Volumes of Revolution

Questions

Q1.

Diagrams not drawn to scale

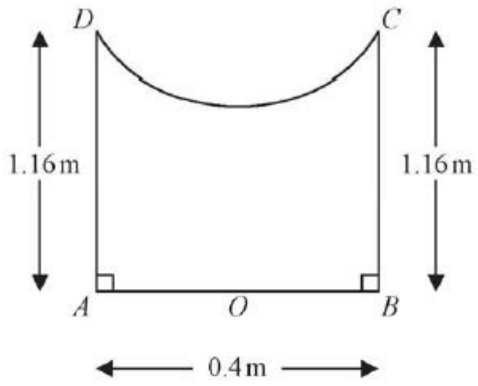

Figure 1

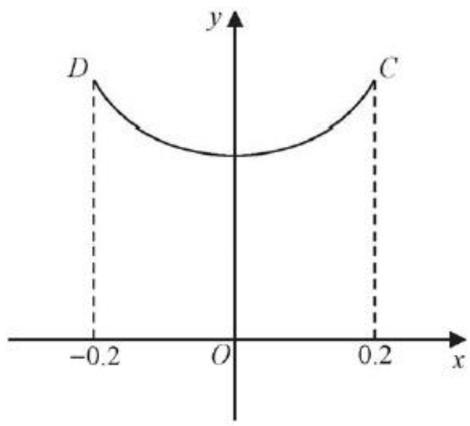

Figure 2

Figure 1 shows the central cross-section *AOBCD* of a circular bird bath, which is made of concrete. Measurements of the height and diameter of the bird bath, and the depth of the bowl of the bird bath have been taken in order to estimate the amount of concrete that was required to make this bird bath.

Using these measurements, the cross-sectional curve *CD*, shown in Figure 2, is modelled as a curve with equation

$$y = 1 + kx^2 \qquad -0.2 \leqslant x \leqslant 0.2$$

where *k* is a constant and where *O* is the fixed origin.

The height of the bird bath measured 1.16 m and the diameter, *AB*, of the base of the bird bath measured 0.40 m, as shown in Figure 1.

(a) Suggest the maximum depth of the bird bath.

(1)

(b) Find the value of *k*.

(2)

(c) Hence find the volume of concrete that was required to make the bird bath according to this model. Give your answer, in m^3, correct to 3 significant figures.

(7)

(d) State a limitation of the model.

(1)

It was later discovered that the volume of concrete used to make the bird bath was 0.127 m^3 correct to 3 significant figures.

(e) Using this information and the answer to part (c), evaluate the model, explaining your reasoning.

(1)

(Total for question = 12 marks)

Q2.

$$f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \qquad x > 0$$

The finite region bounded by the curve $y = f(x)$, the line $x = \frac{1}{8}$, the x-axis and the line $x = 8$ is rotated through θ radians about the x-axis to form a solid of revolution.

Given that the volume of the solid formed is $\frac{461}{2}$ units cubed, use algebraic integration to find the angle θ through which the region is rotated.

(Total for question = 8 marks)

Q3.

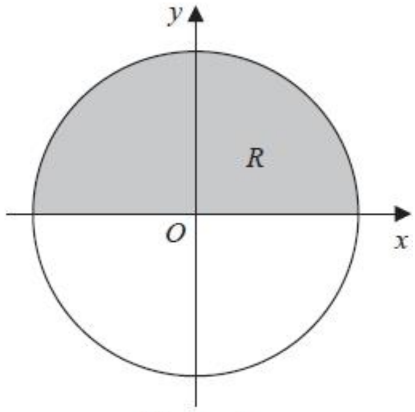

Figure 1

Figure 1 shows a circle with radius *r* and centre at the origin.
The region *R*, shown shaded in Figure 1, is bounded by the *x*-axis and the part of the circle for which $y > 0$
The region R is rotated through 360° about the *x*-axis to create a sphere with volume *V*

Use integration to show that $V = \frac{4}{3}\pi r^3$

(Total for question = 5 marks)

Q4.

