Kinematics Questions

2 A particle P moves with acceleration $(-3\mathbf{i} + 12\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially the velocity of P is $4\mathbf{i}\,\mathrm{m\,s^{-1}}$.

(a) Find the velocity of P at time t seconds. *(2 marks)*

(b) Find the speed of P when $t = 0.5$. *(3 marks)*

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3 (a) A small stone is dropped from a height of 25 metres above the ground.

(i) Find the time taken for the stone to reach the ground. *(2 marks)*

(ii) Find the speed of the stone as it reaches the ground. *(2 marks)*

(b) A large package is dropped from the same height as the stone. Explain briefly why the time taken for the package to reach the ground is likely to be different from that for the stone. *(2 marks)*

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6 A van moves from rest on a straight horizontal road.

(a) In a simple model, the first 30 seconds of the motion are represented by three separate stages, each lasting 10 seconds and each with a constant acceleration.

During the first stage, the van accelerates from rest to a velocity of $4\,\mathrm{m\,s^{-1}}$.

During the second stage, the van accelerates from $4\,\mathrm{m\,s^{-1}}$ to $12\,\mathrm{m\,s^{-1}}$.

During the third stage, the van accelerates from $12\,\mathrm{m\,s^{-1}}$ to $16\,\mathrm{m\,s^{-1}}$.

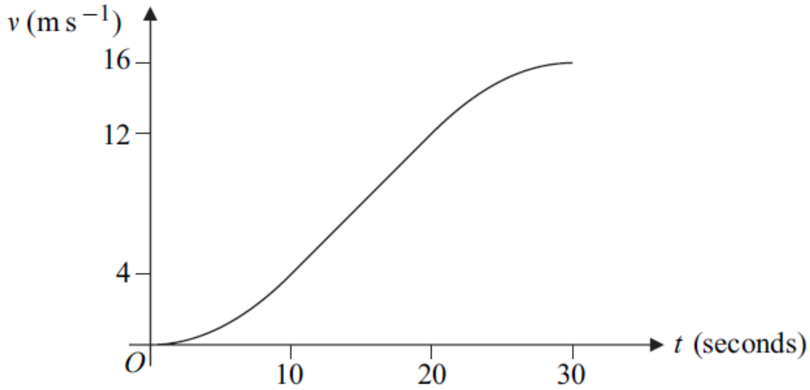
(i) Sketch a velocity–time graph to represent the motion of the van during the first 30 seconds of its motion. *(3 marks)*

(ii) Find the total distance that the van travels during the 30 seconds. *(4 marks)*

(iii) Find the average speed of the van during the 30 seconds. *(2 marks)*

(iv) Find the greatest acceleration of the van during the 30 seconds. *(2 marks)*

(b) In another model of the 30 seconds of the motion, the acceleration of the van is assumed to vary during the first and third stages of the motion, but to be constant during the second stage, as shown in the velocity–time graph below.



The velocity of the van takes the same values at the beginning and the end of each stage of the motion as in part (a).

(i) State, with a reason, whether the distance travelled by the van during the first 10 seconds of the motion in **this** model is greater or less than the distance travelled during the same time interval in the model in part (a). *(2 marks)*

(ii) Give one reason why **this** model represents the motion of the van more realistically than the model in part (a). *(1 mark)*

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1 A stone is dropped from a high bridge and falls vertically.

(a) Find the distance that the stone falls during the first 4 seconds of its motion. *(3 marks)*

(b) Find the average speed of the stone during the first 4 seconds of its motion. *(2 marks)*

(c) State one modelling assumption that you have made about the forces acting on the stone during the motion. *(1 mark)*

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3 A car travels along a straight horizontal road. The motion of the car can be modelled as three separate stages.

During the first stage, the car accelerates uniformly from rest to a velocity of $10\,\mathrm{m\,s^{-1}}$ in 6 seconds.

During the second stage, the car travels with a constant velocity of $10\,\mathrm{m\,s^{-1}}$ for a further 4 seconds.

During the third stage of the motion, the car travels with a uniform retardation of magnitude $0.8\,\mathrm{m\,s^{-2}}$ until it comes to rest.

(a) Show that the time taken for the **third** stage of the motion is 12.5 seconds. *(2 marks)*

(b) Sketch a velocity–time graph for the car during the three stages of the motion. *(4 marks)*

(c) Find the total distance travelled by the car during the motion. *(3 marks)*

(d) State one criticism of the model of the motion. *(1 mark)*

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6 The points A and B have position vectors $(3\mathbf{i} + 2\mathbf{j})$ metres and $(6\mathbf{i} - 4\mathbf{j})$ metres respectively. The vectors $\mathbf{i}$ and $\mathbf{j}$ are in a horizontal plane.

