Polar Coordinates

Questions

Q1.

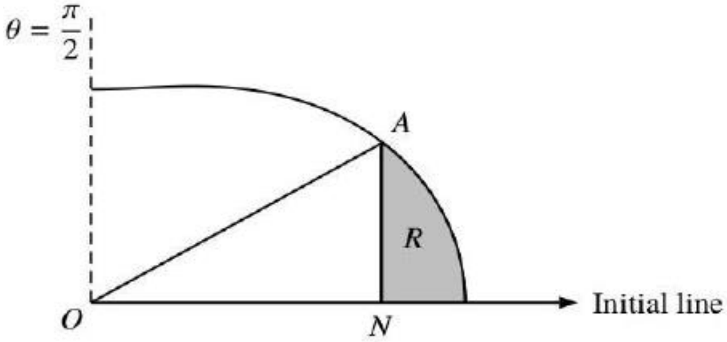


Figure 1

The curve *C* shown in Figure 1 has polar equation

$$r = 4 + \cos 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point *A* on *C*, the value of *r* is $\frac{9}{2}$

The point *N* lies on the initial line and *AN* is perpendicular to the initial line.

The finite region *R*, shown shaded in Figure 1, is bounded by the curve *C*, the initial line and the line *AN*.

Find the exact area of the shaded region *R*, giving your answer in the form $p\pi + q\sqrt{3}$ where *p* and *q* are rational numbers to be found.

(9)

(Total for question = 9 marks)

Q2.

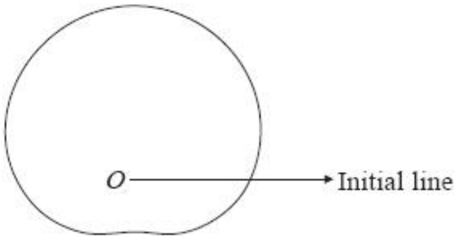


Figure 1

Figure 1 shows a sketch of a curve with polar equation

$$r = 6 + a\sin\theta$$

where $0 < a < 6$ and $0 \leq \theta < 2\pi$

The area enclosed by the curve is $\frac{97\pi}{2}$

Find the value of the constant a.

(8)

(Total for question = 8 marks)

Q3.

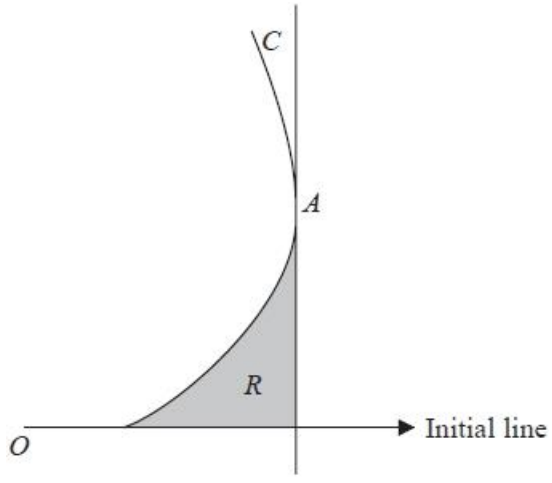


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$r = 1 + \tan\theta \qquad\qquad 0 \leq \theta < \frac{\pi}{3}$$

Figure 1 also shows the tangent to C at the point A.
This tangent is perpendicular to the initial line.

(a) Use differentiation to prove that the polar coordinates of A are $\left(2, \frac{\pi}{4}\right)$

(4)

The finite region R, shown shaded in Figure 1, is bounded by C, the tangent at A and the initial line.

(b) Use calculus to show that the exact area of R is $\frac{1}{2}(1 - \ln 2)$

(6)

(Total for question = 10 marks)

Q4.

