

Algebra & Functions Questions

1 (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. (1 mark)

(ii) Show that $f(-2) = 0$. (1 mark)

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax + b}$$

where a and b are integers. (3 marks)

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of d . (3 marks)

(c) Given that $\frac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ can be written in the form $\frac{A}{(3 - 2x)} + \frac{B}{(1 - x)} + \frac{C}{(1 - x)^2}$,
find the values of A , B and C . (5 marks)

1 (a) The polynomial $p(x)$ is defined by $p(x) = 6x^3 - 19x^2 + 9x + 10$.

(i) Find $p(2)$. (1 mark)

(ii) Use the Factor Theorem to show that $(2x + 1)$ is a factor of $p(x)$. (3 marks)

(iii) Write $p(x)$ as the product of three linear factors. (2 marks)

(b) Hence simplify $\frac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$. (2 marks)

3 (a) Given that $\frac{9x^2 - 6x + 5}{(3x - 1)(x - 1)}$ can be written in the form $3 + \frac{A}{3x - 1} + \frac{B}{x - 1}$, where A
and B are integers, find the values of A and B . (4 marks)

(b) Hence, or otherwise, find $\int \frac{9x^2 - 6x + 5}{(3x - 1)(x - 1)} dx$. (4 marks)

2 The polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 13$.

(a) Use the Remainder Theorem to find the remainder when $f(x)$ is divided by $(2x - 3)$.
(2 marks)

(b) The polynomial $g(x)$ is defined by $g(x) = 2x^3 - 7x^2 + 13 + d$, where d is a constant.

Given that $(2x - 3)$ is a factor of $g(x)$, show that $d = -4$.
(2 marks)

(c) Express $g(x)$ in the form $(2x - 3)(x^2 + ax + b)$.
(2 marks)

4 (a) (i) Express $\frac{3x - 5}{x - 3}$ in the form $A + \frac{B}{x - 3}$, where A and B are integers. (2 marks)

(ii) Hence find $\int \frac{3x - 5}{x - 3} dx$. (2 marks)

(b) (i) Express $\frac{6x - 5}{4x^2 - 25}$ in the form $\frac{P}{2x + 5} + \frac{Q}{2x - 5}$, where P and Q are integers.
(3 marks)

(ii) Hence find $\int \frac{6x - 5}{4x^2 - 25} dx$. (3 marks)

1 (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$. (2 marks)

(b) Simplify the algebraic fraction $\frac{2x^2 + x - 3}{x^2 - 1}$. (3 marks)

(b) Express $\frac{1 + 4x}{(1 + x)(1 + 3x)}$ in partial fractions. (3 marks)

Algebra & Functions Answers

1(a)(i)	$f(1) = 0$	B1	1	
(ii)	$f(-2) = -24 + 8 + 14 + 2 = 0$	B1	1	
(iii)	$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{(x-1)(x+2)}{(x-1)(x+2)(ax+b)}$	B1		Recognising $(x-1), (x+2)$ as factors PI
	$ax^3 = 3x^3 \quad -2b = 2$	B1	3	a
	$a = 3 \quad b = -1$	B1		b
				Or By division M1 attempt started M1 complete division A1 Correct answers
(b)	Use $\frac{1}{3}$	B1		
	$3\left(\frac{1}{3}\right)^3 + 2\left(\frac{1}{3}\right)^2 - 7 \times \frac{1}{3} + d = 2$	M1		Remainder Th ^M with $\pm \frac{1}{3} \pm 3$
	$d = 4$	A1F	3	Ft on $-\frac{1}{3}$ (answer $-\frac{4}{9}$)
				Or by division M1 M1 A1 as above
	Total		8	

5(c)	$2x^2 - 3 =$			
	$A(1-x)^2 + B(3-2x)(1-x) + C(3-2x)$	M1		Or by equating coefficients
	$x=1 \quad -1 = C \times 1 \quad x = \frac{3}{2} \quad \frac{3}{2} = A \times \frac{1}{4}$	M1		M1 same A1 collect terms M1 equate coefficients A1 2 correct A1 3 correct
	$C = -1 \quad A = 6$	A1		Follow on A and C
	$x=0 \quad (-3 = 6 + 3B - 3)$			
	or other value \Rightarrow equation in A, B, C	m1		
	$B = -2$	A1	5	

1 (a)(i)	$p(2) = 0$	B1	1	
(ii)	See $-\frac{1}{2}$	B1		
	$p\left(-\frac{1}{2}\right) = 6 \times \left(-\frac{1}{8}\right) - 19 \times \frac{1}{4} + 9\left(-\frac{1}{2}\right) + 10$ $= 0$	M1 A1	3	Use $\pm \frac{1}{2}$ Arithmetic to show $= 0$ and conclusion. Long division : $0/3$
(iii)	$p(x) = (2x+1)(x-2)(3x-5)$	B1 B1	2	$x-2$ Complete expression
(b)	$\frac{3x(x-2)}{(2x+1)(x-2)(3x-5)}$ $= \frac{3x}{(2x+1)(3x-5)}$	M1 A1	2	For $\frac{3x(x-2)}{\text{their (a)(iii)}}$ Or $\frac{3x}{6x^2 - 7x - 5}$ No ISW on A1
Total			8	

