Basic Algebra Questions – Mainly Quadratics

3 (a) (i) Express $x^2 - 4x + 9$ in the form $(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y = x^2 - 4x + 9$. *(2 marks)*

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4 The quadratic equation $x^2 + (m + 4)x + (4m + 1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

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2 (a) Express $x^2 + 8x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(b) Hence, or otherwise, show that the equation $x^2 + 8x + 19 = 0$ has no real solutions. *(2 marks)*

(c) Sketch the graph of $y = x^2 + 8x + 19$, stating the coordinates of the minimum point and the point where the graph crosses the y-axis. *(3 marks)*

(d) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 8x + 19$. *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k + 1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

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7 The quadratic equation $(k + 1)x^2 + 12x + (k - 4) = 0$ has real roots.

(a) Show that $k^2 - 3k - 40 \leqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

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3 (a) (i) Express $x^2 + 10x + 19$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Write down the coordinates of the vertex (minimum point) of the curve with equation $y = x^2 + 10x + 19$. *(2 marks)*

(iii) Write down the equation of the line of symmetry of the curve $y = x^2 + 10x + 19$. *(1 mark)*

(iv) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 10x + 19$. *(3 marks)*

(b) Determine the coordinates of the points of intersection of the line $y = x + 11$ and the curve $y = x^2 + 10x + 19$. *(4 marks)*

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7 The quadratic equation

$$(2k - 3)x^2 + 2x + (k - 1) = 0$$

where k is a constant, has real roots.

(a) Show that $2k^2 - 5k + 2 \leqslant 0$. *(3 marks)*

(b) (i) Factorise $2k^2 - 5k + 2$. *(1 mark)*

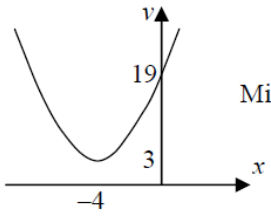
(ii) Hence, or otherwise, solve the quadratic inequality

$$2k^2 - 5k + 2 \leqslant 0$$ *(3 marks)*

Basic Algebra Answers – Mainly Quadratics

3(a)(i)	$(x-2)^2$ $+5$	B1 B1	2	$p=2$ $q=5$
(ii)	Minimum point (2, 5) or $x=2$, $y=5$	B2√	2	B1 for each coordinate correct or ft **Alt method** M1, A1 sketch, differentiation

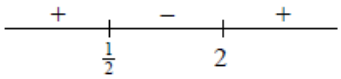
4(a)	$(m+4)^2=m^2+8m+16$	B1		Condone $4m+4m$
	$b^2-4ac=(m+4)^2-4(4m+1)=0$ $m^2+8m+16-16m-4=0$	M1		b^2-4ac (attempted and involving m's and no x's) or $b^2-4ac=0$ stated
	$\Rightarrow m^2-8m+12=0$	A1	3	**AG** (be convinced – all working correct- $=0$ appearing more than right at the end)
(b)	$(m-2)(m-6)=0$	M1		Attempt at factors or quadratic formula
	$m=2$, $m=6$	A1	2	**SC B1** for 2 or 6 only without working
	Total		**5**	

2(a)	$(x+4)^2$ $+3$	B1 B1	2	$p=4$ $q=3$
(b)	$(x+4)^2=-3$ or "their" $(x+p)^2=-q$	M1		Or discriminant = 64 –76
	No real square root of –3	A1	2	Disc < 0 so no real roots (all correct figs)
(c)				
	Minimum (– 4, 3)	B1√		ft their $-p$ and q (or correct)
	graph	B1		Parabola (vertex roughly as shown)
		B1	3	Crossing at y = 19 marked or (0, 19) stated
(d)	Translation (and no additional transf'n)	E1		Not shift, move, transformation, etc
	through $\begin{bmatrix}-4\\3\end{bmatrix}$	M1 A1	3	One component correct eg 3 units up All correct – if not vector – must say 4 units in negative x- direction, to left etc
	Total		**10**	

(ii)	$4(k+1)^2-4(2k^2-7)$	M1		"b^2-4ac" in terms of k (either term correct)
	$4k^2-8k-32=0$ or $k^2-2k-8=0$	A1		$b^2-4ac=0$ correct quadratic equation in k
	$(k-4)(k+2)=0$	m1		Attempt to factorise, solve equation
	$k=-2$, $k=4$	A1	4	SC B1, B1 for –2 , 4 (if M0 scored)

7(a)	$b^2-4ac = 144-4(k+1)(k-4)$	M1		Clear attempt at b^2-4ac Condone slip in one term of expression
	Real roots when $b^2-4ac \geqslant 0$	B1		Not just a statement, must involve k
	$36-(k^2-3k-4)\geqslant 0$ $\Rightarrow k^2-3k-40\leqslant 0$	A1	3	**AG** (watch signs carefully)
(b)	$(k-8)(k+5)$	M1		Factors attempt or formula
	Critical points 8 and –5	A1		
	Sketch or sign diagram **correct**, must have 8 and –5	M1		+ve | - ve | +ve –5 8
	$-5\leqslant k\leqslant 8$ A0 for $-5<k<8$ or two separate inequalities unless word AND used	A1	4	
	Total		**7**	

3(a)(i)	$(x+5)^2$	B1		$p=5$
	–6	B1	2	$q=-6$
(ii)	$x_{\text{vertex}}=-5$ (or their $-p$)	B1√		may differentiate but must have $x=-5$
	$y_{\text{vertex}}=-6$ (or their q)	B1√	2	and $y=-6$. Vertex $(-5,-6)$
(iii)	$x=-5$	B1	1	
(iv)	Translation (not shift, move etc)	E1		and NO other transformation stated
	through $\begin{bmatrix}-5\\-6\end{bmatrix}$ (or 5 left, 6 down)	M1		either component correct
		A1	3	M1, A1 independent of E mark
(b)	$x+11=x^2+10x+19$			quadratic with all terms on one side of equation
	$\Rightarrow x^2+9x+8=0$ or $y^2-13y+30=0$	M1		
	$(x+8)(x+1)=0$ or $(y-3)(y-10)=0$	m1		attempt at formula (1 slip) or to factorise
	$x=-1$, $y=10$ or $x=-8$, $y=3$	A1		both x values correct
		A1	4	both y values correct and linked
				SC $(-1,10)$ B2, $(-8,3)$ B2 no working
	Total		**12**	

7(a)	$b^2-4ac=4-4(k-1)(2k-3)$	M1		(or seen in formula) condone one slip
	Real roots when $b^2-4ac\geqslant 0$	E1		must involve $f(k)\geqslant 0$ (usually M1 must be earned)
	$4-4\left(2k^2-5k+3\right)\geqslant 0$			
	$\Rightarrow -2k^2+5k-3+1\geqslant 0$			at least one step of working justifying $\leqslant 0$
	$\Rightarrow 2k^2-5k+2\leqslant 0$	A1	3	AG
(b)(i)	$(2k-1)(k-2)$	B1	1	
(ii)	(Critical values) $\frac{1}{2}$ and 2	B1√		ft their factors or correct values seen on diagram, sketch or inequality or stated
		M1		use of sketch / sign diagram
	$\Rightarrow 0.5\leqslant k\leqslant 2$	A1	3	M1A0 for $0.5<k<2$ or $k\geqslant 0.5,\ k\leqslant 2$
	Total		7	