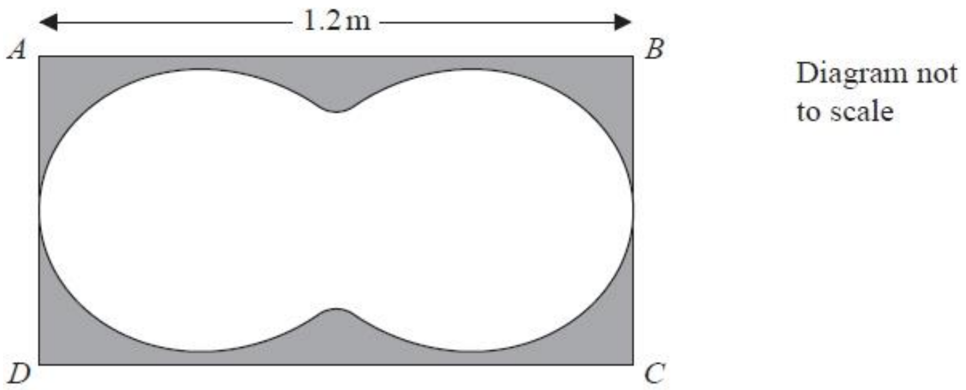


Figure 1

Figure 1 shows the design for a table top in the shape of a rectangle *ABCD*. The length of the table, *AB*, is 1.2 m. The area inside the closed curve is made of glass and the surrounding area, shown shaded in Figure 1, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0.4 + a\cos 2\theta \qquad 0 \leqslant \theta < 2\pi$$

where *a* is a constant.

(a) Show that $a = 0.2$

(2)

Hence, given that *AD* = 60 cm,

(b) find the area of the wooden part of the table top, giving your answer in m^2 to 3 significant figures.

(8)

(Total for question = 10 marks)

Q5.

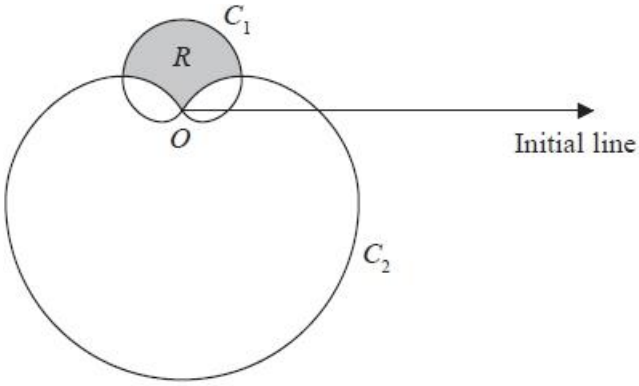


Figure 1

Figure 1 shows a sketch of two curves C_1 and C_2 with polar equations

$$C_1: r = (1 + \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

$$C_2: r = 3(1 - \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

The region R lies inside C_1 and outside C_2 and is shown shaded in Figure 1.

Show that the area of R is

$$p\sqrt{3} - q\pi$$

where p and q are integers to be determined.

(Total for question = 9 marks)

Q6.

(a) (i) Show on an Argand diagram the locus of points given by the values of z satisfying

$$|z - 4 - 3\mathrm{i}| = 5$$

Taking the initial line as the positive real axis with the pole at the origin and given that $\theta \in [\alpha, \alpha + \pi]$,
where $\alpha = -\arctan\left(\frac{4}{3}\right)$,

(ii) show that this locus of points can be represented by the polar curve with equation

$$r = 8\cos\theta + 6\sin\theta$$

(6)

The set of points A is defined by

$$A = \left\{z : 0 \leq \arg z \leq \frac{\pi}{3}\right\} \cap \left\{z : |z - 4 - 3\mathrm{i}| \leq 5\right\}$$

(b) (i) Show, by shading on your Argand diagram, the set of points A.

(ii) Find the **exact** area of the region defined by A, giving your answer in simplest form.

(7)

(Total for question = 13 marks)

Mark Scheme – Polar Coordinates

Q1.

Question	Scheme	Marks	AOs
	$4+\cos 2\theta=\frac{9}{2}\Rightarrow\theta=\ldots$	M1	3.1a
	$\theta=\frac{\pi}{6}$	A1	1.1b
	$\frac{1}{2}\int(4+\cos 2\theta)^2\mathrm{d}\theta=\frac{1}{2}\int(16+8\cos 2\theta+\cos^2 2\theta)\mathrm{d}\theta$	M1	3.1a
	$\cos^2 2\theta=\frac{1}{2}+\frac{1}{2}\cos 4\theta\Rightarrow A=\frac{1}{2}\int\left(16+8\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right)\mathrm{d}\theta$	M1	3.1a
	$=\frac{1}{2}\left[16\theta+4\sin 2\theta+\frac{\sin 4\theta}{8}+\frac{\theta}{2}\right]$	A1	1.1b
	Using limits 0 and their $\frac{\pi}{6}$: $\frac{1}{2}\left[\frac{33\pi}{12}+2\sqrt{3}+\frac{\sqrt{3}}{16}-(0)\right]$	M1	1.1b
	Area of triangle $=\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{81}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}$	M1	3.1a
	Area of $R=\frac{33\pi}{24}+\frac{33\sqrt{3}}{32}-\frac{81\sqrt{3}}{32}$	M1	1.1b
	$=\frac{11}{8}\pi-\frac{3\sqrt{3}}{2}\left(p=\frac{11}{8},\ q=-\frac{3}{2}\right)$	A1	1.1b
			(9 marks)