(a) A particle moves from A to B with constant velocity $(\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. Calculate the time that the particle takes to move from A to B. *(3 marks)*

(b) The particle then moves from B to a point C with a constant acceleration of $2\mathbf{j}\,\mathrm{m\,s^{-2}}$. It takes 4 seconds to move from B to C.

(i) Find the position vector of C. *(4 marks)*

(ii) Find the distance AC. *(2 marks)*

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2 A lift rises vertically from rest with a constant acceleration.

After 4 seconds, it is moving upwards with a velocity of $2\,\mathrm{m\,s^{-1}}$.

It then moves with a constant velocity for 5 seconds.

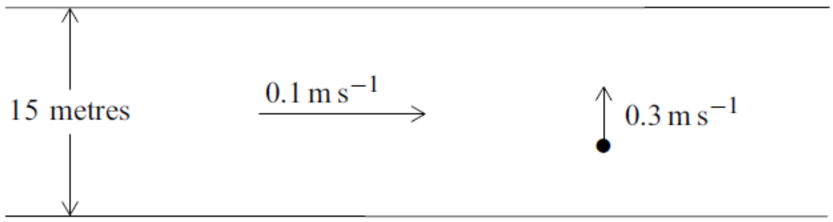
The lift then slows down uniformly, coming to rest after it has been moving for a total of 12 seconds.

(a) Sketch a velocity–time graph for the motion of the lift. *(4 marks)*

(b) Calculate the total distance travelled by the lift. *(2 marks)*

(c) The lift is raised by a single vertical cable. The mass of the lift is 300 kg. Find the maximum tension in the cable during this motion. *(4 marks)*

5 A girl in a boat is rowing across a river, in which the water is flowing at $0.1\ \mathrm{m\,s^{-1}}$. The velocity of the boat relative to the water is $0.3\ \mathrm{m\,s^{-1}}$ and is perpendicular to the bank, as shown in the diagram.



(a) Find the magnitude of the resultant velocity of the boat. *(2 marks)*

(b) Find the acute angle between the resultant velocity and the bank. *(3 marks)*

(c) The width of the river is 15 metres.

(i) Find the time that it takes the boat to cross the river. *(2 marks)*

(ii) Find the total distance travelled by the boat as it crosses the river. *(2 marks)*

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8 A particle is initially at the origin, where it has velocity $(5\mathbf{i} - 2\mathbf{j})\ \mathrm{m\,s^{-1}}$. It moves with a constant acceleration $\mathbf{a}\ \mathrm{m\,s^{-2}}$ for 10 seconds to the point with position vector $75\mathbf{i}$ metres.

(a) Show that $\mathbf{a} = 0.5\mathbf{i} + 0.4\mathbf{j}$. *(3 marks)*

(b) Find the position vector of the particle 8 seconds after it has left the origin. *(3 marks)*

(c) Find the position vector of the particle when it is travelling parallel to the unit vector $\mathbf{i}$. *(6 marks)*

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1 A ball is released from rest at a height h metres above ground level. The ball hits the ground 1.5 seconds after it is released. Assume that the ball is a particle that does not experience any air resistance.

(a) Show that the speed of the ball is $14.7\ \mathrm{m\,s^{-1}}$ when it hits the ground. *(2 marks)*

(b) Find h. *(2 marks)*

(c) Find the distance that the ball has fallen when its speed is $5\ \mathrm{m\,s^{-1}}$. *(3 marks)*

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5 An aeroplane flies in air that is moving due east at a speed of $V\,\mathrm{m\,s^{-1}}$. The velocity of the aeroplane relative to the air is $150\,\mathrm{m\,s^{-1}}$ due north. The aeroplane actually travels on a bearing of 030°.

