Sequences Questions

5 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \qquad u_2 = 150 \qquad u_3 = 120$$

(a) Show that $p = 0.6$ and find the value of q. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L. Write down an equation for L and hence find the value of L. *(3 marks)*

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3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n. *(2 marks)*

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4 (a) The expression $(1 - 2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) Find the coefficient of x in the expansion of $(2 + x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1 - 2x)^4(2 + x)^9$. *(3 marks)*

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5 The second term of a geometric series is 48 and the fourth term is 3.

(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{4}$ and state the other value. *(4 marks)*

(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; *(1 mark)*

(ii) the sum to infinity of the series. *(2 marks)*

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7 (a) The first four terms of the binomial expansion of $(1+2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1+\frac{1}{2}x\right)(1+2x)^8$. *(3 marks)*

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2 The nth term of a geometric sequence is u_n, where

$$u_n = 3 \times 4^n$$

(a) Find the value of u_1 and show that $u_2 = 48$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. *(3 marks)*

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. *(1 mark)*

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4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

Sequences Answers

5(a)	$150 = 200p + q$	M1		Either equation
	$120 = 150p + q$	A1		Both (condone embedded values for the M1A1)
		m1		Valid method to solve two simultaneous eqns in p <u>and</u> q to find either p or q
	$p = 0.6$	A1		AG (condone if left as a fraction)
	$q = 30$	B1	5	
(b)	$u_4 = 102$	B1F√	1	Ft on $(72 + q)$
(c)	$L = pL + q$; $L = 0.6L + 30$	M1		
	$L = \dfrac{q}{1-p}$	m1		
	$L = 75$	A1F√	3	Ft on $2.5q$
	Total		**9**	

3(a)	(Tenth term) $= a + (10-1)\,d$	M1		
	……………. $= 1 + 9(6) = 55$	A1	2	NMS or rep. addn. B2 CAO
				SC if M0 award B1 for $6n-5$ OE
(b)(i)	$S_n = \frac{n}{2}[2 + (n-1)6]$	M1		Formula for $\{S_n\}$ with either $a = 1$ or $d = 6$ substituted
	$\frac{n}{2}[2 + 6n - 6] = 7400$	A1		Eqn formed with some expansion of brackets
	$3n^2 - 2n = 7400 \Rightarrow 3n^2 - 2n - 7400 = 0$	A1	3	CSO AG
(ii)	$(3n+148)(n-50) = 0$	M1		Formula/factorisation OE
	$\Rightarrow n = 50$	A1	2	NMS single ans. 50.. B2 CAO NMS 50 and –49.3(3…) B1 CAO
	Total		**7**	

4(a)	$(1-2x)^4 = (1)^4 + 4(1)^3(-2x) +$ $6(1^2)(-2x)^2 + [4(1)(-2x)^3 + (-2x)^4]$	M1		Any valid method as far as term(s) in x and term(s) in x^2.
	$= [1] - 8x + 24x^2 + [-32x^3 + 16x^4]$	A1		$p = -8$ Accept $-8x$ even within a series.
		A1	3	$q = 24$ Accept $24x^2$ even within a series.
(b)	x term is $\binom{9}{1} 2^8 x$	M1		OE
	Coefficient of x term is $= 9 \times 2^8 = 2304$ $(=k)$	A1	2	Condone $2304x$
(c)	$(1-2x)^4(2+x)^9 = (1+px+..)(2^9+kx...)$	M1		Uses (a) and (b) oe (PI)
	$= \ldots$			
	$= \ldots.. + kx + px(2^9) + \ldots..$	M1		Multiply the two expansions to get x terms
	Coefficient of x is $k + 512p$			
	$= 2304 - 4096 = -1792$	A1ft	3	ft on candidate's values of k and p. Condone $-1792x$
				SC If 0/3 award B1ft for p+k evaluated
	Total		8	

5(a)	$ar = 48;\quad ar^3 = 3$	B1		For either. OE
	$\Rightarrow 16r^2 = 1$	M1		Elimination of a OE
	$r^2 = \frac{1}{16} \Rightarrow r = -\frac{1}{4}$	A1		CSO AG Full valid completion. SC Clear explicit verification (max B2 out of 3.)
	or $r = \frac{1}{4}$	B1	4	
(b)(i)	$a = -192$	B1	1	
(ii)	$\frac{a}{1-r} = \frac{a}{1-\left(-\frac{1}{4}\right)}$	M1		$\frac{a}{1-r}$ used
	$S_\infty = \frac{-768}{5}$ $(= -153.6)$	A1ft	2	Ft on candidate's value for a. i.e. $\frac{4}{5}a$ SC candidate uses $r = 0.25$, gives $a = 192$ and sum to infinity = 256. (max. B0 M1A1)
	Total		7	

7(a)	$(1+2x)^8$ $=1+\binom{8}{1}(2x)^1+\binom{8}{2}(2x)^2+\binom{8}{3}(2x)^3+$	M1		Any valid method. PI by correct value for *a*, *b* or *c*
	$= 1 + 16x + 112x^2 + 448x^3 + \ldots.$	A1A1		A1 for each of *a*, *b*, *c*
	$\{a = 16,\ b = 112,\ c = 448\}$	A1	4	
(b)	x^3 terms <u>from expn.</u> of $\left(1+\frac{1}{2}x\right)(1+2x)^8$			
	are cx^3 and $\frac{1}{2}x\left(bx^2\right)$	M1		Either
	$cx^3 + \frac{1}{2}x\left(bx^2\right)$	A1		*b*, *c* or candidate's values for *b* and *c* from (a)
	Coefficient of x^3 is $c + 0.5\,b = 504$	A1ft	3	Ft on candidate's $(c + 0.5b)$ provided *b* and *c* are positive integers >1
	Total		7	

2(a)	$u_1 = 12$	B1		
	$u_2 = 3 \times 4^2 = 48$	B1	2	CSO AG (be convinced)
(b)	$r = 4$	B1	1	
(c)(i)	$\{S_{12} =\}\dfrac{a\left(1-r^{12}\right)}{1-r}$	M1		OE Using a correct formula with $n = 12$
	$= \dfrac{12\left(1-4^{12}\right)}{1-4}$	A1ft		Ft on answer for u_1 in (a) and *r* in (b)
	$= \dfrac{12\left(1-4^{12}\right)}{-3} = -4(1-4^{12}) = 4^{13} - 4$	A1	3	CAO Accept $k = 13$ for 4^{13} term
(ii)	$\sum_{n=2}^{12} u_n = (4^{13} - 4) - u_1$ $= 67108848$	B1	1	
	Total		7	

4(a)	$\{S_{29} =\}\frac{29}{2}[2a+28d]$	M1		Formula for S_n with $n = 29$ substituted and with a and d
	$29\ (a + 14d) = 1102$	m1		Equation formed then some manipulation
	$a + 14d = \frac{1102}{29} \Rightarrow a + 14d = 38$	A1	3	CSO AG
(b)	$u_2 = a + d \quad u_7 = a + 6d$	B1		Either expression correct
	$u_2 + u_7 = 13 \Rightarrow 2a + 7d = 13$	M1		Forming equation using u_2 & u_7 both in form $a + kd$
	e.g. $21d = 63;\ \ 3a = -12$	m1		Solving $a + 14d = 38$ with candidate's '$2a + 7d = 13$' to at least stage of elimination of either a or d
	$a = -4 \quad d = 3$	A1	4	Both correct
	Total		7	