Notes:

M1: Realises the angle for A is required and attempts to find it.
A1: Correct angle
M1: Uses a correct area formula and squares r to achieve a 3TQ integrand in $\cos 2\theta$
M1: Use of the correct double angle identity on the integrand to achieve a suitable form for integration
A1: Correct integration
M1: Correct use of limits
M1: Identifies the need to subtract the area of a triangle and so finds the area of the triangle
M1: Complete method for the area of R
A1: Correct final answer

Q2.

Question Number	Scheme	Notes	Marks
	$r = 6 + a\sin\theta$		
	$A = \frac{1}{2}\int (6 + a\sin\theta)^2 \, d\theta$	Use of $\frac{1}{2}\int r^2 (d\theta)$ Limits not needed. Can be gained if $\frac{1}{2}$ appears later	B1
	$(6 + a\sin\theta)^2 = 36 + 12a\sin\theta + a^2\sin^2\theta$		
	$(6 + a\sin\theta)^2 = 36 + 12a\sin\theta + a^2\left(\frac{1-\cos 2\theta}{2}\right)$	M1: Squares ($36 + k\sin^2\theta$, where $k = a^2$ or a as min) and attempts to change : $\sin^2\theta$ to an expression in $\cos 2\theta$ A1: Correct expression	M1A1
	$\left(\frac{1}{2}\right)\left[36\theta - 12a\cos\theta + \frac{a^2}{2}\theta - \frac{a^2}{4}\sin 2\theta\right]$	dM1: Attempt to integrate $\cos 2\theta \to \pm\frac{1}{2}\sin 2\theta$ Limits not needed A1: Correct integration limits not needed	dM1A1
	$= 36\pi + \frac{\pi a^2}{2}$	Correct area obtained from correct integration and correct limits. No need to simplify but trig functions must be evaluated.	A1
	$36\pi + \frac{\pi a^2}{2} = \frac{97\pi}{2} \Rightarrow a = ...$	Set their area $= \frac{97\pi}{2}$ and attempt to solve for a (depends on both M marks above) If $\frac{1}{2}$ omitted from the initial formula and area set $= 97\pi$, give the B1 by implication as well as this mark.	ddM1
	$a = 5$	cao and cso $a = \pm 5$ or $a = -5$ scores A0	A1cso
			Total 8
	Alternatives: Splitting the area and so using 2 integrals with different limits.		
	Marks the same as the main scheme.		
1	Limits 0 to π (area above initial line) and limits π to 2π (area below initial line) and add the two results.		
2	Limits 0 to $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ to 2π Twice the sum of the results needed.		

Q3.

Question	Scheme	Marks	AOs
(a)	$x = r\cos\theta = (1+\tan\theta)\cos\theta = \cos\theta + \sin\theta$ $= \cos\theta + \tan\theta\cos\theta$ $\frac{dx}{d\theta} = \alpha(1+\tan\theta)\sin\theta + \beta\sec^2\theta\cos\theta$ or $\frac{dx}{d\theta} = \alpha\sin\theta + \beta\cos\theta$ $\frac{dx}{d\theta} = \alpha\sin\theta + \beta\sec^2\theta\cos\theta + \delta\tan\theta\sin\theta$	M1	3.1a
	$\frac{dx}{d\theta} = -(1+\tan\theta)\sin\theta + \sec^2\theta\cos\theta$ or $\frac{dx}{d\theta} = -\sin\theta + \cos\theta$ $\frac{dx}{d\theta} = -\sin\theta + \sec^2\theta\cos\theta - \tan\theta\sin\theta$ or $\frac{dx}{d\theta} = -\sin\theta + \sec\theta - \tan\theta\sin\theta$	A1	1.1b

