Integration Questions

3 (a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\int \frac{2x^3 + 1}{x^4 + 2x} \, dx$. *(2 marks)*

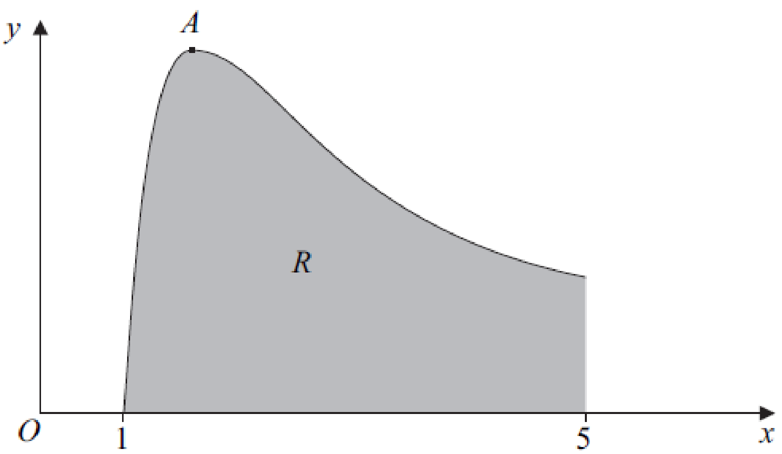
(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1} \, dx = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$ *(3 marks)*

(ii) Hence show that $\int_0^4 x\sqrt{2x+1} \, dx = 19.9$ correct to three significant figures. *(4 marks)*

(b) Using integration by parts, find $\int x^{-2} \ln x \, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

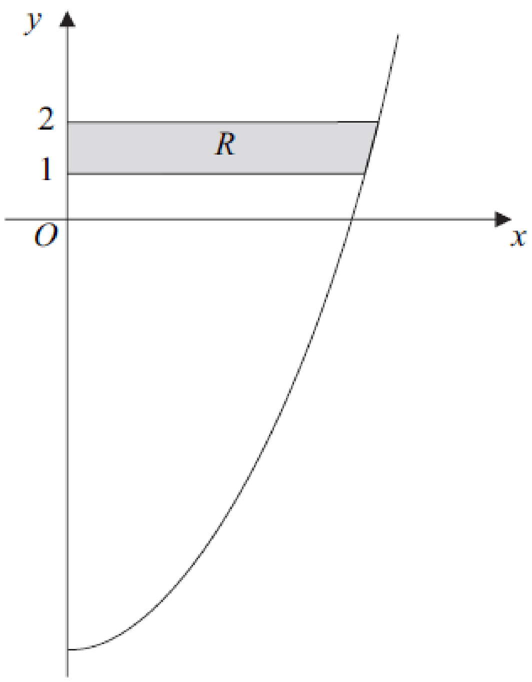
$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

(b) Use the substitution $u = 2x + 1$ to find $\int x(2x+1)^8 \, dx$, giving your answer in terms of x. *(4 marks)*

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2+5} \, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the y-axis**. *(4 marks)*

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6 (a) Use integration by parts to find $\int xe^{5x} \, dx$. *(4 marks)*

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})} \, dx = \int \frac{2}{1+u} \, du$$ *(2 marks)*

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})} \, dx$. *(3 marks)*

Integration Answers

3(a)(i)	$f' = \frac{dy}{dx} = 4x^3 + 2$	B1	1	
(ii)	$\int \frac{2x^3+1}{x^4+2x}\,dx$			
	$= \frac{1}{2}\ln\left(x^4+2x\right)\ (+c)$	M1 A1	 2	For $k\ln\left(x^4+2x\right)$ By substitution $k\ln u$ M1 correct A1
(b)(i)	$u = 2x+1$			
	$du = 2\,dx$	B1		
	$\int x\sqrt{2x+1}\,dx =$			
	$\int \left(\frac{u-1}{2}\right)\sqrt{u}\,\frac{du}{2}$	M1		Must be in terms of u only incl. du
	$= \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$	A1	3	AG

(ii)	$\int_0^4 dx = \int_1^9 du$	B1		Or changing u's to x's at end
	$\frac{1}{4}\int u^{\frac{3}{2}} - u^{\frac{1}{2}} = \frac{1}{4}\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]$	M1 A1		
	$= \frac{1}{4}\left[\left(\frac{2}{5}(9)^{\frac{5}{2}} - \frac{2}{3}(9)^{\frac{3}{2}}\right) - \left(\frac{2}{5} - \frac{2}{3}\right)\right]$			Sight of any of these 3 lines
	$= \frac{1}{4}\left[79.2 + 0.2\dot{6}\right]$			
	$= 19.86$			
	$= 19.9$	A1	4	AG
	Total		**10**	

