Transformations Questions

6 (a) Describe the geometrical transformation that maps the curve with equation $y = \sin x$ onto the curve with equation:

(i) $y = 2\sin x$; *(2 marks)*

(ii) $y = -\sin x$; *(2 marks)*

(iii) $y = \sin(x - 30°)$. *(2 marks)*

(b) Solve the equation $\sin(\theta - 30°) = 0.7$, giving your answers to the nearest 0.1° in the interval $0° \leqslant \theta \leqslant 360°$. *(3 marks)*

(c) Prove that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$. *(4 marks)*

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8 (a) Describe the single geometrical transformation by which the curve with equation $y = \tan\frac{1}{2}x$ can be obtained from the curve $y = \tan x$. *(2 marks)*

(b) Solve the equation $\tan\frac{1}{2}x = 3$ in the interval $\mathbf{0 < x < 4\pi}$, giving your answers in radians to three significant figures. *(4 marks)*

(c) Solve the equation

$$\cos\theta(\sin\theta - 3\cos\theta) = 0$$

in the interval $0 < \theta < 2\pi$, giving your answers in radians to three significant figures. *(5 marks)*

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7 (a) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Write down the **two** solutions of the equation $\tan x = \tan 61°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) (i) Given that $\sin\theta + \cos\theta = 0$, show that $\tan\theta = -1$. *(1 mark)*

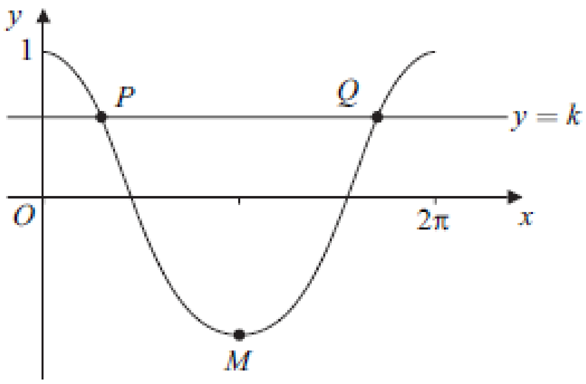
(ii) Hence solve the equation $\sin(x - 20°) + \cos(x - 20°) = 0$ in the interval $0° \leqslant x \leqslant 360°$. *(4 marks)*

(d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20°)$. *(2 marks)*

(e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q. The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of Q in terms of π and α. *(1 mark)*

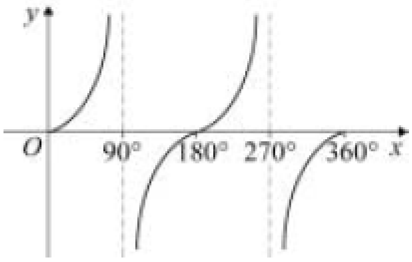
(c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. *(2 marks)*

(d) Solve the equation $\cos 2x = \cos\frac{4\pi}{5}$ in the interval $0 \leqslant x \leqslant 2\pi$, giving the values of x in terms of π. *(4 marks)*

Transformations Answers

Q	Solution	Marks	Total	Comments
6(a)(i)	Stretch (I) in y-direction (II) Scale factor 2 (III)	M1A1	2	>1 transformation is M0. M1 for (I) and either (II) or (III) or (III)
(ii)	Reflection; in x-axis	M1 A1	2	'Reflection'/ 'reflect(ed)' (or in y-axis or $y = 0$ or $x = 0$)
(iii)	Translation; $\begin{bmatrix} 30^\circ \\ 0 \end{bmatrix}$	B1 B1	2	'Translation'/'translate(d)' Accept **full** equivalent in words provided linked to 'translation/move/shift' and **positive** x-direction (Note: B0 B1 is possible)
(b)	$\{\theta - 30^\circ =\}\sin^{-1}(0.7) = 44.4\ldots^\circ$ $\ldots\ldots\ldots = 180^\circ - 44.4^\circ$ $\theta = 74.4^\circ,\ 165.6^\circ$	M1 m1 A1	3	Inverse sine of 0.7 PI eg by sight of 44, 74 or better Valid method for 2nd angle Condone >1dp accuracy
(c)	$\ldots = \cos^2 x + 2\cos x \sin x + \sin^2 x +$ $\cos^2 x - 2\cos x \sin x + \sin^2 x$ $\ldots. = 2\cos^2 x + 2\sin^2 x$ $= 2(\cos^2 x + \sin^2 x) = 2\ (1)$ $= 2$	M1 A1 M1 A1	4	Award for either bracket expanded correctly OE $\cos^2 x + \sin^2 x = 1$ stated or used. AG (be convinced)
	Total		13	