	For example $\left\{\frac{dx}{d\theta}=\right\} -\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \ldots$ $\left\{\frac{dx}{d\theta}=\right\} -\sin\theta + \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = \ldots$ $\left\{\frac{dx}{d\theta}=\right\} -\sin\theta + \cos\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right) = \theta = \ldots$ or $\left\{\frac{dx}{d\theta}=\right\} -(1+\tan\theta)\sin\theta + \sec^2\theta\cos\theta = 0$ $\Rightarrow -\sin\theta - \frac{\sin^2\theta}{\cos\theta} + \frac{1}{\cos\theta} = 0 \Rightarrow -\sin\theta + \frac{1-\sin^2\theta}{\cos\theta} = 0$ $\Rightarrow -\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \ldots$ or $\left\{\frac{dx}{d\theta}=\right\} -\sin\theta - \tan\theta\sin\theta + \sec\theta = 0$ $\Rightarrow -\frac{1}{2}\sin 2\theta - \sin^2\theta + 1 = 0 \Rightarrow \sin 2\theta + 2\sin^2\theta - 1 = 1$ $\Rightarrow \sin 2\theta - \cos 2\theta = 1 \Rightarrow \sqrt{2}\sin\left(2\theta - \frac{\pi}{4}\right) = 1 \Rightarrow \theta = \ldots$ or $\left\{\frac{dx}{d\theta}=\right\} -\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = 0$ $\left\{\frac{dx}{d\theta}=\right\} -\left(1+\tan\left(\frac{\pi}{4}\right)\right)\sin\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = 0$ $\left\{\frac{dx}{d\theta}=\right\} -\sin\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) = 0$	dM1	3.1a
	$r = 1 + \tan\left(\frac{\pi}{4}\right) = 2$ therefore $A\left(2, \frac{\pi}{4}\right)$*	A1*	2.1
		(4)	
	Area bounded by the curve $= \frac{1}{2}\int(1+\tan\theta)^2\ \{d\theta\}$	M1	3.1a

(b)	$=\frac{1}{2}\int(1+2\tan\theta+\tan^2\theta)\ \{d\theta\}$ $=\frac{1}{2}\int(1+2\tan\theta+[\sec^2\theta-1])\ \{d\theta\}=\ldots$		
	$=\frac{1}{2}[2\ln\lvert\sec\theta\rvert+\tan\theta]$ or $\ln\lvert\sec\theta\rvert+\frac{1}{2}\tan\theta$ or $-\ln\cos\theta+\frac{1}{2}\tan\theta$ or $=\frac{1}{2}[-2\ln\lvert\cos\theta\rvert+\tan\theta]$	A1	1.1b
	$=\frac{1}{2}\left[2\ln\left\lvert\sec\left(\frac{\pi}{4}\right)\right\rvert+\tan\left(\frac{\pi}{4}\right)\right]-\frac{1}{2}[2\ln\lvert\sec(0)\rvert+\tan(0)]$ $=\left(\ln\left\lvert\sec\left(\frac{\pi}{4}\right)\right\rvert+\frac{1}{2}\tan\left(\frac{\pi}{4}\right)\right)-\left(\ln\lvert\sec 0\rvert+\frac{1}{2}\tan 0\right)$ $\left\{=\ln\sqrt{2}+\frac{1}{2}\right\}$	dM1	1.1b
	Area of triangle $=\frac{1}{2}xy=\frac{1}{2}\left(2\cos\frac{\pi}{4}\right)\left(2\sin\frac{\pi}{4}\right)=\ldots\left\{\frac{1}{2}\times\sqrt{2}\times\sqrt{2}=1\right\}$ The equation of the tangent is $r=\sqrt{2}\sec\theta$ then applies Area bounded of triangle $=\frac{1}{2}\int_0^{\frac{\pi}{4}}(\sqrt{2}\sec\theta)^2\ \{d\theta\}$	M1	1.1b
	Finds the required area = area of triangle – area bounded by the curve $=1-\left[\ln\sqrt{2}+\frac{1}{2}\right]$ May be seen within an integral $=\frac{1}{2}\int(\sqrt{2}\sec\theta)^2\ \{d\theta\}-\frac{1}{2}\int(1+\tan\theta)^2\ \{d\theta\}$	M1	3.1a
	$=\frac{1}{2}(1-\ln 2)$ * cso	A1*	2.1
		(6)	

