1. i. The first three terms of an arithmetic progression are $2x$, $x + 4$ and $2x - 7$ respectively. Find the value of x.

[3]

ii. The first three terms of another sequence are also $2x$, $x + 4$ and $2x - 7$ respectively.

a. Verify that when $x = 8$ the terms form a geometric progression and find the sum to infinity in this case.

[4]

b. Find the other possible value of x that also gives a geometric progression.

[4]

2. An arithmetic progression u_1, u_2, u_3, … is defined by $u_1 = 5$ and $u_{n+1} = u_n + 1.5$ for $n \geqslant 1$.

i. Given that $u_k = 140$, find the value of k.

[3]

A geometric progression w_1, w_2, w_3, … is defined by $w_n = 120 \times (0.9)$ $n-1$ for $n \geqslant 1$.

ii. Find the sum of the first 16 terms of this geometric progression, giving your answer correct to 3 significant figures.

[2]

iii. Use an algebraic method to find the smallest value of N such that $\sum_{n=1}^{N} u_n > \sum_{n=1}^{\infty} w_n$.

[6]

3. The first term in an arithmetic series is $(5t + 3)$, where t is a positive integer. The last term is $(17t + 11)$ and the common difference is 4. Show that the sum of the series is divisible by 12 when, and only when, t is odd. [7]

4. An ice cream seller expects that the number of sales will increase by the same amount every week from May onwards. 150 ice creams are sold in Week 1 and 166 ice creams are sold in Week 2. The ice cream seller makes a profit of £1.25 for each ice cream sold.

(a) Find the expected profit in Week 10. [3]

(b) In which week will the total expected profits first exceed £5000? [5]

(c) Give two reasons why this model may not be appropriate. [2]

5. (a) Ben saves his pocket money as follows.

Each week he puts money into his piggy bank (which pays no interest). In the first week he puts in 10p. In the second week he puts in 12p. In the third week he puts in 14p, and so on.

How much money does Ben have in his piggy bank after 25 weeks? [4]

(b) On January 1st Shirley invests £500 in a savings account that pays compound interest at 3% per annum. She makes no further payments into this account. The interest is added on 31st December each year.

(i) Find the number of years after which her investment will first be worth more than £600. [4]

(ii) State an assumption that you have made in answering part (b)(i). [1]

6. The first three terms of an arithmetic series are $9p$, $8p - 3$, $5p$ respectively, where p is a constant.

Given that the sum of the first n terms of this series is -1512, find the value of n. [6]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$(x + 4) - 2x = (2x - 7) - (x + 4)$	M1	Attempt to eliminate d to obtain equation in x only	Equate two expressions for d, both in terms of x Could use $a + (n - 1)d$ twice, and then eliminate d Could use $u_1 + u_2 + u_3 = S_3$ or $u_2 = ½\,(u_1 + u_3)$
		i	OR			
		i	$2x + d = x + 4 \quad 2x + 2d = 2x - 7$	A1	Obtain correct equation in just x	Allow unsimplified equation A0 if brackets missing unless implied by subsequent working or final answer
		i	$2x = 15$ $x = 7.5$	A1	Obtain $x = 7.5$	Any equivalent form Allow from no working or T&I
		i				**Alt method:** B1 – state, or imply, $2x + 2d = 2x - 7$, to obtain $d = -3.5$ M1 – attempt to find x from second equation in x and d A1 – obtain $x = 7.5$
		i				<u>Examiner's Comments</u> Many candidates were successful in this part of the question, with the most popular approach being to first find d = -3.5 and then use a second equation to find x. This was usually successful, although sign errors proved a pitfall for some. However, a number of candidates made no further progress beyond finding d, often because they did not consider a third equation. The other common method was to find two expressions for d by considering the difference of consecutive terms