Question	Solution	Marks	Total	Comments
8(a)	Stretch (I) in x-direction (II) scale factor 2 (III)	M1 A1	2	Need(I) and one of (II),(III) M0 if more than one transformation
(b)	$\tan^{-1}3 = 1.2(49….) (=\alpha)$	M1		$\tan^{-1}3$ [PI by 71.(56..)°]
	$\{\frac{1}{2}x=\}\quad \pi+\alpha$;	m1		Correct quadrant; condone degrees or mix
	$\frac{1}{2}x = 1.249…;\ 4.3906…$			
	$x = 2.498... = 2.50$ to 3 sf $x = 8.781... = 8.78$ to 3 sf	A1 A1	4	Condone 2.5 otherwise deduct **max** of 1 mark throughout Q8 from A marks if 'correct' rads. but to 2sf or final answers in degrees. (143°, 503°) As usual, accept greater accuracy answers. Ignore extra values outside the given interval (0 to12.6). If > 2 values inside interval lose an A mark for each one. NB M1m0A1A0 is possible
	SC after M0 for error $\tan x = 6$; Either $x = 1.40(5),\ 4.54(7),\ 7.68(8),\ 10.8(3)$ or $x = 80.5°, 260.5°, 440.5°, 620.5°$ SC B1 (accept each rounded or truncated to 3 sf)			
(c)	$\cos\theta = 0,\quad \sin\theta - 3\cos\theta = 0$	M1		Need both
	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ or $\tan\theta = 3$	M1		$\tan\theta = \frac{\sin\theta}{\cos\theta}$ seen/used
	$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} = 1.57(07…)$	B1		Accept $\frac{\pi}{2}$
	or $\theta = \frac{3\pi}{2} = 4.71(23..)$	B1		Accept $\frac{3\pi}{2}$
	$\tan\theta = 3 \Rightarrow$ $\theta = 1.249..;\ 4.3906… = 1.25,\ 4.39$ to 3sf	A1√	5	If not correct, ft on (b) NB M0M1(B0B0)A1ft is possible 90°; 270°; 71.5(6)°; 251.5(6)°
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)		M1 A1 A1	 3	Correct shape of branch from O {to 90°} or correct shapes of branches from 90°-360° Complete graph for $0^{\circ} \le x \le 360^{\circ}$ (Asymptotes not explicitly required but graphs should show 'tendency') Correct scaling on x-axis $0^{\circ} \le x \le 360^{\circ}$
(b)	61° ; 241°	B1 B1	 2	For 61° For 241° and no 'extras' in the interval $0^{\circ} \le x \le 360^{\circ}$
(c)(i)	$\sin\theta = -\cos\theta \;\Rightarrow \dfrac{\sin\theta}{\cos\theta} = -1$ $\Rightarrow \tan\theta = -1.$	B1	1	AG; be convinced that the identity $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ is known and validly used
(ii)	$\Rightarrow \tan(x - 20°) = -1$ $x - 20° = \tan^{-1}(-1)$ $x - 20° = 135°, 315° \ldots$ $x = 155°$; 335°	M1 m1 A1 A1ft	 4	 Ft on (180 + "155") and no 'extras' in the given interval.
(d)	Translation $\begin{bmatrix} 20 \\ 0 \end{bmatrix}$	B1 B1	 2	'Translation'/'translate(d)' Accept equivalent in words provided linked to 'translation/move/shift' (Note: B0B1 is possible)
(e)	f(x) = tan 4x	B1	1	For tan 4x
	Total		**13**	

8(a)	$\{x =\}\ \cos^{-1}(0.3) = 1.266\ldots.\quad \{= \beta\}$	M1		$\cos^{-1}(0.3)$ PI by eg 72° or 73°
	$\{x =\}\ 2\pi - \beta$	m1		Condone degrees or mix.
	$x = 1.27,\quad 5.02$	A1	3	Accept 1.26 to 1.27 with 5.01 to 5.02 inclusive
(b)(i)	$M(\pi, -1)$	B1;B1	2	B1 for each coordinate
(ii)	$\{x_Q =\}\ 2\pi - \alpha$	B1	1	OE (unsimplified)
(c)	Stretch (I) in x-direction (II) scale	M1		Need(I) & one of (II),(III)
	factor $\frac{1}{2}$ (III)	A1	2	
(d)	$\cos 2x = \cos\frac{4\pi}{5} \Rightarrow 2x = \frac{4\pi}{5}$	B1		OE. (From correct work)
	$\Rightarrow x = \frac{2\pi}{5}\ (= \alpha)$			Condone decimals/degrees
	$x = \pi - \alpha$; OE	M1		OE eg $2x = 2\pi - \frac{4\pi}{5}$ Correct quadrant; condone degrees/decimals/mix
	$x = \pi + \alpha$; $x = 2\pi - \alpha$; OE	m1		Need both (OE for $2x$=) with no extras (quadrants) within the given interval. Condone degrees/decimals/mix
	$x = \frac{2\pi}{5},\ \frac{3\pi}{5},\ \frac{7\pi}{5},\ \frac{8\pi}{5}$	A1	4	Need all 4 solutions for x but condone unsimplified provided in terms of π Ignore extra values outside the given interval.
	Total		**12**	