Probability Questions

2 Xavier, Yuri and Zara attend a sports centre for their judo club's practice sessions. The probabilities of them arriving late are, independently, 0.3, 0.4 and 0.2 respectively.

(a) Calculate the probability that for a particular practice session:

(i) all three arrive late; *(1 mark)*

(ii) none of the three arrives late; *(2 marks)*

(iii) only Zara arrives late. *(2 marks)*

(b) Zara's friend, Wei, also attends the club's practice sessions. The probability that Wei arrives late is 0.9 when Zara arrives late, and is 0.25 when Zara does not arrive late.

Calculate the probability that for a particular practice session:

(i) both Zara and Wei arrive late; *(2 marks)*

(ii) either Zara or Wei, but not both, arrives late. *(3 marks)*

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6 A housing estate consists of 320 houses: 120 detached and 200 semi-detached. The numbers of children living in these houses are shown in the table.

	Number of children				
	None	**One**	**Two**	**At least three**	**Total**
Detached house	24	32	41	23	120
Semi-detached house	40	37	88	35	200
Total	64	69	129	58	320

A house on the estate is selected at random.

D denotes the event 'the house is detached'.
R denotes the event 'no children live in the house'.
S denotes the event 'one child lives in the house'.
T denotes the event 'two children live in the house'.
(D' denotes the event 'not D'.)

(a) Find:

(i) $P(D)$; *(1 mark)*

(ii) $P(D \cap R)$; *(1 mark)*

(iii) $P(D \cup T)$; *(2 marks)*

(iv) $P(D \mid R)$; *(2 marks)*

(v) $P(R \mid D')$. *(3 marks)*

(b) (i) Name two of the events D, R, S and T that are mutually exclusive. *(1 mark)*

(ii) Determine whether the events D and R are independent. Justify your answer. *(2 marks)*

(c) Define, in the context of this question, the event:

(i) $D' \cup T$; *(2 marks)*

(ii) $D \cap (R \cup S)$. *(2 marks)*

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5 Dafydd, Eli and Fabio are members of an amateur cycling club that holds a time trial each Sunday during the summer. The independent probabilities that Dafydd, Eli and Fabio take part in any one of these trials are 0.6, 0.7 and 0.8 respectively.

Find the probability that, on a particular Sunday during the summer:

(a) none of the three cyclists takes part; *(2 marks)*

(b) Fabio is the only one of the three cyclists to take part; *(2 marks)*

(c) exactly one of the three cyclists takes part; *(3 marks)*

(d) either one or two of the three cyclists take part. *(3 marks)*

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2 The British and Irish Lions 2005 rugby squad contained 50 players. The nationalities and playing positions of these players are shown in the table.

		Nationality			
		English	**Welsh**	**Scottish**	**Irish**
Playing position	**Forward**	14	5	2	6
	Back	8	7	2	6

(a) A player was selected at random from the squad for a radio interview. Calculate the probability that the player was:

(i) a Welsh back; *(1 mark)*

(ii) English; *(2 marks)*

(iii) not English; *(1 mark)*

(iv) Irish, given that the player was a back; *(2 marks)*

(v) a forward, given that the player was not Scottish. *(2 marks)*

(b) Four players were selected at random from the squad to visit a school. Calculate the probability that all four players were English. *(3 marks)*

Probability Answers

2(a)	P(X) = 0.3 P(Y) = 0.4 P(Z) = 0.2			
(i)	$P(X \cap Y \cap Z) = 0.3 \times 0.4 \times 0.2 = 0.024$	M1	1	
(ii)	$P(X' \cap Y' \cap Z') = 0.7 \times 0.6 \times 0.8$	M1		At least 2 correct terms
	$= 0.336$	A1	2	CAO
(iii)	$P(X' \cap Y' \cap Z) = 0.7 \times 0.6 \times 0.2$	M1		Correct numerical expression
	$= 0.084$	A1		CAO
(b)	$P(W \mid Z) = 0.9$ $P(W \mid Z') = 0.25$			
(i)	$P(Z \cap W) = 0.2 \times 0.9$	M1		Correct numerical expression
	$= 0.18$	A1	2	CAO
(ii)	$P((Z \cap W') \cup (Z' \cap W))$ **or** $1 - [P((Z \cap W) \cup (Z' \cap W'))]$			
	$= 0.2 \times (1 - 0.9)$ +	M1		0.2×0.9 or (b)(i)
	$(1 - 0.2) \times 0.25$	M1		$(1 - 0.2) \times (1 - 0.25)$
				Cannot score an M1 in both methods
	$= 0.02 + 0.20$			$1 - (0.18 + 0.60)$
	$= 0.22$	A1	3	CAO
	Total		**11**	