(a) Show that $V = 86.6\,\mathrm{m\,s^{-1}}$, correct to three significant figures. *(2 marks)*

(b) Find the magnitude of the resultant velocity of the aeroplane. *(3 marks)*

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8 A boat is initially at the origin, heading due east at $5\,\mathrm{m\,s^{-1}}$. It then experiences a constant acceleration of $(-0.2\mathbf{i} + 0.25\mathbf{j})\,\mathrm{m\,s^{-2}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) State the initial velocity of the boat as a vector. *(1 mark)*

(b) Find an expression for the velocity of the boat t seconds after it has started to accelerate. *(2 marks)*

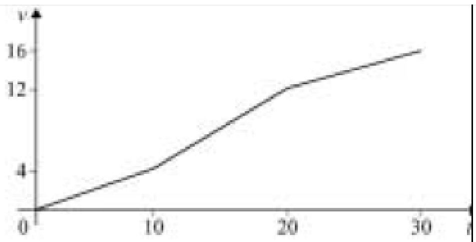
(c) Find the value of t when the boat is travelling due north. *(3 marks)*

(d) Find the bearing of the boat from the origin when the boat is travelling due north. *(6 marks)*

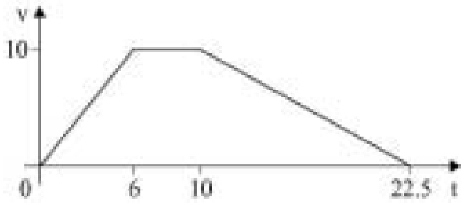
Kinematics Answers

2(a)	$\mathbf{v} = 4\mathbf{i} + (-3\mathbf{i} + 12\mathbf{j})t$	M1 A1	2	use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$
(b)	$t = 0.5,\ \ \mathbf{v} = 2.5\mathbf{i} + 6\mathbf{j}$	B1√		√ 2 terms and t subs
	$\text{Speed} = \sqrt{(2.5^2 + 6^2)}$	M1		2 terms
	$\text{Speed} = 6.5\,\text{m}\,\text{s}^{-1}$	A1√	3	√ 2 terms
	Total		**5**	

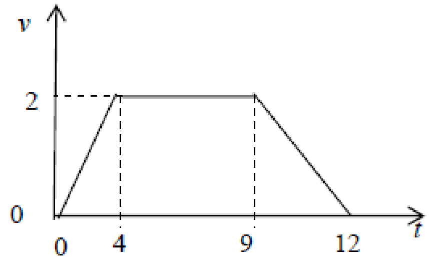
3(a)(i)	$s = ut + \frac{1}{2}at^2$ $25 = 0 + 4.9t^2$ $t = 2.26\,\text{sec}$ $\quad (2.236)(\text{if } g = 10)$ (2.259)	 M1 A1	 2	 full method
(ii)	$v^2 = u^2 + 2as$ $v^2 = 0 + 2 \times 9.8 \times 25$ $v = 22.1\,\text{m}\,\text{s}^{-1}$ $\quad (21.913)$ (22.14)	 M1 A1	 2	
(b)	(Time longer) air resistance slows down motion, links with motion, no contradictions	M1 A1	 2	(or Time less) package large so less distance to travel
	Total		**6**	

6(a)(i)		B1 B1 B1	3	3 straight lines correct end points sensible scales + labelled v/t
(ii)	$s=\frac{1}{2}\times 10\times 4+\frac{1}{2}\times(4+12)\times 10+$ $\frac{1}{2}(12+16)\times 10$ $s=240$ metres	M1 m1 A1 A1$\surd$	4	area attempt / Or equation attempted full method / full method equation correct / all correct $\surd$ one slip / $\surd$ one slip
(iii)	Average speed $=\dfrac{240}{30}$ $=8\,\text{ms}^{-1}$	M1 A1$\surd$	2	$\surd$distance
(iv)	Greatest acceleration $=2^{\text{nd}}$ stage $=\dfrac{12-4}{10}$ $=0.8\,\text{ms}^{2}$	M1 A1	2	cao
(b)(i)	Less area below curve < area below line/velocity lower	B1 B1	2	no additional incorrect statements
(ii)	Change in velocity more gradual oe	B1	1	
	Total		**14**	

1(a)	$s=0+\frac{1}{2}\times 9.8\times 4^2$ $s=78.4$ metres	M1 A1 A1	3	Full method Correct subs, accept ± 9.8 CAO (need positive)
(b)	Average speed $=\dfrac{78.4}{4}$ $=19.6\ \text{ms}^{-1}$	M1 A1F	2	Also accept full method with use of velocities at $t=0$ and 4, or at $t=2$ FT distance
(c)	Only force acting is weight	B1	1	Acc resistance forces negligible or ignored, (not friction, or air friction)
	Total		**6**	

