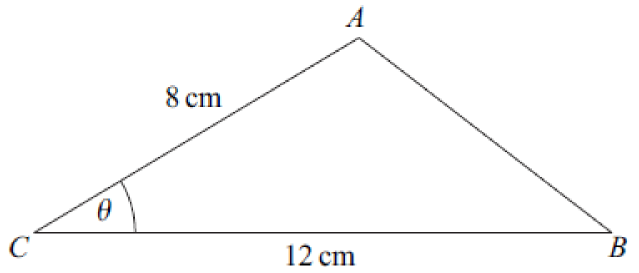
Trigonometry Questions

4 The triangle ABC, shown in the diagram, is such that $AC = 8$ cm, $CB = 12$ cm and angle $ACB = \theta$ radians.

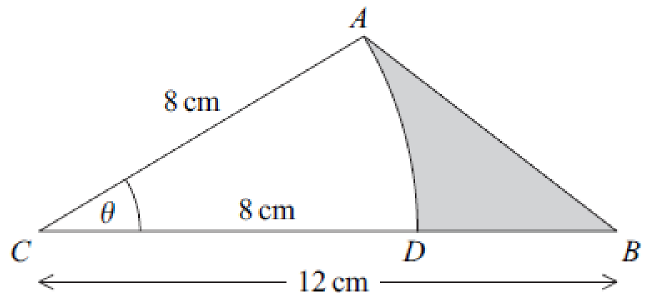


The area of triangle $ABC = 20\,\text{cm}^2$.

(a) Show that $\theta = 0.430$ correct to three significant figures. *(3 marks)*

(b) Use the cosine rule to calculate the length of AB, giving your answer to two significant figures. *(3 marks)*

(c) The point D lies on CB such that AD is an arc of a circle centre C and radius 8 cm. The region bounded by the arc AD and the straight lines DB and AB is shaded in the diagram.

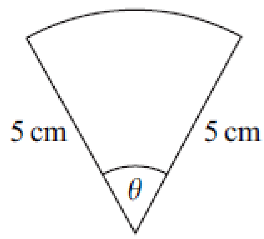


Calculate, to two significant figures:

(i) the length of the arc AD; *(2 marks)*

(ii) the area of the shaded region. *(3 marks)*

1 The diagram shows a sector of a circle of radius 5 cm and angle θ radians.



The area of the sector is $8.1\ \text{cm}^2$.

(a) Show that $\theta = 0.648$. *(2 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

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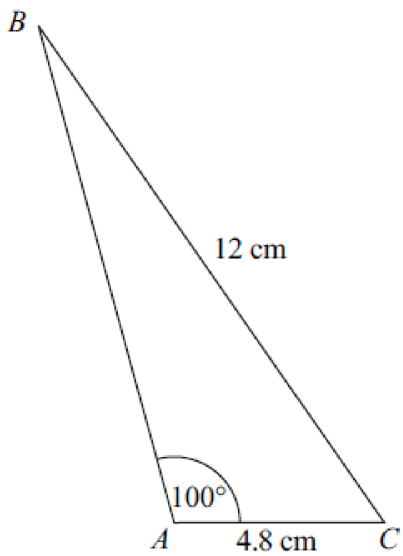
2 The diagram shows a triangle ABC.



The lengths of AC and BC are 4.8 cm and 12 cm respectively.

The size of the angle BAC is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

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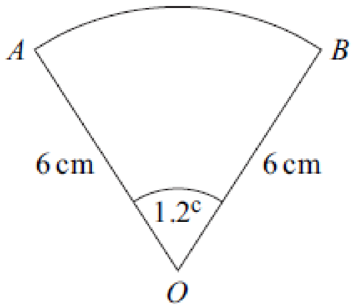
1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle AOB is 1.2 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) Find the perimeter of the sector OAB. *(3 marks)*

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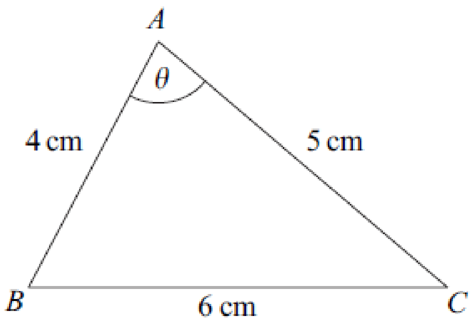
4 The triangle ABC, shown in the diagram, is such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm. The angle BAC is θ.



(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(b) Hence use a trigonometrical identity to show that $\sin\theta = \frac{3\sqrt{7}}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

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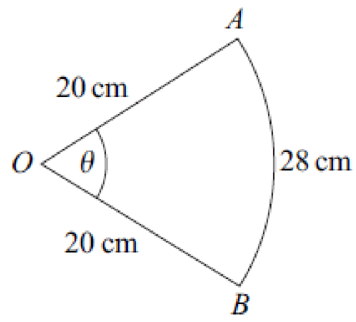
3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.

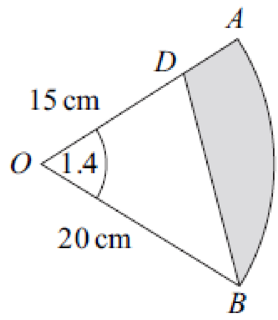


The length of the arc AB is 28 cm.

(a) Show that $\theta = 1.4$. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) The point D lies on OA. The region bounded by the line BD, the line DA and the arc AB is shaded.



The length of OD is 15 cm.