	Alternative Area bounded by the curve $=\frac{1}{2}\int(1+\tan\theta)^2\ \{d\theta\}$ $=\frac{1}{2}\int(1+2\tan\theta+\tan^2\theta)\ \{d\theta\}$ let $u=\tan\theta\Rightarrow\frac{du}{d\theta}=\sec^2\theta$ Leading to $=\frac{1}{2}\int\frac{(1+2u+u^2)}{1+u^2}\{du\}=\frac{1}{2}\int 1+\frac{2u}{1+u^2}\{du\}=\ldots$	M1	3.1a
	$\frac{1}{2}[u+\ln(1+u^2)]$	A1	1.1b
	$\frac{1}{2}[(1+\ln(1+(1)^2))-(0+\ln 1)]$ or $\frac{1}{2}\left[\left(\tan\left(\frac{\pi}{4}\right)+\ln\left(1+\tan^2\left(\frac{\pi}{4}\right)\right)\right)-(\tan(0)+\ln(1+\tan^2(0)))\right]$ $\left\{=\frac{1}{2}\ln 2+\frac{1}{2}\right\}$	dM1	1.1b
	Area of triangle $=\frac{1}{2}xy=\frac{1}{2}\left(2\cos\frac{\pi}{4}\right)\left(2\sin\frac{\pi}{4}\right)=\ldots\left\{\frac{1}{2}\times\sqrt{2}\times\sqrt{2}=1\right\}$	M1	1.1b

	Finds the required area = area of triangle – area bounded by the curve $= 1 - \left[\ln\sqrt{2} + \frac{1}{2}\right]$	M1	3.1a
	$= \frac{1}{2}(1 - \ln 2)$ *	A1*	2.1
		(6)	
			(10 marks)
Notes:			

Notes:

(a)

M1: Substitutes the equation of C into $x = r\cos\theta$ and differentiates to the required form

A1: Fully correct differentiation

dM1: Dependent on previous method mark. Sets their $\frac{dx}{d\theta} = 0$ and uses correct trig identities to find a value for θ. Alternatively substitutes $\theta = \frac{\pi}{4}$ into their $\frac{dx}{d\theta}$ and shows equals 0.

A1*: Shows that $r = 2$ and hence the polar coordinates $\left(2, \frac{\pi}{4}\right)$ from correct working

(b)

M1: Applies area $= \frac{1}{2}\int r^2\theta\ d\theta$, multiplies out, uses the identity $\pm 1 \pm \tan^2\theta = \sec^2\theta$ to get into an integrable form **and** integrates. Condone missing $d\theta$, limits are not required for this mark

A1: Correct integration. Note may include $\theta - \theta$ if the one's were not cancelled earlier.

dM1: Dependent on the first method mark. Applies the limits of $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied

M1: Correct method to find the area of triangle seen. This may be minimal but area = 1 only is M0, they need to show some method.

M1: Finds the required area = area of triangle – area bounded by the curve

A1*: Correct answer, with no errors or omissions. cso

Alternative

M1: Applies area $= \frac{1}{2}\int r^2\theta\ d\theta$, multiplies out, uses the substitution $u = \tan\theta$ to get into an integrable form **and** integrates. Limits are not required for this mark

A1: Correct integration

dM1: Dependent on the first method mark. Applies the limits of $u = 0$ and $u = 1$ or substitutes back using $u = \tan\theta$ and uses the limits $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied

M1: Correct method to find the area of triangle

M1: Finds the required area = area of triangle – area bounded by the curve

A1*: Correct answer, with no errors or omissions. cso

Q4.

Question	Scheme	Marks	AOs
(a)(i)	$2(0.4+a)=1.2$ or $0.4+a=0.6$ or $0.4+a\cos 0=0.6$ $\Rightarrow a=\ldots$	M1	3.4
	$a=0.2$ * cso	A1*	1.1b
		(2)	
(b)	Area of rectangle is $1.2\times 0.6(=0.72)$	B1	1.1b
	Area enclosed by curve $=\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2 (\mathrm{d}\theta)$	M1	3.1a
	$(0.4+0.2\cos 2\theta)^2=0.16+0.16\cos 2\theta+0.04\cos^2 2\theta$ $=0.16+0.16\cos 2\theta+0.04\left(\frac{\cos 4\theta+1}{2}\right)$	M1	2.1
	$\frac{1}{2}\int (0.4+0.2\cos 2\theta)^2\,\mathrm{d}\theta=\frac{1}{2}\left[0.18\theta+0.08\sin 2\theta+0.005\sin 4\theta(+c)\right]$ $=0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta(+c)$ o.e.	A1ft	1.1b
	Area enclosed by curve $=\left[0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta\right]_0^{2\pi}$ or Area enclosed by curve $=2\left[0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta\right]_0^{\pi}$ or Area enclosed by curve $=4\left[0.09\theta+0.04\sin 2\theta+0.0025\sin 4\theta\right]_0^{\pi/2}$	dM1	3.1a
	$=\frac{9}{50}\pi$ or $0.18\pi\,(=0.5654\ldots)$	A1	1.1b
	Area of wood $=1.2\times 0.6-0.18\pi$	M1	1.1b
	$=$ awrt 0.155 (m^2)	A1	1.1b
		(8)	
			(10 marks)