	0 (R)	1 (S)	2 (T)	≥3	T
D (D)	24	32	41	23	120
SD (D′)	40	37	88	35	200
T	64	69	129	58	320

6 (a)(i)	$P(D) = \frac{120}{320}$ or $\frac{3}{8}$ or 0.375	B1	1	CAO; or equivalent
(ii)	$P(D \cap R) = \frac{24}{320}$ or $\frac{3}{40}$ or 0.075	B1	1	CSO; or equivalent
(iii)	$P(D \cup T) = \frac{120+88}{320} = \frac{129+24+32+23}{320}$	M1		
	$= \frac{208}{320}$ or $\frac{13}{20}$ or 0.65	A1	2	CAO; or equivalent
(iv)	$P(D \mid R) = \frac{P(D \cap R)}{P(R)} = \frac{(ii)}{P(R)} = \frac{24/(320)}{64/(320)}$	M1		M0 if independence assumed
	$= \frac{24}{64}$ or $\frac{3}{8}$ or 0.375	A1	2	CAO; or equivalent
(v)	$P(R \mid D') = \frac{P(R \cap D')}{P(D')} = \frac{40/(320)}{200/(320)}$	M1		numerator allow independence assumed
		M1		denominator
	$= \frac{40}{200}$ or $\frac{1}{5}$ or 0.2	A1	3	CAO; or equivalent
(b)(i)	R and S or R and T or S and T	B1	1	not D and D'
(ii)	$P(D) = 0.375 = P(D \mid R)$ or (i) = (iv)	M1		$P(D) \times P(R) = 0.375 \times 0.2$ $= 0.075 = P(D \cap R)$ or (ii) or $P(R \mid D) = P(R) = 0.2$, etc
	so YES	A1	2	
(c)(i)	A semi-detached house or two children (or both)	B1 B1	2	CAO or equivalent
(ii)	A detached house and/with less than two children	B1 B1	2	CAO (0 or 1 must not include 'both')
	Total		**16**	

5(a)	$P(D' \cap E' \cap F') = 0.4 \times 0.3 \times 0.2$	M1		At least 1 probability correct
	$= 0.024$	A1	2	CAO; OE
(b)	$P(D' \cap E' \cap F) = 0.4 \times 0.3 \times 0.8$	M1		At least 2 probabilities correct
	$= 0.096$	A1	2	CAO; OE
(c)	P(One) = (b) + $P(D \cap E' \cap F') + P(D' \cap E \cap F')$	M1		Use of 3 possibilities; ignore multipliers
	=(b) + $(0.6 \times 0.3 \times 0.2) + (0.4 \times 0.7 \times 0.2)$	M1		At least 1 new term correct
	$= 0.096 + 0.036 + 0.056 = 0.188$	A1	3	CAO; OE
(d)	P(One or two) = (c) + (3 terms each of 3 probabilities) **or** = 1 – (a) – (1 term of 3 probabilities)	M1		(c) + P(Two) Used; OE; ignore multipliers 1 – (a) – P(Three)
	$= 0.188 + (0.6 \times 0.7 \times 0.2) +$ $(0.6 \times 0.3 \times 0.8) + (0.4 \times 0.7 \times 0.8)$ $= 0.188 + 0.084 + 0.144 + 0.224$ **or** $= 1 - 0.024 - (0.6 \times 0.7 \times 0.8)$ $= 1 - 0.024 - 0.336$	M1		At least 1 new term correct
	$= 0.64$	A1	3	CAO; OE
	Total		**10**	

2	Ratios: Penalise first occurrence only of a correct answer			
(a)(i)	P(Welsh back) $= \frac{7}{50}$ or 0.14	B1	1	CAO; OE
(ii)	P(English) $= \frac{14+8}{50} =$	B1		Correct expression; PI
	$\frac{22}{50}$ or $\frac{11}{25}$ or 0.44	B1	2	CAO; OE
(iii)	P(not English) = 1 – (ii) =			
	$\frac{28}{50}$ or $\frac{14}{25}$ or 0.56	B1√	1	√ on (ii) if used; $0 < p < 1$
(iv)	P(Irish | back) = $\frac{P(\text{Irish} \cap \text{back})}{P(\text{back})} = \frac{6}{\sum(\text{back})} =$	M1		Used; may be implied by values or answer
	$\frac{6}{23}$ or 0.26 to 0.261	A1	2	CAO/AWFW (6/50 $\Rightarrow$ 0)

(v)	P(forward | not Scottish) = $\dfrac{\text{P}(\text{forward} \cap \text{not Scottish})}{\text{P}(\text{not Scottish})}$ =	M1		Used; OE May be implied by values or answer
	$\dfrac{14+5+6}{50-4} = \dfrac{27-2}{50-4} =$			
	$\dfrac{25}{46}$ or 0.54 to 0.544	A1	2	CAO/AWFW (25/50 ⇒ 0)
(b)	P(4 × English) =			
	$\left(\dfrac{22}{50}\right)\times\left(\dfrac{21}{49}\right)\times\left(\dfrac{20}{48}\right)\times\left(\dfrac{19}{47}\right) =$	M1 M1		Reducing non-tabulated value 4 times Reducing 50 and multiplying 4 terms (ignore multipliers)
	$\dfrac{175560}{5527200}$ or $\dfrac{209}{6580}$			
	or 0.0317 to 0.032	A1	3	CAO/AWFW
	Total		**11**	