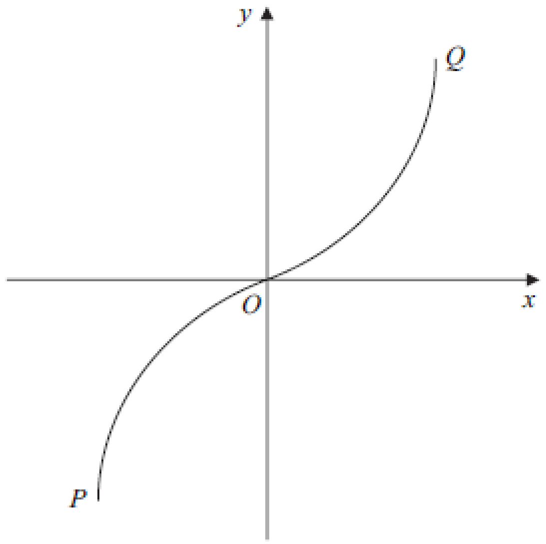
Function Questions

7 (a) The sketch shows the graph of $y = \sin^{-1} x$.



Write down the coordinates of the points P and Q, the end-points of the graph.

(2 marks)

(b) Sketch the graph of $y = -\sin^{-1}(x - 1)$. *(3 marks)*

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8 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2 \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 4$. *(4 marks)*

(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

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4 (a) Sketch and label on the same set of axes the graphs of:

(i) $y = |x|$; *(1 mark)*

(ii) $y = |2x - 4|$. *(2 marks)*

(b) (i) Solve the equation $|x| = |2x - 4|$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|x| > |2x - 4|$. *(2 marks)*

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8 A function f is defined by $f(x) = 2e^{3x} - 1$ for all real values of x.

(a) Find the range of f. *(2 marks)*

(b) Show that $f^{-1}(x) = \frac{1}{3}\ln\left(\frac{x+1}{2}\right)$. *(3 marks)*

(c) Find the gradient of the curve $y = f^{-1}(x)$ when $x = 0$. *(4 marks)*

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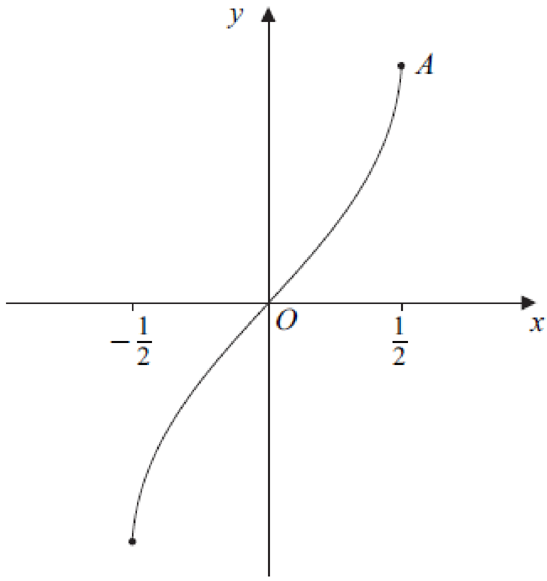
9 The diagram shows the curve with equation $y = \sin^{-1} 2x$, where $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$.



(a) Find the y-coordinate of the point A, where $x = \frac{1}{2}$. *(1 mark)*

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3 The functions f and g are defined with their respective domains by

$$f(x) = 3 - x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \quad \text{for real values of } x, \ x \neq -1$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of g is g^{-1}.

(i) Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) The composite function gf is denoted by h.

(i) Find $h(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h. *(1 mark)*

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7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

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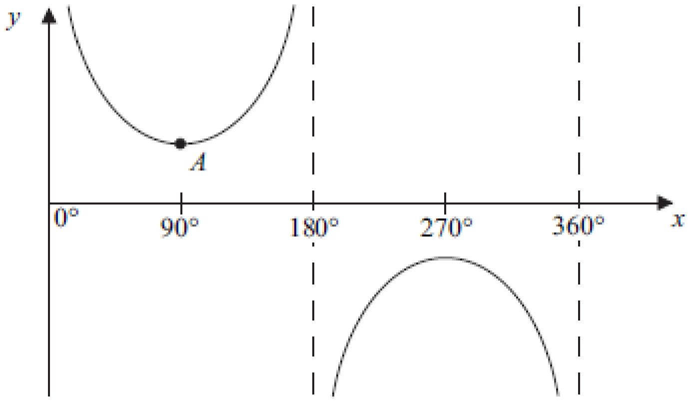
3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0° < x < 360°$.
(2 marks)

(b) The diagram shows the graph of $y = \operatorname{cosec} x$ for $0° < x < 360°$.