3(a)	$9x^2 - 6x + 5$ $= 3(3x-1)(x-1) + A(x-1) + B(3x-1)$	B1		Or $3 + \frac{6x+2}{(3x-1)(x-1)}$
	$x=1 \quad x=\frac{1}{3}$	M1		Substitute $x=1$ or $x=\frac{1}{3}$
	$B=4 \quad A=-6$	A1A1	4	Or equivalent method (equating coefficients, simultaneous equations)
(b)	$\int = \int 3 - \frac{6}{3x-1} + \frac{4}{x-1} dx$	M1		Attempt to use partial fractions
	$= 3x \dots$	B1		
	$- 2 \ln(3x-1) + 4 \ln(x-1) (+c)$	M1		$p \ln(3x-1) + q \ln(x-1)$ Condone missing brackets
		A1F	4	Follow through on A and B ; brackets needed.
Total			8	

2(a)	$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 7\left(\frac{3}{2}\right)^2 + 13$ $= 4$	M1 A1	2	Substitute $\pm\frac{3}{2}$ in $f(x)$
(b)	$g\left(\frac{3}{2}\right) = 0 \Rightarrow d + 4 = 0 \Rightarrow d = -4$	M1A1	2	AG (convincingly obtained) SC Written explanation with $g\left(\frac{3}{2}\right) = 0$ not seen/clear E2,1,0
(c)	$a = -2, b = -3$	B1, B1	2	Inspection expected By division: M1 – complete method A1 CAO Multiply out and compare coefficients: M1 – evidence of use A1 – both a and b correct
Total			6	

4(a)(i)	$\frac{3x-5}{x-3} = 3 + \frac{4}{x-3}$	B1, B1	2	By division: B1 for 3, B1 for $\frac{4}{x-3}$ or $B = 4$ By partial fractions: M1 multiply by $x-3$ and using 2 values of x , A1 both correct
(ii)	$\int 3 + \frac{4}{x-3} dx = 3x + 4\ln(x-3) (+c)$ Alternative: By substitution $u = x-3$ $\int \frac{3x-5}{x-3} dx = \int \frac{3u+4}{u} du$ $= 3(x-3) + 4\ln(x-3)$	M1A1F (M1) (A1)	2	M1 $\int 3 + \frac{4}{x-3} dx$ and attempt at integrals ft on A and B ; condone omission of brackets around $x-3$ Integral in terms of u Correct, in x
(b)(i)	$6x-5 = P(2x-5) + Q(2x+5)$ $x = \frac{5}{2} \quad x = -\frac{5}{2}$ $10 = 10Q \quad -20 = -10P$ $Q = 1 \quad P = 2$	M1 m1 A1	3	Clear evidence of use of cover-up rule M2
(ii)	$\int \frac{2}{2x+5} + \frac{1}{2x-5} dx$	M1		
	$\ln(2x+5) + \frac{1}{2}\ln(2x-5) (+c)$	M1 A1F	3	ft on P and Q ; must have brackets
Total			10	

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<p>1(a) $2\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 3 = -3$</p> <p>Alt algebraic division:</p> $\begin{array}{r} x \\ 2x+1 \overline{) 2x^2+x-3} \\ \underline{2x^2+x} \\ -3 \end{array}$ <p>Alt $\frac{x(2x+1)-3}{2x+1}$</p>	<p>M1A1</p> <p>(M1)</p> <p>(A1)</p> <p>(M1)</p> <p>(A1)</p>	<p>2</p> <p>(2)</p> <p>(2)</p>	<p>use of $\pm\frac{1}{2}$ SC NMS -3 1/2 No ISW, so subsequent answer "3" AO</p> <p>complete division with integer remainder</p> <p>remainder = -3 stated, or -3 highlighted</p> <p>attempt to rearrange numerator with (2x+1) as a factor</p> <p>remainder = -3 stated, or -3 highlighted</p>
<p>(b) $\frac{(2x+3)(x-1)}{(x+1)(x-1)}$ $= \frac{2x+3}{x+1}$</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>3</p>	<p>numerator } not necessarily in fraction denominator }</p> <p>CAO in this form. Not $\frac{2x+3}{x+1} \cancel{x-1}$</p>
<p>(b) Alternative $\frac{2x^2-2+x-1}{x^2-1}$</p> <p>$= 2 + \frac{x-1}{x^2-1}$</p> <p>$= 2 + \frac{x-1}{(x-1)(x+1)}$</p> <p>$= 2 + \frac{1}{x+1}$</p>	<p>(M1)</p> <p>(B1)</p> <p>(A1)</p>	<p>(3)</p>	
Total		5	

<p>(b) $\frac{1+4x}{(1+x)(1+3x)} = \frac{A}{1+x} + \frac{B}{1+3x}$ $1+4x = A(1+3x) + B(1+x)$ $x = -1, x = -\frac{1}{3}$ $A = \frac{3}{2}, B = -\frac{1}{2}$</p> <p>Alt: $\frac{1+4x}{(1+x)(1+3x)} = \frac{A}{1+x} + \frac{B}{1+3x}$ $1+4x = A(1+3x) + B(1+x)$ $A+B=1, 3A+B=4$ $A = \frac{3}{2}, B = -\frac{1}{2}$</p>	<p>M1</p> <p>m1</p> <p>A1</p> <p>(M1)</p> <p>(m1)</p> <p>(A1)</p>	<p>3</p> <p>(3)</p>	<p>correct partial fractions form, and multiplication by denominator</p> <p>Use (any) two values of x to find A and B</p> <p>A and B both correct</p> <p>correct partial fractions form, and multiplication by denominator</p> <p>Set up and solve</p> <p>A and B both correct</p>
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