Indices & Log Questions

3 (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. *(3 marks)*

(b) An infinite geometric series has common ratio r. The sum to infinity of the series is five times the first term of the series.

(i) Show that $r = 0.8$. *(3 marks)*

(ii) Given that the first term of the series is 20, find the least value of n such that the nth term of the series is less than 1. *(3 marks)*

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7 It is given that n satisfies the equation

$$2 \log_a n - \log_a (5n - 24) = \log_a 4$$

(a) Show that $n^2 - 20n + 96 = 0$. *(3 marks)*

(b) Hence find the possible values of n. *(2 marks)*

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5 (a) Given that

$$\log_a x = 2 \log_a 6 - \log_a 3$$

show that $x = 12$. *(3 marks)*

(b) Given that

$$\log_a y + \log_a 5 = 7$$

express y in terms of a, giving your answer in a form not involving logarithms. *(3 marks)*

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3 (a) Write down the values of p, q and r given that:

(i) $64 = 8^p$;

(ii) $\frac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of x for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$ *(2 marks)*

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1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

(b) (i) Find $\int 3x^{\frac{1}{2}}\,\mathrm{d}x$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}}\,\mathrm{d}x$. *(2 marks)*

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8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

(i) express x in terms of a; *(1 mark)*

(ii) express xy in terms of a. *(4 marks)*

Indices & Logarithms Answers

3(a)	$\log 0.8^x = \log 0.05$ / $x = \log_{0.8} 0.05$ **(M1)**	M1		NMS: SC B2 for 13.425 or better (B1 for 13.4 or 13.43; 13.42)
	$x \log_{10} 0.8 = \log_{10} 0.05$ oe	A1		
	$x = 13.425$ to 3dp / 13.425 (**A2**) (else **A1** for 1 or 2dp)	A1	3	Condone greater accuracy
(b)(i)	$\frac{a}{1-r}$	M1		$S_\infty = \frac{a}{1-r}$ <u>used</u>
	$\frac{a}{1-r} = 5a \Rightarrow a = 5a(1-r)$	A1		Or better
	$\Rightarrow 1 = 5(1-r) \Rightarrow r = \frac{4}{5} = 0.8$	A1	3	AG (be convinced)
(ii)	n^{th} term $= 20 \times (0.8)^{n-1}$	M1		Condone $20 \times (0.8)^n$.
	n^{th} term $< 1 \Rightarrow 0.8^{n-1} < \frac{1}{20}$ oe	A1		$0.8^{n-1} < 0.05$ or $0.8^{n-1} = k$, where $k = 0.05$ or k rounds **up** to 0.050
	Least n is 15	A1F	3	If not 15, ft on integer part of [answer (a)+2] provided $n>2$ SC 3/3 for 15 if no error SC n^{th} term$=16^{n-1}$ M1A0A0
	Total		**9**	

7(a)	$2\log_a n - \log_a(5n-24) = \log_a 4$			
	$\Rightarrow \log_a n^2 - \log_a(5n-24) = \log_a 4$	M1		A law of logs used
	$\Rightarrow \log_a\left[\frac{n^2}{5n-24}\right] = \log_a 4$	M1		A second law of logs used leading to both sides being single log terms or single log term on LHS with RHS=0
	$\Rightarrow \frac{n^2}{5n-24} = 4$			
	$\Rightarrow n^2 - 20n + 96 = 0$	A1	3	CSO. AG
(b)	$\Rightarrow (n-8)(n-12) = 0$	M1		Accept alternatives eg formula, completing of sq..
	$\Rightarrow n = 8,\ 12$	A1	2	
	Total		**5**	

5(a)	$\log_a x = \log_a 6^2 - \log_a 3$	M1		One law of logs used correctly
	$\log_a x = \log_a \left(\frac{6^2}{3}\right)$	M1		A second law of logs used correctly
	$\log_a x = \log_a \frac{36}{3} \Rightarrow x = 12$	A1	3	CSO AG
(b)	$\log_a y + \log_a 5 = 7 \Rightarrow \log_a 5y = 7$	M1		
	$\Rightarrow 5y = a^7$ or $y = \frac{1}{5}a^7$ or $a = (5y)^{1/7}$	m1 A1	 3	Eliminates logs Accept these forms
	Total		6	

3(a)(i)	$\{p =\}\ 2$	B1		Condone '$64=8^2$'
(ii)	$\{q =\}\ -2$	B1ft		Ft on '$-p$' if q not correct
(iii)	$\{r =\}\ 0.5$	B1	3	Condone '$\sqrt{8} = 8^{0.5}$'
(b)	$\frac{8^x}{8^{0.5}} = 8^{-2} \Rightarrow 8^{x-0.5} = 8^{-2}$ OE	M1		Using parts (a) & valid index law to stage $8^c = 8^d$ (PI)
	$\Rightarrow x - 0.5 = -2 \quad \Rightarrow x = -1.5$	A1ft	2	Ft on c's $(q + r)$ if not correct (Accept correct answer without working)
	ALT: $\log 8^x = \log k$, $x \log 8 = \log k$; $x = -1.5$			(M1 A1)
	Total		5	

1(a)(i)	x^2	B1	1	
(ii)	$x^{\frac{1}{2}} = \sqrt{x}$	B1	1	Accept either form
(iii)	x^3	B1	1	
(b)(i)	$\int 3x^{\frac{1}{2}}\,dx = \frac{3}{\frac{3}{2}}x^{\frac{3}{2}}\ \{+c\}$	M1 A1		Index raised by 1 Simplification not yet required
	$= 2x^{\frac{3}{2}} + c$	A1	3	Need simplification and the $+ c$ OE
(ii)	$\int_1^9 3x^{\frac{1}{2}}\,dx = (2 \times 9^{\frac{3}{2}}) - (2 \times 1^{\frac{3}{2}})$	M1		F(9) – F(1), where F(x) is candidate's answer to (b)(i) [or clear recovery]
	= 52	A1ft	2	Ft on (b)(i) answer of form $kx^{1.5}$ i.e. $26k$
	Total		8	

	Total		15	
8(a)	$\log_a n = \log_a 3(2n-1)$	M1		OE Log law used PI by next line
	$\Rightarrow\ n = 3(2n-1)$	m1		OE, but must **not** have any logs.
	$\Rightarrow\ 3 = 5n \Rightarrow\ n = \dfrac{3}{5}$	A1	3	
(b)(i)	$\log_a x = 3 \Rightarrow x = a^3$	B1	1	
(ii)	$\log_a y - \log_a 2^3 = 4$	M1		$3\log 2 = \log 2^3$ seen or used any time in **(ii)**
	$\log_a \dfrac{y}{2^3} = 4 \begin{cases} xy = a^7 \times a^{(3\log_a 2)} \\ \text{or} \\ y = a^4 \times a^{(3\log_a 2)} \end{cases}$	M1		Correct method leading to an equation involving y (or xy) and a log but **not** involving + or −
	$\dfrac{y}{2^3} = a^4 \quad \begin{cases} xy = a^7 \times 2^3 \\ \text{or} \\ y = a^4 \times 2^3 \end{cases}$	m1		Correct method to eliminate **ALL** logs e.g. using $\log_a N = k \Rightarrow N = a^k$ or using $a^{\log_a c} = c$
	$by\ =\ a^3 \times 8a^4$ or $8a^7$	A1	4	
	Total		**8**	