(i) The point A on the curve is where $x = 90°$. State the y-coordinate of A.
(1 mark)

(ii) Sketch the graph of $y = |\operatorname{cosec} x|$ for $0° < x < 360°$. *(2 marks)*

(c) Solve the equation $|\operatorname{cosec} x| = 2$, giving all values of x in the interval $0° < x < 360°$.
(2 marks)

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5 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{x-2} \quad \text{for } x \geqslant 2$$

$$\mathrm{g}(x) = \frac{1}{x} \quad \text{for real values of } x,\ x \neq 0$$

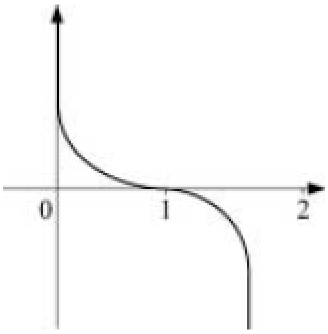
(a) State the range of f. *(2 marks)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

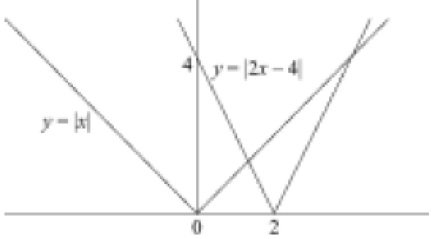
(ii) Solve the equation $\mathrm{fg}(x) = 1$. *(3 marks)*

(c) The inverse of f is f^{-1}. Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

Functions Answers

7(a)	$\left(1, \frac{\pi}{2}\right)$ OE in decimals	B1		Or for -1 and 1
	$\left(-1, -\frac{\pi}{2}\right)$	B1	2	
(b)		M1		Translation in $+ve$ x direction
		M1		Correct shape
		A1	3	Correct Graph Through (1,0) touching $y-$axis

8(a)	(Range of f) $\geqslant 0$	B1	1	
(b)(i)	$\text{fg}(x) = \frac{1}{(x+2)^2}$	B1	1	OE Maybe in part (ii)
(ii)	$\frac{1}{(x+2)^2} = 4$			
	$(x+2)^2 = \frac{1}{4}$	M1		Or $4(x+2)^2 = 1$
	$x+2 = (\pm)\frac{1}{2}$	M1		$(2x+5)(2x+3) = 0$
	$x = -\frac{5}{2}, -\frac{3}{2}$	A1 A1	4	
(c)(i)	Not one to one	E1	1	OE
(ii)	$x = \frac{1}{y+2}$	M1		$x \Leftrightarrow y$
	$y+2 = \frac{1}{x}$	M1		Attempt to isolate
	$y = \frac{1}{x} - 2$ $\left(\frac{1-2x}{x}\right)$	A1	3	
	Total		10	

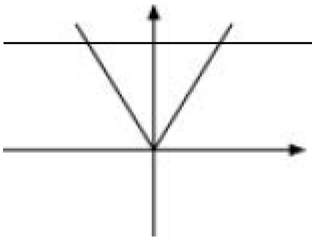
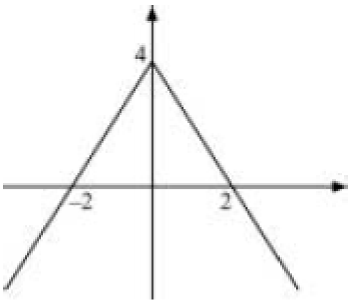


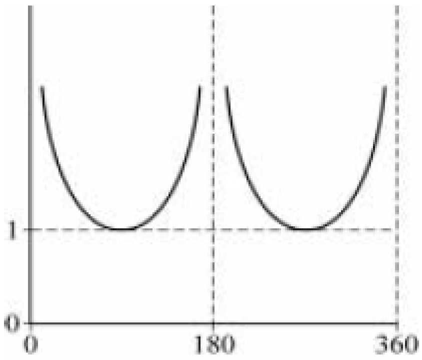
4(a)(i)	(graph above)	B1	1	$y=\lvert x\rvert$
(ii)		M1		2 branches mod graph $x>0$ for $y=0$
		A1	2	for 2, 4
(b)(i)	$x=2x-4,\ x=4$	B1		
	$-x=2x-4$	M1		
	$x=\dfrac{4}{3}$	A1	3	OE one value only
	Alternative:			
	$x^2=(2x-4)^2$	M1		
	$x=4,\ \dfrac{4}{3}$	A1A1		
(ii)	$\dfrac{4}{3}<x<4$	M1		$\dfrac{4}{3}, 4$ (ft) identified as extremes
		A1	2	CAO
	Total		**8**	

