3}{8}\sin 8x$	A1		
			$\mathrm{F}[\frac{1}{8}\pi]-F[\frac{1}{16}\pi]$	M1*dep		
			$\frac{3}{8}\pi+\frac{3}{8}$ oe	A1		allow eg 0.375π + 0.375 or fractions not in lowest terms **Examiner's Comments** Most candidates realised the need to use the appropriate double angle formula and successfully integrated to obtain an expression involving sin8x. Sign errors were quite common, however, and 12x was commonly seen. A few candidates worked with cos^2x and didn't score. A significant minority had no idea how to deal with cos^{2}4x and tried to integrate directly.
			Total	5		

Question			Answer/Indicative content	Marks	Part marks and guidance		
8			$\int(11 - x - 2x^2)\mathrm{d}x = 11x - \frac{1}{2}x^2 - \frac{2}{3}x^3$ $\int 8x^{-3}\mathrm{d}x = -4x^{-2}$ $(22 - 2 - {}^{16}/_3) - (11 - {}^1/_2 - {}^2/_3) = {}^{29}/_6$ $(-1) - (-4) = 3$ ${}^{29}/_6 - 3 = {}^{11}/_6$	M1 A1 M1 A1 M1 M1 A1 [7]	Attempt integration of $11 - x - 2x^2$ Obtain $11x - {}^1/_2x^2 - {}^2/_3x^3$ Attempt integration of $8x^{-3}$ Obtain $-4x^{-2}$ Use limits of $x = 1, 2$ Attempt correct method to find shaded area (at any point) Obtain ${}^{11}/_6$ or exact equiv	Increase in power by 1 for at least 2 terms Obtain correct integral Integrate to kx^{-2} Allow unsimplified coeff In both integrals Must follow clear attempt at integration Must be F(2) – F(1) ie correct order and subtraction M0 if incorrect order of subtraction, even if ${}^{11}/_6$ subsequently appears	

Question			Answer/Indicative content	Marks	Part marks and guidance		
						as final answer M1 can follow M0 for use of limits A0 for decimal answer unless clearly a recurring decimal (but not eg 1.833…) ISW if $^{11}/_{6}$ seen but then followed by eg 1.83 Answer only is 0/7 - need to see evidence of integration, but use of limits does not need to be explicit **Alternative MS for subtracting first:** M1 - attempt subtraction in correct order M1 - attempt integration of $\pm(11 - x - 2x^2)$ A1 - obtain	

Question			Answer/Indicative content	Marks	Part marks and guidance		
						$\pm(11x - {}^{1}/_{2}x^{2} - {}^{2}/_{3}x^{3})$, signs must be consistent with their subtraction M1 - attempt integration of $\pm 8x^{-3}$ A1 - obtain $\mp 4x^{-2}$, sign must be consistent with their subtraction M1 - correct use of limits in entire integral A1 - obtain ${}^{11}/_{6}$ Ignore sight of $11 - x - 2x^{2} = 8x^{-3}$ prior to subtraction occurring Adding functions prior to integration will get max of 5 marks - M0M1A1 M1A1M1A0 (**Alt MS**) – to give same credit as integrating separately, using limits and then adding Multiplying	

Question			Answer/Indicative content	Marks	Part marks and guidance		
						through by x^3 prior to integration can get M1 for use of limits, and possibly M1 if subtraction happens before multiplying through	
					Examiner's Comments This fairly standard integration question was very well answered by many candidates. The integration was usually accurate, especially of the quadratic curve. Candidates were usually able to write the reciprocal curve in an appropriate form, but it was a relatively common error for the index to decrease rather than increase. Candidates were then able to use limits accurately to evaluate their definite integral. The most common approach was to find two separate areas, and then find the difference to get the shaded area. This method invariably resulted in the correct answer, whereas candidate who subtracted before integrating sometimes did so in the incorrect order. A minority of candidates decided to multiply through by x^3 before integrating, whilst the actual integration may be easier they did not appreciate that they were no longer dealing with the		