(i) Find the area of the shaded region, giving your answer to three significant figures. *(3 marks)*

(ii) Use the cosine rule to calculate the length of BD, giving your answer to three significant figures. *(3 marks)*

Trigonometry Answers

4(a)	Area of triangle $=\frac{1}{2}(12)(8)\sin\theta$	M1		Use of $\frac{1}{2}ab\sin C$ or full equivalent
	$\sin\theta = \frac{20}{48}$ [$=0.41(666\ldots)$]	A1		OE (giving 0.412 to 0.42)
	$\Rightarrow \theta = 0.4297(7\ldots) = 0.430$ to 3sf	A1	3	AG(need to see >3sf value)
(b)	$\{AB^2 =\}8^2 + 12^2 - 2\times8\times12\times\cos\theta$	M1		
	$= 64 + 144 - 174.5\ldots$	m1		Accept 33 to 34 inclusive if three values not separate
	$\Rightarrow AB = 5.78\ldots = 5.8$ cm to 2sf	A1	3	If not 2sf condone 5.78 to 5.79 inclusive. Condone ±
(c)(i)	Arc $AD = 8\theta$;	M1;		
	$= 3.44.. = 3.4$ cm to 2sf	A1	2	If not 2sf condone 3.438 to 3.44 inclusive
(ii)	Area of sector $=\frac{1}{2}r^2\theta$	M1		Stated or used [or 13.7(6..) seen]
	Shaded area = Area of triangle – sector area	M1		Difference of areas
	Shaded area $= 20 - 0.5\times8^2\times\theta$			
	$= 6.2$ cm^2 to 2sf	A1	3	Condone 6.24 to 6.2472
	Total		**11**	

1(a)	Area of sector $=\frac{1}{2}r^2\theta = \frac{1}{2}\times5^2\times\theta$	M1		$\frac{1}{2}r^2\theta$ seen or used
	$12.5\theta = 8.1 \Rightarrow \theta = 0.648$	A1	2	AG Condone $\theta = 0.648$ used to show that area = 8.1
(b)	Arc $= 5\theta$;	M1		5θ
	….. $= 3.24$ cm	A1		PI by a correct perimeter
	$\Rightarrow$ Perimeter $= 10 +$ arc $= 13.24$ cm	A1√	3	CSO Condone missing/wrong units; condone 3sf i.e. 13.2 if no obvious error NMS 3/3
	Total		**5**	

2(a)	$\frac{\sin B}{4.8} = \frac{\sin 100}{12}$	M1		Use of the sine rule
	$\sin B = \frac{4.8\sin 100}{12}$ [$= 0.39(392\ldots)$]	m1		Rearrangement
	(angle ABC) $= 23.19(8\ldots)$ {$= 23.2°$.}	A1	3	AG Need >1dp eg 23.19 or 23.20
(b)	Angle $C = 80° - 23.2° = 56.8°$	M1		Valid method to find a relevant angle eg C (PI eg by correct $\sin C$) or 23.2°+10°
	Area of triangle $= 0.5\times12\times4.8\times\sin C$	M1		OE eg $0.5\times4.8\times12\times\cos(B+10)$
	….. $= 24.09\ldots = 24.1$ cm^2. (to 3sf)	A1	3	Condone missing/wrong units
	Total		**6**	

1(a)	{Area of sector =} $\frac{1}{2}r^2\theta$	M1		
	$=0.5\times 36\times 1.2=21.6$ cm^2	A1	2	Condone missing/wrong units throughout the paper
(b)	Arc $= r\theta$	M1		
	$= 6\times 1.2 = 7.2$	A1		
	Perimeter = 12 + 7.2 = 19.2 cm	A1ft	3	Ft on incorrect evaluation of 6×1.2
	Total		**5**	

4(a)	$6^2 = 4^2 + 5^2 - 2(4)(5)\cos\theta$	M1		Use of the cosine rule
	$\cos\theta = \frac{4^2+5^2-6^2}{2(4)(5)}$	m1		Rearrangement
	$\cos\theta = \frac{5}{40} = \frac{1}{8}$	A1	3	CSO **AG** (be convinced)
(b)	$\cos^2\theta + \sin^2\theta = 1$	M1		Stated or used (PI)
	$\sin^2\theta = \frac{63}{64}$	A1		Or better
	$\sin\theta = \frac{\sqrt{63}}{8} = \frac{\sqrt{9\times 7}}{8} = \frac{3\sqrt{7}}{8}$	A1	3	**AG** (be convinced)
(c)	Area of triangle = $0.5\times 4\times 5\times \sin\theta$.	M1		
	….. = $\frac{30\sqrt{7}}{8}$ cm^2.	A1	2	OE (Condone 9.92)
	Total		**8**	

3(a)	Arc = $r\theta$	M1		For $r\theta$ or 20θ or PI by 20×1.4
	$28 = 20\theta \Rightarrow \theta = 1.4$	A1	2	AG
(b)	Area of sector = $\frac{1}{2}r^2\theta$	M1		$\frac{1}{2}r^2\theta$ OE seen
	$=\frac{1}{2}20^2(1.4) = 280$ (cm^2.)	A1	2	Condone absent cm^2.
(c)(i)	Area triangle $=\frac{1}{2}\times 15\times 20\times \sin 1.4$ (= 147.8….)	M1		Use of $\frac{1}{2}ab\sin C$ OE
	Shaded area = Area of sector – area of triangle	M1		
	= 280 – 147.8 = 132 (cm^2.) (3sf)	A1ft	3	Ft on [ans (b) – 147.8…] to 3sf provided [….] > 0
(ii)	$\{BD^2 =\}15^2 + 20^2 - 2\times 15\times 20\cos 1.4$	M1		RHS of cosine rule used
	$= 225 + 400 - 101.98...$	m1		Correct order of evaluation
	$\Rightarrow BD = \sqrt{523.019...} = 22.86..$ = 22.9 (cm) to 3 sf	A1	3	Condone absent cm
	Total		**10**	