(b)	$\int x^{-2}\ln x\,dx \qquad u=\ln x \qquad dv = x^{-2}$	M1		Attempt at integration by parts
	$du=\frac{1}{x} \qquad v=-x^{-1}$	A1		
	$\int = -\frac{1}{x}\ln x + \int x^{-2}\,dx$	A1		
	$= -\frac{1}{x}\ln x - \frac{1}{x}(+c)$	A1	4	

(ii)	$R=\left[-\frac{1}{x}(\ln x+1)\right]_1^5$	M1		$R=\left[\text{ Their (b)}\right]_1^5$
	$=-\frac{1}{5}(\ln 5+1)+(\ln 1+1)$	A1		OE
	$=\frac{1}{5}(4-\ln 5)$	A1	3	convincing argument; AG

(b)	$\int x(2x+1)^8\,dx$ $u=2x+1$ $du=2\,dx$	B1		OE
	$\int = \int\left(\frac{u-1}{2}\right)u^8\left(\frac{du}{2}\right)$	M1		all in terms of u. Condone omission of du
	$=\frac{1}{4}\int u^9-u^8\,du$			
	$=\frac{1}{4}\left[\frac{u^{10}}{10}-\frac{u^9}{9}\right]$	B1		$p\frac{u^{10}}{10}+q\frac{u^9}{9}$
	$=\frac{(2x+1)^{10}}{40}-\frac{(2x+1)^9}{36}\,(+c)$	A1	4	OE; CAO SC: correct answer, no working/parts in x (B1)

4(a)	$\int x\sin x\,dx \quad u = x$			
	$\frac{dv}{dx} = \sin x$	M1		For differentiating one term and integrating other
	$\frac{du}{dx} = 1 \quad v = -\cos x$			
	$\int = -x\cos x - \int -\cos x\,(dx)$	m1		For correctly substituting their terms into parts formula
		A1		
	$= -x\cos x + \sin x\ (+c)$	A1	4	CSO
(b)	$u = x^2 + 5$			
	$du = 2x\,dx$			
	$\int = \int \frac{1}{2}u^{\frac{1}{2}}(du)$	M1		$\int ku^{\frac{1}{2}}(du)$ condone omission of du but M0 if dx
		A1		$k = \frac{1}{2}$ OE
	$= \frac{u^{\frac{3}{2}}}{3}$	A1√		Ft $\int ku^{\frac{1}{2}}du$
	$= \frac{1}{3}\sqrt{(x^2+5)^3}\ (+c)$	A1	4	CSO SC $\frac{2}{6}\sqrt{(x^2+5)^3}$ with no working B3
(c)	$y = x^2 - 9$			
	$x^2 = y + 9$			
	$V = \pi\int x^2\,dy$	B1		Must have π and x^2, condone omission of dy, but B0 if dx
	$= \pi\int (y+9)\,dy$			
	$= (\pi)\left[\frac{y^2}{2} + 9y\right]_1^2$ or $(\pi)\left[\frac{(y+9)^2}{2}\right]_1^2$	M1		$\int$"their x^2"dy integrated; Limits 2 and 1 substituted in correct order including – sign } π not necessary
	$= (\pi)\left[20 - 9\frac{1}{2}\right]$	m1		
	$= 10\frac{1}{2}\pi$	A1	4	CSO
	Total		**12**	

6(a)	$\int x\mathrm{e}^{5x}\mathrm{d}x$			
	$u = x \quad \mathrm{d}v = \mathrm{e}^{5x}$	M1		integrate one term, differentiate one term
	$\mathrm{d}u = 1 \quad v = \frac{1}{5}\mathrm{e}^{5x}$	A1		
	$\int = \frac{1}{5}x\mathrm{e}^{5x} - \int \frac{1}{5}\mathrm{e}^{5x}\mathrm{d}x$	A1		
	$= \frac{1}{5}x\mathrm{e}^{5x} - \frac{1}{25}\mathrm{e}^{5x} (+c)$	A1	**4**	
(b)(i)	$u = x^{\frac{1}{2}}$			
	$\mathrm{d}u = \frac{1}{2}x^{-\frac{1}{2}}\,\mathrm{d}x$	M1		
	$\int = \int \frac{1}{1+u} \times 2\,\mathrm{d}u$	A1	2	correct with no errors; AG
(ii)	$\int_1^9 \mathrm{d}x = \int_1^3 \frac{2}{1+u}\mathrm{d}u$	m1		correct limits used in correct expression, ignoring k
	$= \left[2\ln(1+u)\right]_1^3$	M1		for $k\ln(1+u)$
	$= 2\ln 4 - 2\ln 2$ $(= \ln 4)$	A1	3	ISW OE
	Total		**9**	