Numerical Measures Questions

3 When an alarm is raised at a market town's fire station, the fire engine cannot leave until at least five fire-fighters arrive at the station. The call-out time, X minutes, is the time between an alarm being raised and the fire engine leaving the station.

The value of X was recorded on a random sample of 50 occasions. The results are summarised below, where $\bar{x}$ denotes the sample mean.

$$\sum x = 286.5 \qquad \sum (x - \bar{x})^2 = 45.16$$

(a) Find values for the mean and standard deviation of this sample of 50 call-out times. *(2 marks)*

(b) Hence construct a 99% confidence interval for the mean call-out time. *(4 marks)*

(c) The fire and rescue service claims that the station's mean call-out time is less than 5 minutes, whereas a parish councillor suggests that it is more than $6\frac{1}{2}$ minutes.

Comment on **each** of these claims. *(2 marks)*

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4 The time, x seconds, spent by each of a random sample of 100 customers at an automatic teller machine (ATM) is recorded. The times are summarised in the table.

Time (seconds)	Number of customers
$20 < x \leqslant 30$	2
$30 < x \leqslant 40$	7
$40 < x \leqslant 60$	18
$60 < x \leqslant 80$	27
$80 < x \leqslant 100$	23
$100 < x \leqslant 120$	13
$120 < x \leqslant 150$	7
$150 < x \leqslant 180$	3
Total	**100**

(a) Calculate estimates for the mean and standard deviation of the time spent at the ATM by a customer. *(4 marks)*

(b) Kirk attends darts coaching sessions for three months. He then claims that he has a probability of 0.4 of winning any game, and that the outcome of each game is independent of the outcome of every other game.

(i) Assuming this claim to be true, calculate the mean and standard deviation for the number of games won by Kirk in a match of 15 games. *(3 marks)*

(ii) To assess Kirk's claim, Les keeps a record of the number of games won by Kirk in a series of 10 matches, each of 15 games, with the following results:

8 5 6 3 9 12 4 2 6 5

Calculate the mean and standard deviation of these values. *(2 marks)*

(iii) Hence comment on the validity of Kirk's claim. *(3 marks)*

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1 The times, in seconds, taken by 20 people to solve a simple numerical puzzle were

17 19 22 26 28 31 34 36 38 39
41 42 43 47 50 51 53 55 57 58

(a) Calculate the mean and the standard deviation of these times. *(3 marks)*

(b) In fact, 23 people solved the puzzle. However, 3 of them failed to solve it within the allotted time of 60 seconds.

Calculate the median and the interquartile range of the times taken by all 23 people. *(4 marks)*

(c) For the times taken by all 23 people, explain why:

(i) the mode is **not** an appropriate numerical measure;

(ii) the range is **not** an appropriate numerical measure. *(2 marks)*

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4 A library allows each member to have up to 15 books on loan at any one time.

The table shows the numbers of books currently on loan to a random sample of 95 members of the library.

Number of books on loan	0	1	2	3	4	5–9	10–14	15
Number of members	4	13	24	17	15	11	5	6

(a) For these data:

(i) state values for the mode and range; *(2 marks)*

(ii) determine values for the median and interquartile range; *(4 marks)*

(iii) calculate estimates of the mean and standard deviation. *(4 marks)*

(b) Making reference to your answers to part (a), give a reason for preferring:

(i) the median and interquartile range to the mean and standard deviation for summarising the given data; *(1 mark)*

(ii) the mean and standard deviation to the mode and range for summarising the given data. *(1 mark)*

Numerical Measures Answers

3(a)	Mean $= \frac{286.5}{50} = 5.73$	B1		CAO
	Standard deviation $= \sqrt{\frac{45.16}{49 \text{ or } 50}} =$			
	0.95 to 0.961	B1	2	AWFW
(b)	99% $\Rightarrow z = 2.57$ to 2.58	B1		AWFW 2.5758
	CI for μ is $\bar{x} \pm z \times \frac{(\sigma \text{ or } s)}{\sqrt{n}}$	M1		Use of Must have $\left(\div\sqrt{n}\right)$ with $n > 1$
	Thus $5.73 \pm 2.5758 \times \frac{(0.95 \text{ to } 0.961)}{\sqrt{50}}$	A1√		√ on z and $s^2 > 0$ but not on $\bar{x}$ Accept only 50 or 49 for n
	$5.73 \pm (0.34 \text{ to } 0.36)$	↑		Dependent
	5.37 to 5.39, 6.07 to 6.09)	A1	4	AWFW
(c)	**CI excludes both values** of 5 and 6½ so **Neither claim** appears **valid**	B1√ ↑ B1√		√ on (b); OE Dependent √ on (b); OE
	or			
	CI excludes 5 so claim not valid and CI excludes 6½ so claim not valid	(B1√) (B1√)	2	√ on (b); OE √ on (b); OE
	Total		**8**	

