Differentiation Questions

7 The volume, V m^3, of water in a tank at time t seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{dV}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2V}{dt^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in m^3 s^{-1}, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

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3 A curve has equation $y = 7 - 2x^5$.

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) Find an equation for the tangent to the curve at the point where $x = 1$. *(3 marks)*

(c) Determine whether y is increasing or decreasing when $x = -2$. *(2 marks)*

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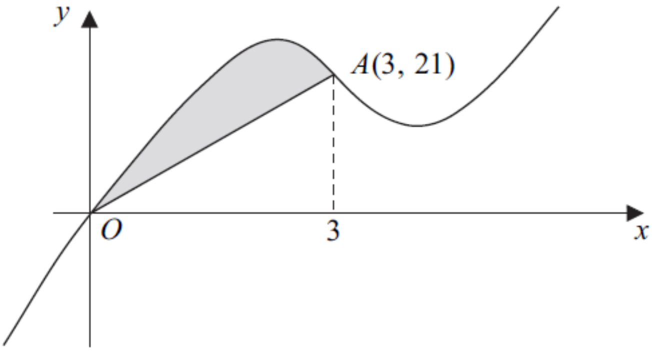
5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a stationary point when $x = 2$ and find the x-coordinate of the other stationary point. *(4 marks)*

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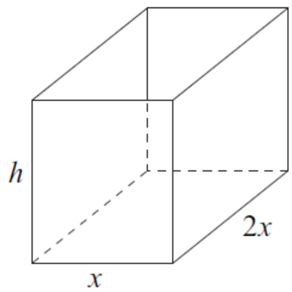
5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, V m^3, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3}$$ *(1 mark)*

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

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4 A model helicopter takes off from a point O at time $t = 0$ and moves vertically so that its height, y cm, above O after time t seconds is given by

$$y = \frac{1}{4}t^4 - 26t^2 + 96t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\frac{dy}{dt}$; *(3 marks)*

(ii) $\frac{d^2y}{dt^2}$. *(2 marks)*

(b) Verify that y has a stationary value when $t = 2$ and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of y with respect to t when $t = 1$. *(2 marks)*

(d) Determine whether the height of the helicopter above O is increasing or decreasing at the instant when $t = 3$. *(2 marks)*

Differentiation Answers

7(a)(i)	$\frac{dV}{dt} = 2t^5 - 8t^3 + 6t$	M1 A1 A1	3	One term correct unsimplified Further term correct unsimplified All correct unsimplified (no + c etc)
(ii)	$\frac{d^2V}{dt^2} = 10t^4 - 24t^2 + 6$	M1 A1	2	One term FT correct unsimplified **CSO**. All correct simplified
(b)	Substitute $t = 2$ into their $\frac{dV}{dt}$	M1		
	$(= 64 - 64 + 12)\ = 12$	A1	2	**CSO**. Rate of change of volume is $12\text{m}^3\ \text{s}^{-1}$
(c)(i)	$t = 1 \Rightarrow \frac{dV}{dt} = 2 - 8 + 6$	M1		Or putting their $\frac{dV}{dt} = 0$
	$= 0 \Rightarrow$ Stationary value	A1	2	**CSO**. Shown to = 0 **AND** statement (If solving equation must obtain $t = 1$)
(ii)	$t = 1 \Rightarrow \frac{d^2V}{dt^2} = -8$	M1		Sub $t = 1$ into their second derivative or equivalent full test.
	Maximum value	A1√	2	Ft if their test implies minimum
	Total		**11**	