Notes
(a) M1: Interprets the information from the model and realises that the maximum value of r gives half the length of the table top (or equivalent) and solves to find a value for a. Use $\theta = 0$ and $r = 0.6$ or $\theta = \pi$ and $r = -0.6$ to find a value for a. Using $\theta = 2\pi$ is M0 A1*: Correct value for a. **Alternative** M1: Uses $a = 0.2$ and $\theta = 0$ to find a value for r A1: Finds $r = 0.6$ and concludes that $a = 0.2$ (b) B1: 1.2×0.6 or 0.72 M1: A correct strategy identified for finding an area enclosed by the polar curve using a correct formula with r substituted. Attempt at area $= \frac{1}{2}\int (0.4 + 0.2\cos 2\theta)^2 \, d\theta = ...$ Look for $= \lambda \times \frac{1}{2}\int (0.4 + 0.2\cos 2\theta)^2 \, d\theta = ...$ If the $\frac{1}{2}$ is not explicitly seen then look at the limits and it must be either $= \int_0^{\pi} (0.4 + 0.2\cos 2\theta)^2 \, d\theta = ...$ or $= 2\int_0^{\frac{\pi}{2}} (0.4 + 0.2\cos 2\theta)^2 \, d\theta = ...$ Condone missing $d\theta$ M1: Squares to achieve three terms and uses $\cos^2 2\theta = \frac{\pm 1 \pm \cos 4\theta}{2}$ to obtain an expression in an integrable form. A1ft: Correct follow through integration as long as the previous two method marks have been awarded. dM1: Dependent of first method mark. Finds the required area enclosed by the curve using the correct limits. There are only three cases either $\frac{1}{2}\int_0^{2\pi} (0.4 + 0.2\cos 2\theta)^2 \, d\theta$ or $\int_0^{\pi} (0.4 + 0.2\cos 2\theta)^2 \, d\theta$ or $2\int_0^{\frac{\pi}{2}} (0.4 + 0.2\cos 2\theta)^2 \, d\theta$ The use of the limit 0 can be implied if it gives 0 but the use of 0 must been seen or implied if it does not result in 0 (just writing 0 is insufficient)

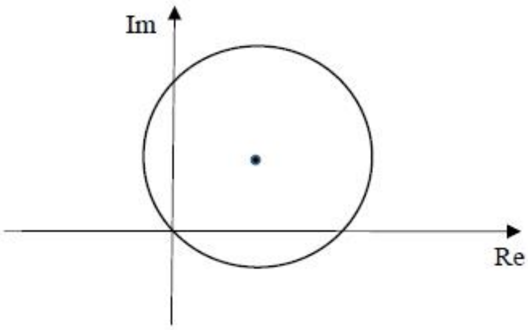
A1: Correct area of the glass following fully correct working. **Do not award for the correct answer following incorrect working.** M1: Subtracts their area of the glass from their area of the rectangle, as long as it does not give a negative area A1: awrt 0.155 or awrt 0.155m^2 (If the units are stated they must be correct) **Note:** Using a calculator to find the area scores a maximum of B1M0M0A0M0A0M1A1

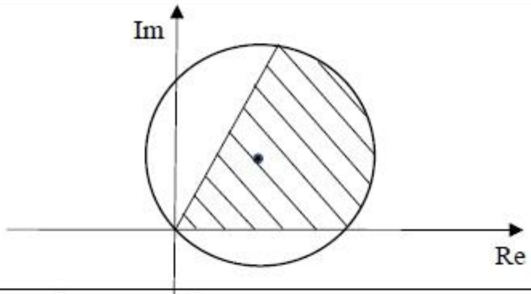
Q5.