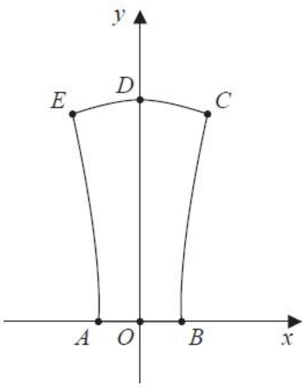

Figure 2

Figure 2 shows the vertical cross-section, *AOBCDE*, through the centre of a wax candle.

In a model, the candle is formed by rotating the region bounded by the *y*-axis, the line *OB*, the curve *BC*, and the curve *CD* through 360° about the *y*-axis.

The point *B* has coordinates (3, 0) and the point *C* has coordinates (5, 15).

The units are in centimetres.

The curve *BC* is represented by the equation

$$y = \frac{\sqrt{225x^2 - 2025}}{a} \qquad 3 \leqslant x < 5$$

where *a* is a constant.

(a) Determine the value of *a* according to this model.

(2)

The curve *CD* is represented by the equation

$$y = 16 - 0.04x^2 \qquad 0 \leqslant x < 5$$

(b) Using algebraic integration, determine, according to the model, the exact volume of wax that would be required to make the candle.

(9)

(c) State a limitation of the model.

(1)

When the candle was manufactured, 700 cm^3 of wax were required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

(Total for question = 13 marks)

Q5.

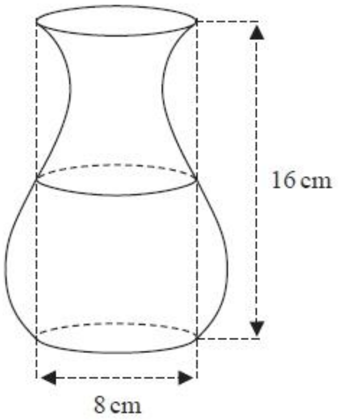

Figure 1

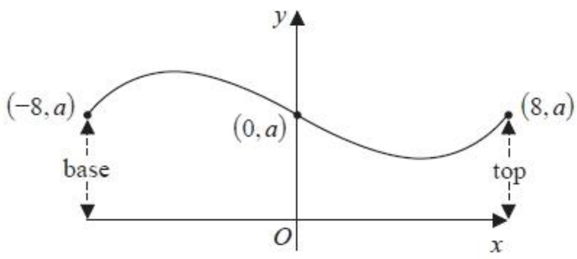

Figure 2

Figure 1 shows a sketch of a 16 cm tall vase which has a flat circular base with diameter 8 cm and a circular opening of diameter 8 cm at the top.

A student measures the circular cross-section halfway up the vase to be 8 cm in diameter.

The student models the shape of the vase by rotating a curve, shown in Figure 2, through 360° about the *x*-axis.

(a) State the value of *a* that should be used when setting up the model.

(1)

Two possible equations are suggested for the curve in the model.

Model A $\quad y = a - 2\sin\left(\frac{45}{2}x\right)^{\circ}$

Model B $\quad y = a + \frac{x(x-8)(x+8)}{100}$

For each model,

(b) (i) find the distance from the base at which the widest part of the vase occurs,

(ii) find the diameter of the vase at this widest point.

(7)

The widest part of the vase has diameter 12 cm and is just over 3 cm from the base.

(c) Using this information and making your reasoning clear, suggest which model is more appropriate.

(1)

(d) Using algebraic integration, find the volume for the vase predicted by Model B.
You must make your method clear.

(5)

The student pours water from a full one litre jug into the vase and finds that there is 100 ml left in the jug when the vase is full.

(e) Comment on the suitability of Model B in light of this information.

(1)

(Total for question = 15 marks)

Mark Scheme – Volumes of Revolution

Q1.