3(a)	$v = u + at$ $0 = 10 + (-0.8) \times t$	M1		Full method with u, v used correctly Accept ± 0.8
	$t = 12.5$ sec	A1	2	CAO (correct subs and answer)
(b)		B1 B1 B1		each line, straight and correct end points SC: B1 for 3 lines giving correct shape but no values shown SC: first error in labelling times loses B1, repeated errors no further penalty
		B1	4	axes labelled v, t
(c)	distance $= \frac{1}{2} \times 10 \times (4 + 22.5)$	M1		Full correct method
		A1F		Correct subs, FT graph if final $t = 12.5$
	$= 132.5$ metres	A1F	3	FT one slip, AWRT 133
(d)	Acceleration unlikely to: change so abruptly or be constant or velocity unlikely to be constant	B1	1	
	Total		**10**	

6(a)	$\mathbf{d} = 3\mathbf{i} - 6\mathbf{j}$	B1		Accept $\pm\mathbf{d}$ or displacements of 3, 6 shown on a diagram
	$3\mathbf{i} - 6\mathbf{j} = (\mathbf{i} - 2\mathbf{j})t$	M1		Or equivalent method for t Accept ratio of vectors leading directly to ± 3
	$t = 3$	A1	3	CAO
(b)(i)	$\mathbf{r} = (\mathbf{i} - 2\mathbf{j}) \times 4 + \frac{1}{2} \times 2\mathbf{j} \times 16$	M1		Full method for vector expression giving change in position
		A1		For correct subs (gives $4\mathbf{i} + 8\mathbf{j}$)
	$+6\mathbf{i} - 4\mathbf{j}$	M1		
	$= 10\mathbf{i} + 4\mathbf{j}$	A1F	4	FT slip provided obtain vector expression $(\mathbf{u} = 0$ gives $6\mathbf{i} + 12\mathbf{j})$
(ii)	$A(3,2)$ $\quad C(10,4)$ $\mathbf{d} = 7\mathbf{i} + 2\mathbf{j}$	M1		Attempt to find vector $\overline{AC}$ or $\overline{CA}$ (using candidate's C
	$\lvert\mathbf{d}\rvert = \sqrt{7^2 + 2^2}$ $AC = \sqrt{53} = 7.28$	A1F	2	FT $\mathbf{d}$ provided two non-zero components Accept $\sqrt{53}$
	Total		**9**	

2(a)		B1 B1 B1 B1	 4	Starts and finishes at rest Correct shape Correct values on t-axis Correct values on v-axis Condone omission of the origin
(b)	$s=\frac{1}{2}(5+12)\times 2$ **or** $s=\frac{1}{2}\times 2\times 4+5\times 2+\frac{1}{2}\times 2\times 3=17$ $=17$	M1 A1	 2	Use of the area under the graph (or equivalent) to find s Correct distance **SC** When 21 used instead of 12 allow full marks for $s=26$
(c)	$\max a=\frac{2}{4}=0.5$ $300\times 0.5=T-300\times 9.8$ $T=2940+150=3090$	B1 M1 A1 A1	 4	Maximum acceleration Three term equation of motion using their a Correct equation using $a=0.5$ Correct tension
			10	

5(a)	$v=\sqrt{0.3^2+0.1^2}=\sqrt{0.1}=0.316\text{ ms}^{-1}$	M1A1	2	Use of Pythagoras to find v. Correct v
(b)	$\alpha=\tan^{-1}\left(\frac{0.3}{0.1}\right)=71.6°$	M1A1 A1	 3	Use of trigonometry with reasonable choice of sides to find α. Correct expression Correct angle **CAO**
(i)	$t=\frac{15}{0.3}=50\text{s}$	M1 A1	 2	Use of s/v to find t with s and t consistent Correct t
(ii)	$s=50\times\sqrt{0.1}=15.8\text{m}$	M1A1	2	Use of their t in $t\times v$ to find s or the use of trigonometry. Correct distance **CAO**
	Total		**9**	