Question			Answer/Indicative content	Marks	Part marks and guidance	
					original functions.	
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance		
9		a	(a) when $x = 0$, $t = 0$ and hence $y = 0$	E1(AO2.4) [1]	Justify (0, 0) convincingly		
		a	(b) when $x = 1$, $t = 1$ and hence $y = 0.5$	B1(AO1.1) [1]	Obtain $y = 0.5$		
		b	$\frac{dx}{dt} = \frac{2}{(1+t)^2}$	M1(AO2.1)	Attempt $\frac{dx}{dt}$	Using quotient rule, or other valid method	
				A1(AO2.1)	Obtain correct derivative		
			$\int \frac{t^2}{1+t} dx = \int \frac{t^2}{1+t} \times \frac{2}{(1+t)^2} dt$	M1(AO2.1)	Use $\int y dx = \int y \frac{dx}{dt} dt$		
			$= \int \frac{2t^2}{(1+t)^3} dt$	A1(AO2.1)	Obtain given answer		
				B1(AO2.4) [5]	Justify t -limits from $x = 0, 1$	$x = 0$: $\frac{2t}{1+t} = 0$ so $t = 0$ $x = 1$: $\frac{2t}{1+t} = 1$ *$2t = 1 + t$ so $t = 1$*	

Question			Answer/Indicative content	Marks	Part marks and guidance		
		c	DR use $u = 1 + t$ giving $\mathrm{d}u = \mathrm{d}t$ $\int \frac{2t^2}{(1+t)^3}\,\mathrm{d}t = \int \frac{2(u-1)^2}{u^3}\,\mathrm{d}u$ $= \int 2u^{-1} - 4u^{-2} + 2u^{-3}\,\mathrm{d}u$ $= \left[2\ln u + 4u^{-1} - u^{-2}\right]_1^2$ = (2ln2 + 2 – 0.25) – (2ln1 + 4 – 1) $= 2\ln 2 - \frac{5}{4}$	E1(AO1.1a) M1(AO1.1a) A1(AO1.1) M1(AO1.1a) M1(AO1.1a) A1(AO1.1) [6]	Must be stated explicitly Attempt to change integrand to function of u Obtain correct integrand Attempt integration Attempt use of limits u = 1, 2 Obtain correct exact area	Any equivalent form Allow any exact equiv	
			Total	13			

Question			Answer/Indicative content	Marks	Part marks and guidance
10			$5x - x^2 = 2x$ ($x^2 - 3x = 0$ or $x(x - 3) = 0$)	M1(AO3.1a)	
			$x = 0$ or 3	A1(AO1.1)	
			$\int_0^3 (5x - x^2)\mathrm{d}x$ or $5x - x^2 - 2x$ (= 13.5)	M1(AO1.1a)	
			$'13.5' - \frac{1}{2} \times 3 \times 6$ or $\int_0^3 (3x - x^2)\mathrm{d}x$	M1(AO1.1)	
			= 4.5 = 4.5	A1(AO1.1) [5]	
			Total	5	

Question			Answer/Indicative content	Marks	Part marks and guidance		
11			<u>Summary of marks:</u>				
			Attempt find x at intersection of curves	M1 (AO 3.1a)	Can be implied		
			$x = 1$	A1 (AO 1.1)			
			Correct integral, any limits	M1 (AO 3.1a)			
			Correct numerical result	A1 (AO 1.1)	from correct limits		
			Attempt area of part or all of 2 × 2 square	M1 (AO 1.1)			
			Wholly correct method	M1 (AO 2.1)			
			$\frac{44}{3}$	A1 (AO 1.1) [7]			
			<u>Examples of methods:</u>				
			<u>Method 1</u> $3 - 2x^2 = x$ $2x^2 + x - 3 = 0$	M1		or $3 - 2x^2 = -x$ $2x^2 - x - 3 = 0$	
			$x = 1$	A1		$x = -1$	
			$\int_0^1 (3-2x^2)\mathrm{d}x$ or $\int_{-1}^1 (3-2x^2)\mathrm{d}x$	M1	Ignore other root		
			$= \left[3x - \frac{2x^3}{3}\right]_0^1$ or $\left[3x - \frac{2x^3}{3}\right]_{-1}^1$				
			$= \frac{7}{3}$ or $\frac{14}{3}$	A1	Correct integrand with any limits		
			$"\frac{7}{3}" - 1 \left(= \frac{4}{3}\right)$ or $"\frac{14}{3}" - 2 \left(= \frac{8}{3}\right)$	M1		or $"\frac{14}{3}" - 1 \left(= \frac{11}{3}\right)$	
			$8 \times "\frac{4}{3}" + 4$ or $4 \times "\frac{8}{3}" + 4$	M1	Attempt area above $y = 1$ or above $y = x$	$4 \times \frac{11}{3}$	
			$= \frac{44}{3}$	A1	Complete correct method		
			<u>Method 2</u> $3 - 2x^2 = x$	M1			
			$x = 1$	A1			
			$\int_1^3 \left(\frac{y-3}{2}\right)^{\frac{1}{2}}\mathrm{d}y$	M1			
			= …….				

