

Binomial Questions

6 Plastic clothes pegs are made in various colours.

The number of red pegs may be modelled by a binomial distribution with parameter p equal to 0.2.

The contents of packets of 50 pegs of mixed colours may be considered to be random samples.

(a) Determine the probability that a packet contains:

- (i) less than or equal to 15 red pegs; *(2 marks)*
- (ii) exactly 10 red pegs; *(2 marks)*
- (iii) more than 5 but fewer than 15 red pegs. *(3 marks)*

(b) Sly, a student, claims to have counted the number of red pegs in each of 100 packets of 50 pegs. From his results the following values are calculated.

$$\text{Mean number of red pegs per packet} = 10.5$$

$$\text{Variance of number of red pegs per packet} = 20.41$$

Comment on the validity of Sly's claim. *(4 marks)*

5 Kirk and Les regularly play each other at darts.

(a) The probability that Kirk wins any game is 0.3, and the outcome of each game is independent of the outcome of every other game.

Find the probability that, in a match of 15 games, Kirk wins:

- (i) exactly 5 games; *(3 marks)*
 - (ii) fewer than half of the games; *(3 marks)*
 - (iii) more than 2 but fewer than 7 games. *(3 marks)*
-

- 2 A hotel has 50 single rooms, 16 of which are on the ground floor. The hotel offers guests a choice of a full English breakfast, a continental breakfast or no breakfast. The probabilities of these choices being made are 0.45, 0.25 and 0.30 respectively. It may be assumed that the choice of breakfast is independent from guest to guest.
- (a) On a particular morning there are 16 guests, each occupying a single room on the ground floor. Calculate the probability that exactly 5 of these guests require a full English breakfast. *(3 marks)*
- (b) On a particular morning when there are 50 guests, each occupying a single room, determine the probability that:
- (i) at most 12 of these guests require a continental breakfast; *(2 marks)*
- (ii) more than 10 but fewer than 20 of these guests require no breakfast. *(3 marks)*
- (c) When there are 40 guests, each occupying a single room, calculate the mean and the standard deviation for the number of these guests requiring breakfast. *(4 marks)*
-

- 6 Each weekday, Monday to Friday, Trina catches a train from her local station. She claims that the probability that the train arrives on time at the station is 0.4, and that the train's arrival time is independent from day to day.
- (a) Assuming her claims to be true, determine the probability that the train arrives on time at the station:
- (i) on at most 3 days during a 2-week period (10 days); *(2 marks)*
- (ii) on more than 10 days but fewer than 20 days during an 8-week period. *(3 marks)*
- (b) (i) Assuming Trina's claims to be true, determine the mean and standard deviation for the number of times during a week (5 days) that the train arrives on time at the station. *(3 marks)*
- (ii) Each week, for a period of 13 weeks, Trina's travelling colleague, Suzie, records the number of times that the train arrives on time at the station. Suzie's results are
- 2 2 4 1 2 3 3 2 2 0 3 2 0
- Calculate the mean and standard deviation of these values. *(3 marks)*
- (iii) Hence comment on the likely validity of Trina's claims. *(2 marks)*
-

Binomial Answers

6(a)(i)	$B(50, 0.2)$ $P(R \leq 15) = 0.969$ to 0.97	M1 A1	2	Use of in (a) AWFW 0.9692
(ii)	$P(R = 10) = P(R \leq 10) - P(R \leq 9)$ or $P(R = 10) = \binom{50}{10} (0.2)^{10} (0.8)^{40}$ $= 0.5836 - 0.4437 = 0.139$ to 0.141	M1 A1	 2	Stated or implied Stated or implied AWFW 0.1399
(iii)	$P(5 < R < 15) =$ $P(R \leq 14 \text{ or } 15) = 0.9393$ or 0.9692 minus $P(R \leq 5 \text{ or } 4) = 0.0480$ or 0.0185 $= 0.89$ to 0.893 or $B(50, 0.2)$ expressions stated for at least 3 of $5 \leq R \leq 15$ <div style="text-align: right;">Answer</div>	M1 M1 A1 (M1) (A2)	 3	Accept values to 3 dp Accept values to 3 dp AWFW 0.8913 Or implied by a correct answer
(b)	Mean, $\mu = np = 50 \times 0.2 = 10$ or Estimate of p , $\hat{p} = 0.21$ Variance, $\sigma^2 = np(1-p) = 10 \times 0.8 = 8$ Mean or Estimate of p is similar to that expected but Variance (standard deviation) is different from that expected Reason to doubt validity of Sly's claim	B1 B1 B1 B1	 4	Either; CAO CAO 10.5 and 10 or 0.21 and 0.2 Either point 20.41 and 8 or 4.5 and 2.8 Must be based on both 10 or 0.2 and 8 or on both 10 or 0.2 and 2.8 correctly
Total			11	