Question	Scheme		Marks	AOs
(a)	Depth = 0.16 (m)		B1	2.2b
			(1)	
(b)	$y=1+kx^2 \Rightarrow 1.16=1+k(0.2)^2 \Rightarrow k=\ldots$		M1	3.3
	$\Rightarrow k=4$ **cao** $\left\{\text{So } y=1+4x^2\right\}$		A1	1.1b
			(2)	
(c)	$\frac{\pi}{4}\int(y-1)\mathrm{d}y$	$\frac{\pi}{4}\int y\,\mathrm{d}y$	B1ft	1.1a
	$=\left\{\frac{\pi}{4}\right\}\int_1^{1.16}(y-1)\mathrm{d}y$	$=\left\{\frac{\pi}{4}\right\}\int_0^{0.16}y\mathrm{d}y$	M1	3.3
	$=\left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}-y\right]_1^{1.16}$	$=\left\{\frac{\pi}{4}\right\}\left[\frac{y^2}{2}\right]_0^{0.16}$	M1 A1	1.1b 1.1b
	$=\frac{\pi}{4}\left(\left(\frac{1.16^2}{2}-1.16\right)-\left(\frac{1}{2}-1\right)\right)\{=0.0032\pi\}$	$=\frac{\pi}{4}\left(\left(\frac{0.16^2}{2}\right)-(0)\right)\{=0.0032\pi\}$		
	$V_{\text{cylinder}}=\pi(0.2)^2(1.16)\ \{=0.0464\pi\}$		B1	1.1b
	Volume $=0.0464\pi-0.0032\pi\ \{=0.0432\pi\}$		M1	3.4
	$=0.1357168026\ldots=0.136(\mathrm{m}^3)$ (3sf)		A1	1.1b
			(7)	
(d)	Any one of e.g. The measurements may not be accurate. The inside surface of the bowl may not be smooth. There may be wastage of concrete when making the bird bath.		B1	3.5b
			(1)	
(e)	Some comment consistent with their values. We do need a reason. e.g. $\left[\left(\frac{0.136-0.127}{0.127}\right)\times 100=7.0866\ldots\right]$ so not a good estimate because the volume of concrete needed to make the bird bath is approximately 7% lower than that predicted by the model. **or** We might expect the actual amount of concrete to exceed that which the model predicts due to wastage, so the model does not look suitable since it predicts more concrete than was used.		B1ft	3.5a
			(1)	
			(12 marks)	

	Question	Notes
(a)	B1	Infers that the maximum depth of the bird bath could be 0.16 (m).
(b)	M1	Substitutes $y = 1.16$ and $x = 0.2$ or $x = -0.2$ into $y = 1 + kx^2$ and rearranges to give $k = ...$
	A1	$k = 4$ **cao**
(c)	B1ft	Uses the model to obtain either $\frac{\pi}{(\text{their } k)}\int(y-1)\,\mathrm{d}y$ or $\frac{\pi}{(\text{their } k)}\int y\,\mathrm{d}y$
	M1	Chooses limits that are appropriate to their model.
	M1	Integrates y (with respect to y) to give $\pm\lambda y^2$, where $\lambda \neq 0$ is a constant.
	A1	Uses their model correctly to give either $y - 1 \to \frac{y^2}{2} - y$ or $y \to \frac{y^2}{2}$
	B1	$V_{\text{cylinder}} = \pi(0.2)^2(1.16)$ or 0.0464π or $\frac{29}{625}\pi$, o.e.
	M1	Depends on **both** previous M marks. Uses the model to find $V_{\text{their cylinder}}$ − their integrated volume.
	A1	0.136 **cao**
(d)	B1	States an acceptable limitation of the model.
(e)	B1ft	Compares the actual volume with their answer to (c). Makes an assessment of the model. E.g. evaluates the percentage error and uses this to make a sensible comment about the model with a reason.

Q2.