4(a)	$\Sigma fx = 8025$ $\Sigma fx^2 = 739975$			
	Mean $(\bar{x}) = 80.2$ to 80.3	B2		AWFW 80.25
	Standard Deviation $(s_n, s_{n-1}) = 30.9$ to 31.2	B2		AWFW 30.97882 or 31.13489
	MPs (x): 25, 35, 50, 70, 90, 110, 135, 165	(B1)		At least 4 correct
	Mean $(\bar{x}) = \frac{\Sigma fx}{100}$	(M1)	4	Use of

(b)(i)	Mean, $\mu = np = 15 \times 0.4 = 6$	B1		CAO
	Variance, $\sigma^2 = np(1-p) = 6 \times 0.6 = 3.6$	M1		use of $\sigma^2 = np(1-p)$
	Standard deviation = $\sqrt{3.6} = 1.89$ to 1.9	A1	3	AWFW; or equivalent
(ii)	Mean, $\bar{x} = 6$	B1		CAO ($\Sigma x = 60$) CSO if evidence of $np(1-p)$ or 1.9
	Standard deviation, s or $\sigma = 2.82$ to 2.99	B1	2	AWFW; or equivalent. ($\Sigma x^2 = 440$)
(iii)	Means are same/equal	B1√		√ on 2 means; accept $\frac{6}{15} = 0.4$ if not contradicted by $\bar{x}$ in (ii)
	Standard deviations are different	B1 dep		dependent on 2 correct SDs
	Reason to doubt validity of Kirk's claim	B1 dep	3	dependent on 2 correct SDs

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1(a)	Mean $(\bar{x})$ = 39.3 to 39.4	B1		AWFW (39.35)
	Standard Deviation (s_n, s_{n-1}) = 12.3 to 12.7	B2	3	AWFW (12.358 or 12.679)
	If **neither** correct **but** working shown, then			$\sum x = 787$ $\sum x^2 = 34023$
	Mean $(\bar{x}) = \frac{\sum x}{20}$	(M1)		Used
(b)	Median = 42	B2		CAO
	Median = 41.5 or 39 or 40	(B1)		CAO
	Interquartile Range = 55 – 31 = 24	B2	4	CAO; allow B1 for identification of 31 and 55; B0 if method shown is incorrect
	Interquartile Range = 21 to 27	(B1)		AWFW
(c)(i)	**Mode:** eg Does not exist If exists, must be > 60 or 58 All / too many different values Sparse data	B1		OE
(ii)	**Range:** eg **Maximum value** is unknown / > 60 or 58	B1	2	OE; accept 'slowest' but not 'smallest'
		Total	**9**	

4(a)(i)	Mode = 2	B1		CAO
	Range = 15	B1	2	CAO
(ii)	CF: 4 17 41 58 73 84 89 95 x: 0 1 2 3 4 9 14 15			
	Median (48^{th}) = 3	B2		CAO; B0 if shown method is incorrect
	Interquartile Range ($72^{nd} - 24^{th}$) = 4 – 2 = 2	B2		CAO Allow B1 for identification of 4 and 2 B0 if shown method is incorrect
	If neither correct but CF attempted and matched correctly with ≥ 5 x–values	(M1) (A1)	4	Allow for median = $2 + \frac{x}{17}$
(iii)	Mean $(\bar{x})$ = 4.2	B2		CAO $\sum fx = 399$
	Standard Deviation $(s_n,\ s_{n-1})$ = 3.88 to 3.91	B2		$\sum fx^2 = 3111$ AWFW (3.887 or 3.907)
	If neither correct but mid-points of 7 and 12 seen	(B1)		
	and use of mean $(\bar{x}) = \frac{\sum fx}{95}$	(M1)	4	Allow for $4.1 \le \bar{x} \le 4.3$
(b)(i)	Unknown values (16) have no effect on median and IQR **or** median and IQR are exact values but $\bar{x}$ and s are estimates	B1	1	
(ii)	Use all available data or Enable further analyses	B1	1	
	Total		**12**	