8(a)	$\mathrm{f}(x)=2e^{3x}-1$			
	Range: $\mathrm{f}(x)>-1$ (or $y>-1$ or $\mathrm{f}>-1$)	M1		for -1 only
		A1	2	exactly correct
(b)	$y=2e^{3x}-1$			
	$x=2e^{3y}-1$	M1		$x\leftrightarrow y$
	$2e^{3y}=x+1$			
	$e^{3y}=\dfrac{x+1}{2}$	M1		attempt to isolate
	$y=\dfrac{1}{3}\ln\left(\dfrac{x+1}{2}\right)$	A1	3	all correct with no error AG (be convinced)
(c)	$\mathrm{f}'^{-1}(x)=\dfrac{1}{3}\left(\dfrac{2}{x+1}\right)\times\dfrac{1}{2}$ OE	M1		for differentiation of ln; $\dfrac{k}{\text{their}(x\pm1)}$
		A1		for $\dfrac{1}{2}$
		A1		all correct
	$x=0$			
	$\mathrm{f}'^{-1}(x)=\dfrac{1}{3}$	A1	4	CSO
	Alternative			
	$\mathrm{f}^{-1}(x)=\dfrac{1}{3}\ln(x+1)-\dfrac{1}{3}\ln 2$	M1A1		
	$\mathrm{f}'^{-1}(x)=\dfrac{1}{3(x+1)}$	A1		
	$\mathrm{f}'^{-1}(0)=\dfrac{1}{3}$	A1		CSO
	Total		**9**	

9(a)	$x=\frac{1}{2}$ $\quad y=\frac{\pi}{2}$ $\left(\text{or } 1.57, \sin^{-1}1\right)$	B1	1	ignore 90°

3(a)	$\text{f}(x)\le 3$	M1A1	2	M1 for f <3, $x\le 3$ Condone y, f, range
(b)(i)	$y=\frac{2}{x+1}$			
	$x+1=\frac{2}{y}$	M1		Attempt to obtain x as a function of y or y as a function of x
	$x=\frac{2}{y}-1$	M1		$x\leftrightarrow y$ at any stage
	$y/\text{g}^{-1}(x)=\frac{2}{x}-1=\frac{2-x}{x}$	A1	3	Any correct form
(ii)	$\left(\text{g}^{-1}(x)\right)\ne -1$	B1	1	
(c)(i)	$\text{h}(x)=\frac{2}{3-x^2+1}$	M1		
	$=\frac{2}{4-x^2}=\frac{2}{(2-x)(2+x)}$	A1	2	
(ii)	$(x\in\mathbb{R})$, $x\ne +2$, $x\ne -2$	B1	1	Condone omit 'x is real' Allow $x^2\ne 4$
	Total		9	

7(a)		B1	1	
(b)		M1 A1 A1	 3	Shape inverted V in all four quadrants Symmetrical about y axis Coordinates
(c)	$4-\lvert 2x\rvert=x$			
	$4-2x=x \qquad x=\frac{4}{3}$	M1 A1		Attempt to solve
	$4+2x=x \qquad x=-4$	A1	3	And no others
(d)	$-4<x<\frac{4}{3}$	M1 A1	 2	Either correct Other solution and no extras SC $-4\le x\le\frac{4}{3}$ B1

3(a)	$\text{cosec } x = 2$			
	$\Rightarrow \sin x = \frac{1}{2}$	M1		$30°$ scores M1 implied
	$x = 30, 150$	A1	2	and no extras in range
(b)(i)	1	B1	1	
(ii)		M1		all positive, 2 U shapes
		A1	2	minima consistent > 0, not intersecting with each other or y-axis
(c)	$x = 30, 150, 210, 330$	B1F		3 correct values from their (a), which must be $\theta, 180 - \theta$
		B1	2	all correct and no extras in range

5(a)	$f(x) \geq 0$	allow $y \geq 0$	M1		> 0 or $f \geq 0$ or ≥ 0
			A1	2	
(b)(i)	$\sqrt{\frac{1}{x} - 2}$		B1	1	
(ii)	$\frac{1}{x} - 2 = 1$		M1		squaring their (b)(i) in an equation
	$\frac{1}{x} = 3$	OE	A1		
	$x = \frac{1}{3}$		A1	3	CSO
(c)	$y = \sqrt{x - 2}$				
	$y^2 = x - 2$		M1		attempt to isolate; condone 1 slip
	$x^2 = y - 2$		M1		reverse $x \Leftrightarrow y$
	$y = x^2 + 2$		A1	3	
		Total		**9**	