Question	Scheme	Marks	AOs
	A correct overall strategy, an attempt at integrating y^2 with respect to x combine in some way with the volume of revolution formula (use of $\pi\int y^2\mathrm{d}x$ or $\alpha\int y^2\mathrm{d}x$ for any variable α is fine) followed by attempt to find an angle/form an equation in θ	M1	3.1a
	$y^2 = kx^{\frac{2}{3}} + \ldots + \dfrac{m}{x^{\frac{4}{3}}}$ or $y^2 = kx^{\frac{2}{3}} + \ldots + mx^{-\frac{4}{3}}$ where … is one or two more terms.	M1	1.1b
	$y^2 = 4x^{\frac{2}{3}} + 4x^{-\frac{1}{3}} + x^{-\frac{4}{3}}$ or $y^2 = 4x^{\frac{2}{3}} + 2x^{-\frac{1}{3}} + x^{-\frac{4}{3}} + 2x^{-\frac{1}{3}}$ (oe)	A1	1.1b
	$\int y^2\mathrm{d}x = \int 4x^{\frac{2}{3}} + \dfrac{4}{x^{\frac{1}{3}}} + \dfrac{1}{x^{\frac{4}{3}}}\mathrm{d}x = \alpha x^{\frac{5}{3}} + \beta x^{\frac{2}{3}} + \gamma x^{-\frac{1}{3}}$	M1	1.1b
	$= \dfrac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \dfrac{3}{x^{\frac{1}{3}}}$ (oe)	A1ft A1	1.1b 1.1b
	$\dfrac{\theta}{2}\left[\dfrac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \dfrac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \dfrac{461}{2}$ $\Rightarrow \dfrac{\theta}{2}\left[\left(\dfrac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \dfrac{3}{8^{\frac{1}{3}}}\right) - \left(\dfrac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\dfrac{1}{8}\right)^{\frac{2}{3}} - \dfrac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = \dfrac{461}{2} \Rightarrow \theta = \ldots$ OR $\pi\left[\dfrac{12x^{\frac{5}{3}}}{5} + 6x^{\frac{2}{3}} - \dfrac{3}{x^{\frac{1}{3}}}\right]_{\frac{1}{8}}^{8} = \pi\left[\left(\dfrac{12\times 8^{\frac{5}{3}}}{5} + 6\times 8^{\frac{2}{3}} - \dfrac{3}{8^{\frac{1}{3}}}\right) - \left(\dfrac{12\times\left(\frac{1}{8}\right)^{\frac{5}{3}}}{5} + 6\times\left(\dfrac{1}{8}\right)^{\frac{2}{3}} - \dfrac{3}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}\right)\right] = ..$ followed by $\dfrac{\theta}{2\pi}\times\ldots = \dfrac{461}{2} \Rightarrow \theta = \ldots$	M1	3.1a
	$\theta = \dfrac{40}{9}$ (radians)	A1	1.1b
		(8)	
			(8 marks)

Notes		
	M1	A correct overall strategy, either finding full volume rotated by 2π first, then performing some kind of scaling, or using $\alpha\int y^2\mathrm{d}x$ for a variable α (ideally $\frac{\theta}{2}$, but for the strategy accept with any variable multiple), to form an equation in just the angle.
	M1	Attempting to square y to a three or four term expression. Look for correct powers on first and last term with some term(s) in the middle.
	A1	Correct expansion in three or four terms – award when first seen.
	M1	Integrates y^2 w.r.t. x. Must have at least two terms in their y^2 with fractional indices. Power to be increased by 1 in at least two terms.
	A1ft	Two terms of integral correct. Follow through on their expansion. Need not be simplified.
	A1	Fully correct integral. Need not be simplified. May still be four terms
	M1	Either : Substitutes limits and subtracts correct way round (must be seen or implied by the answer), and equates to $\frac{461}{2}$ if using $\frac{1}{2}\theta\int y^2\mathrm{d}x$ and proceeds to find θ. Or : Substitutes limits and subtracts correct way round (seen or implied) and multiplies by π to get the full volume AND then multiplies the result by $\frac{\theta}{2\pi}$ before equating to $\frac{461}{2}$. **The method must be correct for this mark – so they must be using $\frac{\theta}{2}\int y^2\mathrm{d}x$ directly or $\pi\int y^2\mathrm{d}x$ and scale by $\frac{\theta}{2\pi}$ when setting equal to $\frac{461}{2}$**
	A1	Correct angle found. Accept $\frac{40}{9}$, awrt 4.44 or awrt $255°$ (as long as the degrees units are made clear – do not accept just 255) isw once a correct value of θ is found.

<u>Special case</u> The question specified that algebraic integration must be used, so use of a calculator to find the integral cannot score the marks for integration but may be allowed the strategy and answer marks. A maximum of M1M0A0M0A0A0M1A1 is available in such cases.
Expanding y^2 first but showing no integration can score the second M and first A (if earned) as well.