						which could then be equated and solved. This was an elegant and concise method, but a lack of brackets resulted in errors being made. Other, more creative, solutions were also seen including adding the sum of the three terms and equating this to an expression for S_3.
		ii	terms are 16, 12, 9 $^{12}/_{16} = 0.75$, $^{9}/_{12} = 0.75$	B1	List 3 terms	Ignore any additional terms given Must show two values of 0.75, or unsimplified fractions Must state, or imply, that ratio has been checked twice, using both pairs of consecutive terms
		ii	common ratio of 0.75 so GP	B1	Convincing explanation of why it is a GP	No need to show actual division of terms to justify 0.75, so allow eg arrows from one term to the next with 'x0.75' **SR B2** if 16, 12, 9 never stated explicitly in a list but are soi in a convincing method for $r = 0.75$ twice
		ii	$S_\infty = {}^{16}/_{1-0.75} = 64$	M1	Attempt use of $^{a}/_{1-r}$	Must be correct formula Could be implied by method Allow if used with their incorrect a and / or r Allow if using $a = 8$, even if 16 given correctly in list
		ii		A1	Obtain 64	A0 if given as 'approximately 64'
		ii				<u>Examiner's Comments</u> Virtually all of the candidates gained the first mark for stating the three relevant terms, and most also gained the final two marks for finding the sum to infinity, though a few used $\frac{4}{3}$ as their ratio. It was the second mark that proved to be the most challenging. Candidates had been asked to verify that the terms did form a geometric progression, and were expected to provide a convincing proof

						that considered the ratio between two pairs of terms, or an equivalent justification. Whilst some candidates did provide this explanation, far too many assumed that it was a geometric progression and simply found the ratio from a single pair of terms.
		iii	${}^{(2x-7)}/_{(x+4)} = {}^{(x+4)}/_{2x}$ $4x^2 - 14x = x^2 + 8x + 16$	M1*	Attempt to eliminate r to obtain equation in x only	Equate two expressions for r, both in terms of x Could use ar^{n-1} twice, and then eliminate r from simultaneous eqns
		iii	OR			
			$2xr = x + 4$ $2xr^2 = 2x - 7$			
		iii	$3x^2 - 22x - 16 = 0$ $(3x + 2)(x - 8) = 0$ $x = {}^{-2}/_3$, $x = 8$	A1	Obtain $3x^2 - 22x - 16 = 0$	Allow $6x^2 - 44x - 32 = 0$ Allow $3x^3 - 22x^2 - 16x = 0$, or a multiple of this Allow any equivalent form, as long as no brackets and like terms have been combined Condone no = 0, as long as implied by subsequent work
		iii		M1d*	Attempt to solve quadratic	Dependent on first M1 for valid method to eliminate r See guidance sheet for acceptable methods
		iii		A1	Obtain $x = {}^{-2}/_3$	Allow recurring decimal, but not rounded or truncated Condone $x = 8$ also given Allow from no working or T&I
		iii				<u>**Examiner's Comments**</u> This proved to be a challenging question for many candidates. Whilst most were able to make some attempt at it, it was often not enough to gain even the first mark. The most efficient solution was to equate two algebraic expressions for the ratio, and then rearrange them to get a quadratic which could then be solved. Some candidates were able

						to provide a concise and elegant solution in this way. Some candidates did embark on this method, but then attempted to first simplify their fractions which invariably went wrong. Others started with the generic equations for the *n*th term of a geometric progression so that when they eliminated *r* their equation involved the square or square root of a rational expression.
			Total	11		
2		i	$u_k = 5 + 1.5(k - 1)$	M1*	Attempt *n*th term of an AP, using $a = 5$ and $d = 1.5$	Must be using correct formula, so M0 for $5 + 1.5k$ Allow if in terms of *n* not *k* Could attempt an *n*th term definition, giving $1.5k + 3.5$
		i	$5 + 1.5(k - 1) = 140$ $k = 91$	M1d*	Equate to 140 and attempt to solve for *k*	Must be valid solution attempt, and go as far as an attempt at *k* Allow equiv informal methods Answer only gains full credit
		i		A1	Obtain 91	**Examiner's Comments** This proved to be a straightforward question for many candidates, and the majority gained full credit. Most candidates used the formula for the *n*th term of an arithmetic progression and another effective method was to generate an *n*th term expression for the sequence. Informal methods were rarely correct, and the other common error was to use the *n*th term as $5 + 1.5n$ or even $n + 1.5$.
		ii	$S_{16} = \frac{120(1-0.9^{16})}{1-0.9}$ = 978	M1	Attempt to find the sum of 16 terms of GP, with $a = 120$, $r = 0.9$	Must be using correct formula