5(a)	B(15, 0.3)	M1		use of in (a)
(i)	$P(K = 5) = P(K \leq 5) - P(K \leq 4)$	M1		may be implied
	$P(K = 5) = \binom{15}{5}(0.3)^5(0.7)^{10}$	A1	3	AWFW (0.2061)
	$= 0.7216 - 0.5155 = 0.2055$ to 0.2065			
(ii)	(Fewer than) half $\Rightarrow 7$ or $7\frac{1}{2}$ or 8	B1		stated or implied
	Thus require $P(K \leq 7$ or $< 8)$	M1		used or implied by correct answer
	$= 0.9495$ to 0.9505	A1	3	AWFW (0.9500)
(iii)	$P(2 < K < 7) = 0.8689$ or 0.9500	M1		
	minus 0.1268 or 0.2969	M1		
	$= 0.7415$ to 0.7425	A1	3	AWFW (0.7421)
	or			
	B(15, 0.3) expressions stated for at least 3 terms within $2 \leq K \leq 7$	M1		or implied by a correct answer
	Answer	A2		
<hr/>				
2(a)	Use of binomial in (a), (b) or (c)	M1		Can be implied
	$P(E = 5) = \binom{16}{5}(p)^5(1-p)^{11}$	M1		Allow $p = 0.45, 0.25, 0.30$ or $\frac{1}{3}$
	$= 0.112$	A1	3	AWRT (0.1123)
(b)(i)	B(50, 0.25)	B1		Used; can be implied
	$P(C \leq 12) = 0.511$	B1	2	AWRT (0.5110)
(ii)	$P(10 < B' < 20) = 0.9152$ or 0.9522	M1		Allow 3 dp accuracy
	minus 0.0789 or 0.1390	M1		Allow 3 dp accuracy
	$= 0.836$	A1	3	AWRT (0.8363)
	or			
	B(50, 0.30) expressions stated for at least 3 terms within $10 \leq B' \leq 20$	(M1)		Or implied by a correct answer
	Answer = 0.836	(A2)		AWRT
(c)	$n = 40, p = 0.7$	B1		Both used; can be implied
	Mean $\mu = np = 28$	B1 \checkmark		CAO; \checkmark on p only
	Variance $\sigma^2 = np(1-p) = 8.4$	M1		Use of $np(1-p)$ even if SD
	Standard deviation = $\sqrt{8.4}$	A1	4	CAO; AWFW
	or = 2.89 to 2.9			
Total			12	

6(a)	Use of binomial in (a) or (b)(i)	M1		PI	
(i)	$P(T_{10} \leq 3) = 0.38$ to 0.383	B1	2	AWFW	(0.3823)
(ii)	$P(10 < T_{40} < 20) = 0.8702$ or 0.9256	M1		Allow 3 dp accuracy	
	minus 0.0352 or 0.0156	M1		Allow 3 dp accuracy	
	= 0.83 to 0.84	A1		AWFW	(0.835)
	OR				
	B(40, 0.40) expressions stated for at least 3 terms within $10 \leq T_{40} \leq 20$	(M1)		Or implied by a correct answer	
	Answer = 0.83 to 0.84	(A2)	3	AWFW	
(b)(i)	$n = 5$ $p = 0.4$				
	Mean, $\mu = np = 2$	B1		CAO	
	Variance, $\sigma^2 = np(1-p) = 1.2$	M1		Use of $np(1-p)$ even if SD	
	Standard deviation = $\sqrt{1.2}$ or = 1.09 to 1.1	A1	3	CAO AWFW	
(ii)	Mean (\bar{x}) = 2	B1		CAO	$\sum x = 26$
	Standard Deviation (s_n, s_{n-1}) = 1.1 to 1.16	B2		AWFW	$\sum x^2 = 68$ (1.1094 or 1.1547)
	If neither correct but use of mean (\bar{x}) = $\frac{\sum x}{13}$	(M1)	3		
(iii)	Means are same and SDs are similar/same Means are same but SDs are different so Trina's claims appear valid / invalid	B1 \uparrow Dep \uparrow B1			Must have scored full marks in (b)(i) and (b)(ii)
	Total		13		