8(a)	$75\mathbf{i} = (5\mathbf{i} - 2\mathbf{j}) \times 10 + \frac{1}{2}\mathbf{a} \times 10^2$	M1		Equation to find $\mathbf{a}$ from $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
		A1		Correct expression
	$\mathbf{a} = \frac{75\mathbf{i} - 50\mathbf{i} + 20\mathbf{j}}{50} = 0.5\mathbf{i} + 0.4\mathbf{j}$	A1	3	**AG** Correct $\mathbf{a}$ from correct working
(b)	$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j}) \times 8 + \frac{1}{2}(0.5\mathbf{i} + 0.4\mathbf{j}) \times 8^2$	M1		Expression for $\mathbf{r}$ using $t = 8$ with no extra terms
		A1		Correct expressions
	$= 56\mathbf{i} - 3.2\mathbf{j}$	A1	3	Correct position vector
(c)	$\mathbf{v} = (5 + 0.5t)\mathbf{i} + (0.4t - 2)\mathbf{j}$	M1A1		Expression for $\mathbf{v}$. Correct expression
	$0.4t - 2 = 0$	dM1		$\mathbf{j}$ component equal to zero
	$t = \frac{2}{0.4} = 5$	A1		Correct t
	$\mathbf{r} = (5\mathbf{i} - 2\mathbf{j}) \times 5 + \frac{1}{2}(0.5\mathbf{i} + 0.4\mathbf{j}) \times 5^2$	dM1		Expression for $\mathbf{r}$ using t from $\mathbf{j}$ component equal to zero
	$= 31.25\mathbf{i} - 5\mathbf{j}$			
	$= 31.3\mathbf{i} - 5\mathbf{j}$	A1	6	Correct position vector
	Total		**12**	

1(a)	$v = 0 + 1.5 \times 9.8$	M1		Use of constant acceleration equation to find v
	$= 14.7 \text{ ms}^{-1}$	A1	2	AG Correct v from correct working $1.5 \times 9.8 = 14.7$ is not enough on its own
(b)	$h = \frac{1}{2} \times 9.8 \times 1.5^2$	M1		Use of constant acceleration equation with $a = 9.8$ to find h
	$= 11.0$ m (to 3 sf)	A1	2	Correct h Allow 11 m; ignore negative signs
(c)	$5^2 = 0^2 + 2 \times 9.8s$	M1		Use of constant acceleration equation with $u = 0$ to find s
		A1		Correct equation
	$s = \frac{25}{19.6} = 1.28$ m (to 3 sf)	A1	3	Correct s Accept 1.27
	OR $t = \frac{5}{9.8} = 0.510$ $s = \frac{1}{2}(0 + 5)\frac{5}{9.8} = 1.28$ m **OR** $s = 0 + \frac{1}{2} \times 9.8 \times \left(\frac{5}{9.8}\right)^2 = 1.28$ m			
	Total		7	

5(a)	$V = 150\tan 30°$	M1		Using trigonometry (usually tan or sine rule) to find V
	$= 86.6\ \text{ms}^{-1}$	A1	2	AG Correct answer from correct working (Division by 2 only acceptable if sin30° or cos60° seen)
	OR $\dfrac{V}{\sin 30°} = \dfrac{150}{\sin 60°}$ AG $V = 86.6\ \text{ms}^{-1}$			
(b)	$\dfrac{150}{v} = \cos 30°$	M1		Using trigonometry or Pythagoras to find v
		A1		Correct expression
	$v = \dfrac{150}{\cos 30°} = 173\ \text{ms}^{-1}$ (to 3sf)	A1	3	Correct answer
	Total		**5**	

8(a)	$\mathbf{u} = 5\mathbf{i}$ or $\begin{bmatrix}5\\0\end{bmatrix}$	B1	1	Correct velocity
(b)	$\mathbf{v} = 5\mathbf{i} + (-0.2\mathbf{i} + 0.25\mathbf{j})t$	M1		Use of constant acceleration equation, with **u** and **a** not zero
		A1	2	Correct velocity M1A0 for using 5**j** or just 5
	OR $\mathbf{v} = \begin{bmatrix}5-0.2t\\0.25t\end{bmatrix}$			
(c)	$5 - 0.2t = 0$	M1		Easterly component zero
		A1		Correct equation
	$t = \dfrac{5}{0.2} = 25$ seconds	A1	3	Correct t
(d)	$\mathbf{r} = 5\mathbf{i}\times 25 + \dfrac{1}{2}(-0.2\mathbf{i} + 0.25\mathbf{j})\times 25^2$	M1		Use of constant acceleration equation with t from part (c)
		A1F		Correct expression based on t from part (c)
	$= 62.5\mathbf{i} + 78.125\mathbf{j}$	A1		Correct simplification CAO
	$\theta = \tan^{-1}\left(\dfrac{62.5}{78.125}\right)$	dM1		Using tan to find the angle
		A1F		Correct expression based on t from part (c), with correct two values(either way)
	$= 038.7°$	A1	6	Correct angle Accept 38.6° or 039°
	OR $\mathbf{r} = \dfrac{1}{2}(5\mathbf{i} + 6.25\mathbf{j})\times 25$	(M1) (A1F) (A1)		
	$\theta = \tan^{-1}\left(\dfrac{5}{6.25}\right) = 038.7°$	(dM1) (A1F) (A1)		
	Total		**12**	