Numerical Methods Questions

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. *(2 marks)*

(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \dfrac{3 - x^3}{4}$. *(1 mark)*

(c) (i) Use the iteration $x_{n+1} = \dfrac{3 - x_n^3}{4}$ with $x_1 = 0.5$ to find x_3, giving your answer to two decimal places. *(3 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \dfrac{3 - x^3}{4}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(3 marks)*
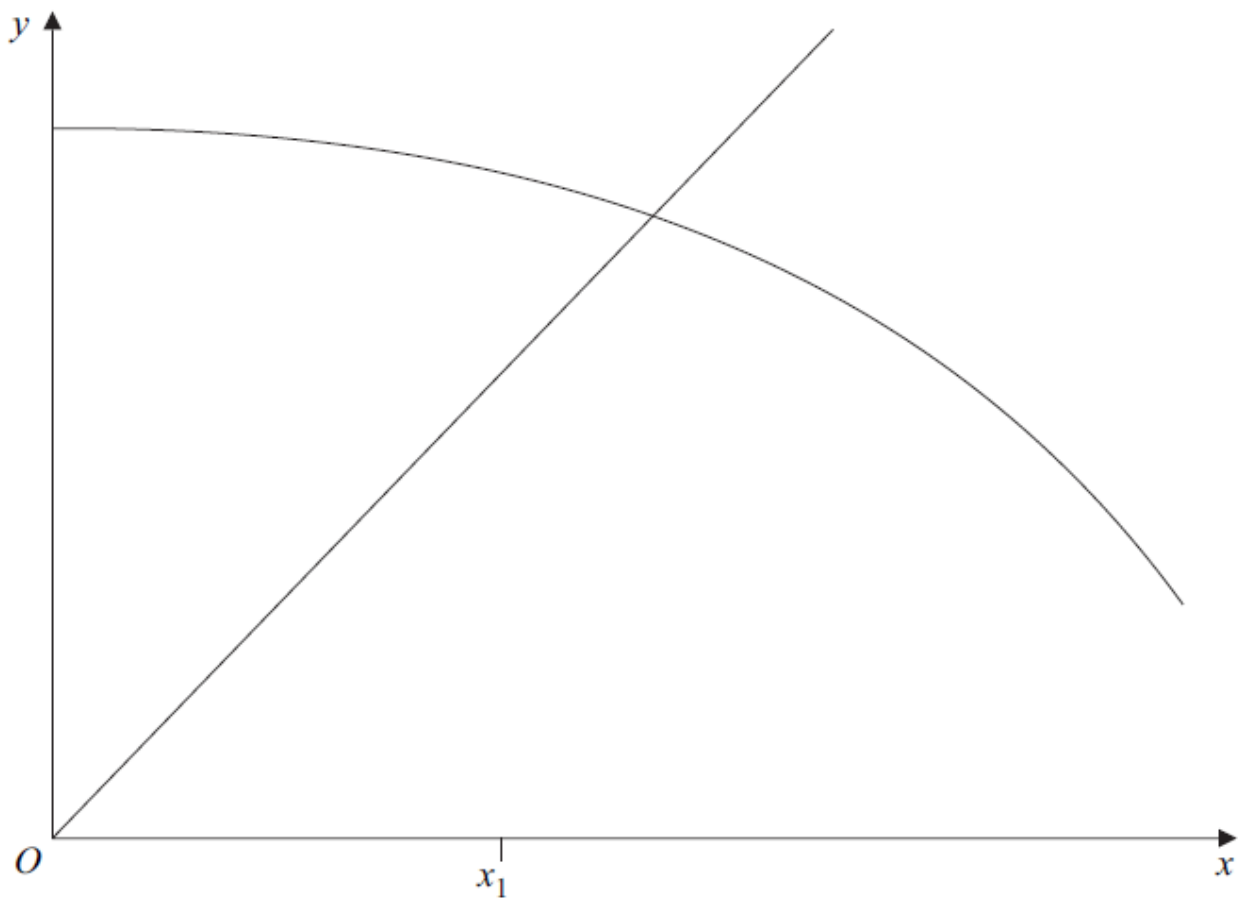
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1 The curve $y = x^3 - x - 7$ intersects the x-axis at the point where $x = \alpha$.

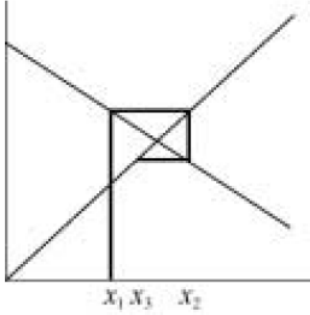(a) Show that α lies between 2.0 and 2.1. *(2 marks)*

(b) Show that the equation $x^3 - x - 7 = 0$ can be rearranged in the form $x = \sqrt[3]{x + 7}$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \sqrt[3]{x_n + 7}$ with $x_1 = 2$ to find the values of x_2, x_3 and x_4, giving your answers to three significant figures. *(3 marks)*

Figure 1 (for Question 6)

Numerical Methods Answers

6(a)	$f(0.5) = -0.875$	M1			
	$f(1) = 2$				
	Change of sign $\therefore$ root	A1	2		
(b)	$x^3 + 4x - 3 = 0$				
	$4x = 3 - x^3$	B1	1		
	$x = \dfrac{3 - x^3}{4}$			AG	
(c)(i)	$x_1 = 0.5$	M1			
	$x_2 = 0.71875 \quad 0.72 \quad$ AWRT	A1			
	$x_3 = 0.66$	A1	3		

(ii)

M1		For cobweb, x_1 to curve		
A1		For x_2		
A1	3	All correct		

$x_1 \, x_3 \quad x_2$

		Total		**9**	

1(a)	$f(2) = -1$	M1		both attempted	
	$f(2.1) = +0.161$				
	change of sign $\therefore 2 < \alpha < 2.1$	A1	2		
(b)	$x^3 - x - 7 = 0$				
	$x^3 = x + 7$				
	$x = \sqrt[3]{x+7}$	B1	1	AG	
(c)	$x_1 = 2$	M1			
	$x_2 = 2.0801...$	A1		AWRT 2.08	
	$x_3 = 2.0862...$			AWRT 2.09	
	$x_4 = 2.09$	A1	3		
		Total		**6**	