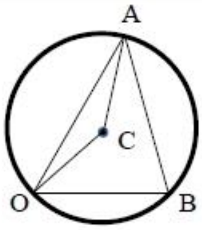
Question	Scheme	Marks	AOs
	$3(1-\sin\theta)=1+\sin\theta \Rightarrow \sin\theta=\frac{1}{2} \Rightarrow \theta=...$	M1	3.1a
	$\theta=\frac{\pi}{6}\left(\text{or}\frac{5\pi}{6}\right)$	A1	1.1b
	Use of $\frac{1}{2}\int(1+\sin\theta)^2\,\mathrm{d}\theta$ or $\frac{1}{2}\int\{3(1-\sin\theta)\}^2\,\mathrm{d}\theta$	M1	1.1a
	$\left(\frac{1}{2}\right)\int\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta$ $=\left(\frac{1}{2}\right)\int\left[1+2\sin\theta+\sin^2\theta-9+18\sin\theta-9\sin^2\theta\right]\mathrm{d}\theta$ or $\int(1+\sin\theta)^2\,\mathrm{d}\theta=\int(1+2\sin\theta+\sin^2\theta)\,\mathrm{d}\theta$ **and** $\int 9(1-\sin\theta)^2\,\mathrm{d}\theta=9\int(1-2\sin\theta+\sin^2\theta)\,\mathrm{d}\theta$	M1 A1	2.1 1.1b
	$\int\sin^2\theta\,\mathrm{d}\theta=\frac{1}{2}\int(1-\cos 2\theta)\,\mathrm{d}\theta \Rightarrow$ $\int\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta=2\sin 2\theta-12\theta-20\cos\theta$	M1 A1	3.1a 1.1b
	$A=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta$ or $A=2\times\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left[(1+\sin\theta)^2-9(1-\sin\theta)^2\right]\mathrm{d}\theta$ $=\frac{1}{2}\left\{\left(-\sqrt{3}-10\pi+10\sqrt{3}\right)-\left(\sqrt{3}-2\pi-10\sqrt{3}\right)\right\}=...$	DM1	3.1a
	$=9\sqrt{3}-4\pi$	A1	1.1b
		(9)	
			(9 marks)

Notes
M1: Realises that the angles at the intersection are required and solves $C_1 = C_2$ to obtain a value for θ A1: Correct value for θ. Must be in radians – if given in degrees you may need to check later to see if they convert to radians before substitution. M1: Evidence selecting the correct polar area formula on either curve M1: Fully expands both expressions for r^2 either as parts of separate integrals or as one complete integral. (Can be scored from incorrect polar area formula, e.g. missing the ½) A1: Correct expansions for both curves (may be unsimplified) M1: Selects the correct strategy by applying the correct double angle identity in order to reach an integrable form and attempting the integration of at least one of the curves. A1: Correct integration (of both integrals if done separately), FYI: If done separately the correct integrals are $\int (1+\sin\theta)^2 \, d\theta = \theta - 2\cos\theta + \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) = \frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta$ and $\int 9(1-\sin\theta)^2 \, d\theta = 9\theta + 18\cos\theta + \frac{9}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right) = \frac{27}{2}\theta + 18\cos\theta - \frac{9}{4}\sin 2\theta$ DM1: Depends on all previous M's. For a fully correct strategy with appropriate limits correctly applied to their integral or integrals and terms combined if necessary. Make sure that if limits of $\frac{\pi}{6}$ and $\frac{\pi}{2}$ are used that the area is doubled as part of the strategy. A1: Correct area

Q6.

Question	Scheme	Marks	AOs
(a)(i)		M1	1.1b
		A1	1.1b
(a)(ii)	$\lvert z-4-3i\rvert = 5 \Rightarrow \lvert x+iy-4-3i\rvert = 5 \Rightarrow (x-4)^2+(y-3)^2 = ...$	M1	2.1
	$(x-4)^2+(y-3)^2 = 25$ or any correct form	A1	1.1b
	$(r\cos\theta-4)^2+(r\sin\theta-3)^2 = 25$ $\Rightarrow r^2\cos^2\theta - 8r\cos\theta + 16 + r^2\sin^2\theta - 6r\sin\theta + 9 = 25$ $\Rightarrow r^2 - 8r\cos\theta - 6r\sin\theta = 0$	M1	2.1
	$\therefore r = 8\cos\theta + 6\sin\theta$*	A1*	2.2a
		(6)	