		ii		A1	Obtain 978, or better	If > 3sf, allow answer rounding to 977.6 with no errors seen Answer only, or listing and summing 16 terms, gains full credit **Examiner's Comments** The majority of candidates were equally successful here, with solutions being mostly fully correct. Despite being told that it was a geometric progression, many candidates did not recognise w_n as being of the form a × r^{n-1} and instead generated the first few terms of the sequence to find the values of the first term and the common ratio, not always correctly.
		iii	$\frac{1}{2}N(10 + (N-1) \times 1.5) > \frac{120}{1-0.9}$	B1	Correct sum to infinity stated	Could be 1200 or unsimplified expression
		iii	$N(1.5N + 8.5) > 2400$ $3N^2 + 17N - 4800 > 0$ $N = 38$	B1	Correct S_N stated	Any correct expression, including unsimplified
		iii		M1*	Link S_N of AP to S_∞ of GP and attempt to rearrange	Must be recognisable attempt at S_N of AP and S_∞ of GP, though not necessarily fully correct Allow any (in)equality sign, including < Must rearrange to a three term quadratic, not involving brackets
		iii		A1	Obtain correct 3 term quadratic	aef - not necessary to have all algebraic terms on the same side of the (in)equation Allow any (in)equality sign
		iii		M1d*	Attempt to solve quadratic	See additional guidance for acceptable methods May never consider the negative root M1 could be implied by sight of 37.3, as long as from correct quadratic

		iii		A1	Obtain $N = 38$ (must be equality)	A0 for $N \geq 38$ or equiv in words eg 'N is at least 38' Allow A1 if 38 follows =, > or ≥ being used but A0 if 38 follows < or ≤ being used A0 if second value of N given in final answer Must be from an algebraic method - at least as far as obtaining the correct quadratic **Examiner's Comments** The majority of candidates could identify that the sum to infinity was required, and correctly state this. There was then some uncertainty as to what was required on the left-hand side, with both the sum of the geometric progression and the nth term of the arithmetic progression being common errors. However many candidates could make a reasonable attempt at both of the summations, but there were a surprising number of errors when attempting to simplify their inequality. The most common errors included only multiplying one side by 2 in an attempt to remove the fraction or incorrect expansion of brackets. Candidates then had to solve the quadratic with both completing the square and use of the quadratic formula being seen, though the latter was by far the most common. A few candidates clearly anticipated that the quadratic would factorise and gave up when they realised that this was not the case. Some candidates, with an incorrect quadratic equation, simply wrote down two solutions with no method shown. In these circumstances, Examiners cannot speculate as to what method may have been used and no credit can be awarded. To gain full credit in this question, candidates had to appreciate that N had to be a positive integer and hence discard