Note that $\int_{1/8}^{8}\left(2x^{\frac{1}{3}}+x^{-\frac{2}{3}}\right)^2\,\mathrm{d}x=\frac{4149}{40}=103.725$ but just this alone is worth **no marks**. There must be an attempt to incorporate this within a strategy to gain access to marks.

Q3.

Question	Scheme	Marks	AOs
	$x^2 + y^2 = r^2$	B1	1.2
	$\{V\} = \pi \int_{-r}^{r} r^2 - x^2 \, dx$ or $\{V\} = 2\pi \int_{0}^{r} r^2 - x^2 \, dx$	B1	2.1
	Integrates to the form $\alpha x \pm \beta x^3$ $\left[\text{note: the correct integration gives } r^2 x - \frac{1}{3}x^3\right]$	M1	1.1b
	Substitutes limits of $-r$ and r and subtracts the correct way round $\left(r^2(r) - \frac{1}{3}(r)^3\right) - \left(r^2(-r) - \frac{1}{3}(-r)^3\right)$ or Substitutes limits of 0 and r and subtracts the correct way round with twice the volume. Note the limit of 0 can be implied if gives and answer of 0 $\left(r^2(r) - \frac{1}{3}(r)^3\right) - (0)$	dM1	1.1b
	$V = \frac{4}{3}\pi r^3$ * cso	A1*	1.1b
		(5)	
		(5 marks)	

Notes:

B1: Correct equation of the circle, may be implied by correct integral

B1: Correct expression for the volume, including limits, dx may be implied and if using limits r and 0 the 2 could appear later with reasoning

M1: Integrates to the form $\alpha x \pm \beta x^3$. Do not award if $r^2 \to \lambda r^3$

dM1: Dependent on previous method mark. Correct use of limits r and r or limits of 0 and r with twice the volume.

A1*: $V = \frac{4}{3}\pi r^3$ * cso

Note: rotation about the y-axis all marks are available, however for the final accuracy mark must refer to symmetry

Q4.

Question	Scheme	Marks	AOs
(a)	$(5, 15) \Rightarrow 15 = \dfrac{\sqrt{225\times 5^2 - 2025}}{a} \Rightarrow a = ...$	M1	3.3
	$a = 4$	A1	1.1b
		(2)	
(b)	Evidence of the use of $\pi\int x^2\,\mathrm{d}y$ for the curve BC or the curve CD	M1	3.1b
	For BC $V_1 = \dfrac{\pi}{225}\int(16y^2+2025)\,\mathrm{d}y$ or $\pi\int\left(\dfrac{16}{225}y^2+9\right)\mathrm{d}y$	A1ft	1.1b
	For CD $V_2 = 25\pi\int(16-y)\,\mathrm{d}y$ or $\pi\int(400-25y)\,\mathrm{d}y$	A1	1.1b
	$V_1 = \dfrac{\pi}{225}\int_0^{15}(16y^2+2025)\,\mathrm{d}y$ or $\pi\int_0^{15}\left(\dfrac{16}{225}y^2+9\right)\mathrm{d}y$	M1	3.3
	$V_2 = 25\pi\int_{15}^{16}(16-y)\,\mathrm{d}y$ or $\pi\int_{15}^{16}(400-25y)\,\mathrm{d}y$	M1	3.3
	$V_1 = \dfrac{\{\pi\}}{225}\left[\dfrac{16y^3}{3}+2025y\right]_0^{15}$ or $\{\pi\}\left[\dfrac{16y^3}{675}+9y\right]_0^{15}$	A1ft	1.1b
	$V_2 = 25\{\pi\}\left[16y-\dfrac{y^2}{2}\right]_{15}^{16}$ or $\{\pi\}\left[400y-\dfrac{25y^2}{2}\right]_{15}^{16}$	A1ft	1.1b
	$V = V_1+V_2 = \dfrac{\pi}{225}(18000+30375)+25\pi\left(128-\dfrac{255}{2}\right)$ $V = V_1+V_2 = 215\pi+12.5\pi$	M1	3.4
	$V = \dfrac{455\pi}{2}\,\mathrm{cm}^3$ or $227.5\pi\,\mathrm{cm}^3$	A1	2.2b
		(9)	