3(a)	$\frac{dy}{dx} = -10x^4$	M1 A1	2	kx^4 condone extra term Correct derivative unsimplified
(b)	When $x = 1$, gradient $= -10$	B1√		FT their gradient when $x = 1$
	Tangent is	M1		Attempt at y & tangent (**not** normal)
	$y - 5 = -10(x - 1)$ *or* $y + 10x = 15$ etc	A1	3	CSO Any correct form
(c)	When $x = -2$ $\frac{dy}{dx} = -160$ (or < 0)	B1√		Value of their $\frac{dy}{dx}$ when $x = -2$
	($\frac{dy}{dx} < 0$ hence) y is **decreasing**	E1√	2	ft Increasing if their $\frac{dy}{dx} > 0$
	Total		**7**	

5(a)(i)	$\frac{dy}{dx} = 3x^2 - 20x + 28$	M1 A1 A1	3	One term correct Another term correct All correct (no + c etc)
(ii)	Their $\frac{dy}{dx} = 0$ for stationary point	M1		Or realising condition for stationary pt
	$(x - 2)(3x - 14) = 0$	m1		Attempt to solve using formula/ factorise
	$\Rightarrow x = 2$	A1		Award M1, A1 for verification that
	or $x = \frac{14}{3}$	A1	4	$x = 2 \Rightarrow \frac{dy}{dx} = 0$ then may earn m1 later

5(a)(i)	$2x^2+2xh+4xh \quad (=54)$ $\Rightarrow x^2+3xh=27$	M1 A1	2	Attempt at surface area (one slip) **AG CSO**
(ii)	$h=\dfrac{27-x^2}{3x}$ or $h=\dfrac{9}{x}-\dfrac{x}{3}$ etc	B1	1	Any correct form
(iii)	$V=2x^2h \quad = 18x-\dfrac{2x^3}{3}$	B1	1	**AG** (watch fudging) condone omission of brackets
(b)(i)	$\dfrac{dV}{dx}=18-2x^2$	M1 A1	2	One term correct "their" V All correct unsimplified $18-6x^2/3$
(ii)	Sub $x=3$ into their $\dfrac{dV}{dx}$	M1		Or attempt to solve their $\dfrac{dV}{dx}=0$
	Shown to equal 0 plus **statement** that this implies a stationary point if verifying	A1	2	**CSO** Condone $x=\pm3$ or $x=3$ if solving
(c)	$\dfrac{d^2V}{dx^2}=-4x$ $(=-12)$ $\dfrac{d^2V}{dx^2}<0$ at stationary point $\Rightarrow$ maximum	B1√ E1√	2	**FT** their $\dfrac{dV}{dx}$ **FT** their second derivative conclusion If "their" $\dfrac{d^2y}{dx^2}>0 \Rightarrow$ minimum etc
	Total		**10**	

4(a)(i)	$t^3-52t+96$	M1 A1 A1	3	one term correct another term correct all correct (no + c etc)
(ii)	$3t^2-52$	M1 A1√	2	ft one term correct ft all "correct"
(b)	$\dfrac{dy}{dt}=8-104+96$ $=0 \Rightarrow$ stationary value	M1 A1		substitute $t=2$ into their $\dfrac{dy}{dt}$ CSO; shown $=0$ + statement
	Substitute $t=2$ into $\dfrac{d^2y}{dt^2}$ $(=-40)$	M1		any appropriate test, e.g. $y'(1)$ and $y'(3)$
	$\dfrac{d^2y}{dt^2}<0 \Rightarrow$ max value	A1	4	all values (if stated) must be correct
(c)	Substitute $t=1$ into their $\dfrac{dy}{dt}$	M1		must be their $\dfrac{dy}{dt}$ NOT $\dfrac{d^2y}{dt^2}$
	Rate of change $=45$ $\left(\text{cm s}^{-1}\right)$	A1√	2	ft their $y'(1)$

(d)	Substitute $t = 3$ into their $\frac{dy}{dt}$ $(27 - 156 + 96 = -33 < 0)$ $\Rightarrow$ decreasing when $t = 3$	M1 E1√	 2	interpreting their value of $\frac{dy}{dt}$ allow increasing if their $\frac{dy}{dt} > 0$
	Total		**13**	