Question			Answer/Indicative content	Marks	Part marks and guidance	
			$= \frac{4}{3}$	A1	Method 3 $3 - 2x^2 = x$ $x = 1$ $\int_0^1 (3 - 2x^2 - 1)\,dx$ $= \left[2x - \frac{2x^3}{3}\right]_0^1$ $= \frac{4}{3}$	
			$\frac{4}{3} + \frac{1}{2}$ $(= \frac{11}{6})$	M1	$\frac{4}{3} + \frac{1}{2}$ $(= \frac{11}{6})$	
			$8 \times \frac{11}{6}$	M1	$8 \times \frac{11}{6}$	Other correct methods may be seen
			$= \frac{44}{3}$	A1	$= \frac{44}{3}$	
					Examiner's Comments A large variety of correct methods were seen. Some were unnecessarily long. Examples of correct, although long, methods were these: 1. Find the inverse function in order to find the equation of the reflection of the given curve in $y = x$. Then solve this with the given function in order to find the point of intersection, C. 2. Rearrange the given function to make x the subject and then find $\int x\,dy$. Some candidates found the points where the given curve cuts the x-axis and hence integrated with incorrect limits. Many made mistakes in trying either to add or subtract all or part of the area of the middle	

Question			Answer/Indicative content	Marks	Part marks and guidance	
					square. Perhaps the neatest method was $4 \times \int_{-1}^{1} (3 - 2x^2 - 1)\mathrm{d}x + 4$.	
			Total	7		

Question			Answer/Indicative content	Marks	Part marks and guidance	
12		a	$x = \ln(t^2 - 4) \Rightarrow \frac{dx}{dt} = \frac{2t}{t^2 - 4}$	M1 (AO 1.1)	Attempt differentiation of x using chain rule – must be of the form $\frac{kt}{t^2 - 4}$	
			$\text{Area} = \int \frac{4}{t^2}\left(\frac{2t}{t^2 - 4}\right) dt$	M1 (AO 1.1a)	Use of $\int y \frac{dx}{dt}\, dt$ with their $\frac{dx}{dt}$	
			$= \int \frac{8}{t(t^2 - 4)}\, dt$	A1 (AO 2.2a)	AG	
			$a = 3$, $b = 4$	B1 (AO 2.2a)	Correct limits	
				[4]		

Question			Answer/Indicative content	Marks	Part marks and guidance	
		b	DR $\frac{8}{t(t^2-4)} \equiv \frac{A}{t} + \frac{B}{t-2} + \frac{C}{t+2}$ 8 = $A(t-2)(t+2) + Bt(t+2) + Ct(t-2)$ $A = -2$, $B = 1$, $C = 1$ $\int\left(-\frac{2}{t}+\frac{1}{t-2}+\frac{1}{t+2}\right)\mathrm{d}t = -2\ln t + \ln(t-2) + \ln(t+2)$ (–2ln4 + ln2 + ln6) – (–2ln3 + ln1 + ln5) l $\left(\frac{27}{20}\right)$ n	B1 (AO 3.1a) M1 (AO 1.1a) A2 (AO 1.1,1.1) M1* (AO 1.1) M1dep* (AO 1.1) M1 (AO 2.1) A1 (AO 2.2a) [8]	Correct form of partial fractions Cover up, substituting or equating coefficients – must be a complete method for finding one of A, B or C A1 for one correct Attempt to integrate all three terms – must be of the form $\alpha \ln t + \beta \ln(t-2) + \gamma \ln(t+2)$ Applying their limits correctly Correctly combining all their log terms – dependent on both previous M marks $k = \frac{27}{20}$	

Question			Answer/Indicative content	Marks	Part marks and guidance	
		c	$t^2 = \frac{4}{y} \Rightarrow x = \ln\left(\frac{4}{y} - 4\right)$ $e^x = \frac{4}{y} - 4 \Rightarrow y = \ldots$ $y = \frac{4}{e^x + 4}$ Alternative solution $e^x = t^2 - 4$ $t^2 = e^x + 4 \Rightarrow y = \ldots$ $y = \frac{4}{e^x + 4}$	M1* (AO 3.1a) M1dep* (AO 1.1) A1 (AO 1.1) M1* M1dep* A1 [3]	Re-arrange and eliminate *t* Remove logs and attempt to make *y* the subject Remove logs Rearrange and eliminate *t*	
			Total	15		