(c)	E.g. • The equation of the curve may not be a suitable model • The sides of the candle will not be perfectly curved/smooth • There will be a whole in the middle for the wick	B1	3.5b
		(1)	
(d)	Makes an appropriate comment that is consistent with their value for the volume and 700 cm^3. E.g. a good estimate as 700 cm^3 is only 15 cm^3 less than 715 cm^3	B1ft	3.5a
		(1)	
			(13 marks)

Notes

(a)
M1: Substitutes (5, 15) into the equation modelling the curve in an attempt to find the value of a
A1: Infers from the data in the model, the value of a
(b)
M1: Uses either model to obtain x^2 in terms of y and applies $\pi\int x^2\,dy$
A1ft: Correct expression for the volume generated by the curve BC (follow through their a value)
A1: Correct expression for the volume generated by the curve CD
M1: Chooses limits appropriate to their model for the curve BC
M1: Chooses limits appropriate to their model for the curve CD
A1ft: Correct integration (follow through their a value)
A1ft: Correct integration follow through on their volume as long it is of the form $Ay - By^2$
M1: Uses the model to find the sum of volumes
A1: $\frac{455\pi}{2}$
Note: Use of calculator for integration maximum score M1 A1ft A1 M1 M1 A0ft A0ft M1 A1
(c)
B1: States an acceptable limitation of the model
(d)
B1ft: Compares the actual volume to their answer to part (b) and makes an assessment of the model with a reason.

Q5.

Question	Scheme	Marks	AOs
(a)	$a = 4$	B1	3.3
		(1)	
(b)	Model A: (i) Widest point will be 4 (cm) from the base	B1	3.4
	(ii) Width at widest point is 12 (cm) $\quad (2\times('a'+2)\text{ ft})$	B1ft	3.4
	Model B: (i) $y = 4 + \frac{x^3 - 64x}{100} \Rightarrow \frac{dy}{dx} = \frac{3x^2 - 64}{100}$	M1	3.1b
	$\frac{dy}{dx} = 0 \Rightarrow x = \pm\sqrt{\frac{64}{3}} = \pm\frac{8\sqrt{3}}{3} = \pm\text{awrt}4.62$	A1	1.1b
	So max width is a distance $8 - \frac{8}{\sqrt{3}} = 8 - \frac{8\sqrt{3}}{3} \approx 3.38$ (cm) from base.	A1	3.4
	(ii) $y\|_{-4.61..} = 4 + \frac{(-4.62...)^3 - 64(-4.62...)}{100} = ...$	dM1	3.4
	$= 5.97...$ so diameter is approximately 11.9 (cm) $\quad [2a + 3.94...\text{ ft}]$	A1ft	3.2a
		(7)	
(c)	Model A and model B both have diameters closed to 12 Model B distance from base is closer to 3 than Model A so is more appropriate.	B1ft	3.5b
		(1)	

(d)	$V_B = \pi\int_{-8}^{8} y^2\,dx = \pi\int_{-8}^{8}\left(4 + \frac{x^3 - 64x}{100}\right)^2 dx = ...$	B1	1.1b
	$= \frac{\{\pi\}}{10000}\int_{(-8)}^{(8)} 400^2 + x^6 + 64^2x^2 + 2\left(400x^3 - 400\times 64x - 64x^4\right)dx$ $= \frac{\{\pi\}}{10000}\int_{(-8)}^{(8)} 160000 + x^6 + 4096x^2 + 800x^3 - 51200x - 128x^4\,dx$ $= \{\pi\}\int_{(-8)}^{(8)} 16 + \frac{x^6}{10000} + \frac{4096}{10000}x^2 + \frac{8}{100}x^3 - \frac{512}{100}x - \frac{128}{10000}x^4\,dx$ $= \{\pi\}\int_{(-8)}^{(8)} 16 + \frac{x^6}{1000} + \frac{256}{625}x^2 + \frac{2}{25}x^3 - \frac{128}{25}x - \frac{8}{625}x^4\,dx$ $= \{\pi\}\int_{(-8)}^{(8)} 16 + \frac{8x(x-8)(x+8)}{100} + \left(\frac{x(x-8)(x+8)}{100}\right)^2 dx$	M1	1.1b
	$= \frac{\{\pi\}}{10000}\left[160000x + \frac{x^7}{7} + 4096\frac{x^3}{3} + 800\frac{x^4}{4} - 51200\frac{x^2}{2} - 128\frac{x^5}{5}\right]_{(-8)}^{(8)}$	dM1	1.1b