(b)(i)		B1	1.1b
		B1ft	1.1b
(b)(ii)	$A=\frac{1}{2}\int r^2\,\mathrm{d}\theta=\frac{1}{2}\int\left(8\cos\theta+6\sin\theta\right)^2\mathrm{d}\theta$ $=\frac{1}{2}\int\left(64\cos^2\theta+96\sin\theta\cos\theta+36\sin^2\theta\right)\mathrm{d}\theta$	M1	3.1a
	$=\frac{1}{2}\int\left(32\left(\cos 2\theta+1\right)+96\sin\theta\cos\theta+18\left(1-\cos 2\theta\right)\right)\mathrm{d}\theta$	M1	1.1b
	$=\frac{1}{2}\int\left(14\cos 2\theta+50+48\sin 2\theta\right)\mathrm{d}\theta$	A1	1.1b
	$=\frac{1}{2}\left[7\sin 2\theta+50\theta-24\cos 2\theta\right]_0^{\frac{\pi}{3}}=\frac{1}{2}\left\{\left(\frac{7\sqrt{3}}{2}+\frac{50\pi}{3}+12\right)-\left(-24\right)\right\}$	M1	2.1
	$=\frac{7\sqrt{3}}{4}+\frac{25\pi}{3}+18$	A1	1.1b
		(7)	

	 		
	Alternative: **Candidates may take a geometric approach** **E.g. by finding sector + 2 triangles**		
	Angle $ACB = \left(\frac{2\pi}{3}\right)$ so area sector $ACB = \frac{1}{2}(5)^2\frac{2\pi}{3}$ Area of triangle $OCB = \frac{1}{2}\times 8\times 3$	M1	3.1a
	Sector area ACB + triangle area $OCB = \frac{25\pi}{3} + 12$	A1	1.1b
	Area of triangle OAC: Angle $ACO = 2\pi - \frac{2\pi}{3} - \cos^{-1}\left(\frac{5^2+5^2-8^2}{2\times5\times5}\right)$ so area $OAC = \frac{1}{2}(5)^2\sin\left(\frac{4\pi}{3} - \cos^{-1}\left(\frac{-7}{25}\right)\right)$	M1	1.1b
	$= \frac{25}{2}\left(\sin\frac{4\pi}{3}\cos\left(\cos^{-1}\left(\frac{-7}{25}\right)\right) - \cos\frac{4\pi}{3}\sin\left(\cos^{-1}\left(\frac{-7}{25}\right)\right)\right)$ $= \frac{25}{2}\left(\left(\frac{7\sqrt{3}}{50}\right) + \frac{1}{2}\sqrt{1-\left(\frac{7}{25}\right)^2}\right) = \frac{7\sqrt{3}}{4} + 6$ Total area $= \frac{25\pi}{3} + \frac{1}{2}\times8\times3 + 6 + \frac{7\sqrt{3}}{4}$	M1	2.1
	$= \frac{7\sqrt{3}}{4} + \frac{25\pi}{3} + 18$	A1	1.1b
			(13 marks)

Notes
(a)(i) M1: Draws a circle which passes through the origin A1: Fully correct diagram. (a)(ii) M1: Uses $z = x + iy$ in the given equation and uses modulus to find equation in x and y only A1: Correct equation in terms of x and y in any form – may be in terms of r and θ M1: Introduces polar form, expands and uses $\cos^2\theta + \sin^2\theta = 1$ leading to a polar equation A1*: Deduces the given equation (ignore any reference to $r = 0$ which gives a point on the curve) (b)(i) B1: Correct pair of rays added to their diagram B1ft: Area between their pair of rays and inside their circle from (a) shaded, as long as there is an intersection. (b)(ii) M1: Selects an appropriate method by linking the diagram to the polar curve in (a), evidenced by use of the polar area formula M1: Uses double angle identities A1: Correct integral M1: Integrates and applies limits A1: Correct area (b)(ii) Alternative: M1: Selects an appropriate method by finding angle ACB and area of sector ACB and finds area of triangle OCB to make progress towards finding the required area A1: Correct combined area of sector ACB + triangle OCB M1: Starts the process of finding the area of triangle OAC by calculating angle ACO and attempts area of triangle OAC M1: Uses the addition formula to find the exact area of triangle OAC and employs a full correct method to find the area of the shaded region A1: Correct area