						their negative root and round up their positive root. Some candidates spoilt an otherwise correct solution by failing to do so.
			Total	11		
3			$(5t + 3) + 4(n - 1) = (17t + 11)$ $n = 3t + 3$ $S_N = \frac{1}{2}(3t+3)\{(5t+3)+(17t+11)\}$ $S_N = \frac{1}{2}(3t+3)(22t+14) = 3(t+1)(11t+7)$ When t is odd, $t = 2k + 1$ so $S_N = 3(2k + 2)(22t + 18)$ $= 12(k + 1)(11k + 9)$ hence multiple of 12 When t is even, $t = 2k$ so $S_N = 3(2k + 1)(22k + 7)$ hence always odd	M1(AO3.1a) A1(AO2.1) M1(AO2.1) A1(AO2.1) E1(AO2.2a) E1(AO2.4) E1(AO2.4) [7]	Attempt to use $a + (n - 1)d = l$ Obtain $n = 3t + 3$ Attempt to find sum of AP Obtain $S_N = 3(t + 1)(11t + 7)$ oe Consider S_N when t is odd Fully correct and convincing proof | Allow consideration of odd and even factors	

					Allow worded eg 3 × odd × odd		
			Total	7			
4		a	u_{10} = 150 + 9 × 16 = 294 ice creams profit = 294 × £1.25 = £367.50	B1(AO3.1b) M1(AO1.1) A1FT(AO3.2a) [3]	Identify AP, with a = 150 and d = 16 Correct u_{10} Correct profit for their u_{10}	Units required	
		b	£5000 ÷ £1.25 = 4000 $S_N = 0.5N(300 + (N - 1)16)$ $150N + 8N(N - 1) > 4000$ $8N^2 + 142N - 4000 > 0$ $N = 15.18$ (and possibly – 32.9) Week 16	B1(AO3.1b) M1(AO3.4) A1(AO3.1a) M1(AO1.1) A1(AO3.2a) [5]	Identify that 4000 sales are reqd Attempt S_N of AP, with a = 150 and d = 16 Link to 4000 (any sign) and rearrange to 3 term quadratic	Or use d = £20 Or d = 20 Or link to 5000 (any sign) and rearrange to 3 term quadratic	

					Attempt to solve quadratic Conclude with Week 16 only	**BC** Allow ‘during Week 16’	
		c	Sales cannot continue to increase for ever Weekly sales could fluctuate depending on the Weather	E1(AO3.5b) E1(AO3.5b) [2]	Refer to trend not continuing Refer to changes week by week	Any two different reasons	
			Total	10			
5		a	$a = 10$, $d = 2$ $S_n = \frac{25}{2}(2 \times 10 + 24 \times 2)$ = 850 After 25 weeks he has £8.50	B1 (AO1.1a) B1 (AO1.1) A1 (AO1.1) A1 (AO3.2a) [4]	soi Subst their *a* and *d* into correct formula Correct money		

					notation, 2 dp only	
		b	(i) $500 \times 1.03^n > 600$ $1.03^n > 1.2$ $n > \log_{1.03} 1.2$ $n > 6.17$ First worth > 600 after 7 years	M1 (AO3.1b) M1 (AO1.1) A1 (AO1.1) A1 (AO3.2a) [4]	Allow "=" throughout Or attempt log both sides to same base so $n > \frac{\log_a 1.2}{\log_a 1.03}$ BC	
			(ii) E.g. Assume interest rate does not change.	E1 (AO3.5b) [1]		
			Total	9		
6			$(8p - 3) - 9p = 5p - (8p - 3)$ $p = 3$ $a = 27,\ d = -6$ $\frac{n}{2}\left[2(27) + (n-1)(-6)\right] = -1512$	M1 (AO 3.1a) A1 (AO 1.1) A1FT (AO 1.1) M1 (AO 2.1) M1 (AO 1.1)	Setting up an equation to find p — Allow a single sign error Using their value of p to calculate a and d	

			$n^2 - 10n - 504 = 0 \Rightarrow (n - 28)(n + 18) = 0$ $n = 28$ only	A1 (AO 2.2a) [6]	Setting up an equation using the correct formula for the sum of an AP equated to −1512 Expand and attempt to solve 3-term quadratic equation in n This mark should be withheld if $n = -18$ appears as part of the final answer | Solving of 3-term quadratic may be done **BC**	
			Total	6		