	$= \{\pi\}\left[16x + \frac{x^7}{70000} + \frac{256}{1875}x^3 + \frac{1}{50}x^4 - \frac{64}{25}x^2 - \frac{8}{3125}x^5\right]_{(-8)}^{(8)}$		
	$= \frac{\{\pi\}}{10000}(620583.00.... - -2258983.01...) \approx \frac{2879566\pi}{10000}$	M1	3.4
	$=$ awrt $905\left(\text{cm}^3\right)$ cso	A1	1.1b
		(5)	
(e)	Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately.	B1ft	3.5a
		(1)	
			(15 marks)

Notes:

Units not required in this question

(a)

B1: For $a = 4$, ignore any reference to units.

(b)

B1: Correct distance from base for Model A is 4

B1ft: Correct width at widest point. Follow through their 'a', so $2\times('a'+2)$.

M1: Attempts the derivative for Model B's equation, reduce any power by 1

A1: Sets $\frac{dy}{dx} = 0$ and finds correct x coordinate of the stationary point (accept $\pm$)

A1: For $8 - \frac{8}{\sqrt{3}}$ or awrt 3.38 cso

dM1: Dependent on previous M mark. Uses their value of x to find the value of y. If no working shown the value of y must come from their x value.

Note using $x = 4.62$ give $y = 2.029...$

A1: Correct diameter, awrt 11.9 follow through their 'a', so $[2a + 3.94... \text{ft}]$

Note: Correct answers with no working send to review

Trial and error approach

Candidates could score B1 B1 for model A however if working in integers it is unlikely that they will find the correct value for x (they are using $x = -5$) not a valid method M0A0A0dM0A0

(c)

B1ft: They must have answers for all parts in (b). Accept any well-reasoned comment that follows their answers to (b) If the answers are correct, they must conclude that model B is more appropriate.

- If answers for one model are correct ish but other incorrect, or one value is clearly closer For example

	Distance (3)	Diameter (12)	Distance (3)	Diameter (12)
A	9.4	9.05	4	6
B	3.38	12.06	4.62	4.06
Conclusion	Selects B as distance/diameter closet		Select A as diameter closest	

- If distances and diameters are similar selects the model which has the most appropriate value for distance or diameter
  For example

	Distance (3)	Diameter (12)	Distance (3)	Diameter (12)

A	0.76	6.8	4	20
B	1.28	10.5	3.38	19.94
Conclusion	selects B as the diameter is closet		Selects B as distance is closet	

- If all values of the distances and diameters are varied any sensible reason stated for selecting a model.

(d)

B1: Applies $\pi\int_{-8}^{8} y^2\,dx$ to the model. Must have π and correct limits, with y substituted in.

Alternatively attempts to square y first and then substitute in.

M1: Attempts to expand y^2 this can be a poor attempt but must include at least a constant and x^6 terms as long a clear attempt at y^2 (Limits not required for this mark.)

dM1: Attempts the integration, must first be rearranged to an integrable form then look for power increasing by at least 1 in at least two terms. (Limits not required for this mark.)

M1: Applies correct limits to their integral following an attempt at y^2 with at least a constant and x^6 terms.

If there is no working shown, allow this method mark if the correct answer appears from a calculator as it implies correct limits have been applied the correct way round. (So M0dM0M1 is possible.)

A1: awrt 905 cso note it must come from a fully correct solution

Note: For answers that appear from calculator B1M0dM0M1A0 is possible, the question specifies algebraic integration to be used so the integration needs to be seen to score the other marks.

(e)

B1ft: Compares their volume to 900 or compares their volume + 100 to 1 litre or 1000 and comments appropriately. Correct answer in (d) needs to conclude that it is suitable.