Central Limit Theorem

Questions

Q1.

A courier delivers parcels. The random variable X represents the number of parcels delivered successfully each day by the courier where $X \sim B(400, 0.64)$

A random sample X_1, X_2, ... X_{100} is taken.

Estimate the probability that the mean number of parcels delivered each day by the courier is greater than 257

(4)

(Total for question = 4 marks)

Q2.

A biased spinner can land on the numbers 1, 2, 3, 4 or 5 with the following probabilities.

Number on spinner	1	2	3	4	5
Probability	0.3	0.1	0.2	0.1	0.3

The spinner will be spun 80 times and the mean of the numbers it lands on will be calculated.
Find an estimate of the probability that this mean will be greater than 3.25.

(Total for question = 6 marks)

Q3.

A random sample of 100 observations is taken from a Poisson distribution with mean 2.3

Estimate the probability that the mean of the sample is greater than 2.5

(4)

(Total for question = 4 marks)

Mark Scheme – Central Limit Theorem

Q1.

Question	Scheme	Marks	AOs
	$\overline{X} \approx \mathrm{N}(256,...)$ oe	M1	3.1a
	$\overline{X} \approx \mathrm{N}(256, 0.9216)$	A1	1.1b
	$\mathrm{P}(\overline{X} > 257) = \mathrm{P}\left(Z > \dfrac{257-256}{\sqrt{\text{"}0.9216\text{"}}}\right)$ [= awrt 1.04]	dM1	3.4
	$p = 0.1492...$	A1	1.1b
		(4)	
			(4 marks)

Notes:

M1:	For realising the need to use the CLT with correct mean	
A1:	For a correct normal stated	
dM1:	Dep on previous Method mark. Use of the normal model to find $\mathrm{P}(\overline{X} > 257)$ If final answer is incorrect then we need to see the standardisation using their σ.	
A1:	awrt 0.149 (0.14878… from calculator)	
	NB Allow awrt 0.148 if a continuity correction is used.	

Q2.

Qu	Scheme	Marks	AO
	{Let X = the number when the spinner is spun} $\quad \mu = \underline{3}$	B1	1.1b
	$\left[\mathrm{E}(X^2)=\right] 0.3+4\times0.1+9\times0.2+16\times0.1+25\times0.3 \quad [=11.6 \text{ or } \frac{58}{5}]$	M1	1.1b
	$\sigma^2\left[=11.6-3^2=\right] \underline{2.6}$	A1	1.1b
	$\bar{X} \approx \sim \mathrm{N}\left(\text{"3"}, \sqrt{\dfrac{\text{"2.6"}}{80}}^{\,2}\right)$	M1	2.1
		A1ft	1.1b
	$\mathrm{P}(\bar{X}>3.25)=[\mathrm{P}(Z>1.3867...)=]0.0827589...\text{(calc)}$ $\quad$ awrt $\underline{0.0828}$	A1	3.4
		(6 marks)	

	Notes
	B1 for stating or using mean = 3
	1st M1 for using the given model to attempt $\mathrm{E}(X^2)$ with at least 3 correct products seen
	1st A1 for $\mathrm{Var}(X)=2.6$ or $\sigma=\sqrt{2.6}=1.6124...$ (awrt 1.61)
ALT	**Use of pgf** (B1 when mean = 3 seen) (M1 when correct $\mathrm{G}''(t)$ seen with attempt at $\mathrm{G}''(1)$) $\mathrm{G}(t)=0.3t+0.1t^2+0.2t^3+0.1t^4+0.3t^5$ $\mathrm{G}'(t)=0.3+0.2t+0.6t^2+0.4t^3+1.5t^4$ $\mathrm{G}''(t)=0.2+1.2t+1.2t^2+6t^3$ leading to $\mathrm{G}''(1)=8.6$
	2nd M1 for use of CLT – must use $\bar{X}$ and normal <u>or</u> sight of $\mathrm{N}\left(\text{"3"}, \sqrt{\dfrac{\text{"2.6"}}{80}}^{\,2}\right)$ with any letter
	2nd A1ft for a correct mean and variance, ft their 3 and their 2.6 This M1A1ft may be implied by sight of correct st. dev. used in a standardisation leading to $\mathrm{P}(Z>1.39)$ Must see correct use of Z NB $\dfrac{2.6}{80}=0.0325$ and $\sqrt{\dfrac{2.6}{80}}=0.18027...$ so allow e.g. $\mathrm{N}(3, \text{awrt } (0.180)^2)$
	3rd A1 for using the normal model to find probability awrt 0.0828
ALT	**Use of** $\sum X$ (If see clear attempt at $\mathrm{P}(\Sigma X>260)$ condone $\mathrm{P}(\Sigma X>260.5)$ then: 2nd M1 for $\Sigma X \sim \mathrm{N}(\ldots)$ <u>or</u> any letter $\sim\mathrm{N}(\text{"240"}, \sqrt{\text{"2.6"}\times 80}^{\,2})$ 2nd A1ft for mean = "3" × 80 = 240 <u>and</u> variance = "2.6" × 80 = 208 May see $\mathrm{P}(\Sigma X>260.5)=0.077597\ldots$ but it will only score 2nd M1 2nd A1ft and **3rd A0**

Q3.

Question	Scheme	Marks	AOs
	Po(2.3) $n = 100$ $\mu = 2.3$ $\sigma^2 = 2.3$		
	CLT $\Rightarrow \bar{X} \approx \mathrm{N}\left(2.3, \frac{2.3}{100}\right)$	M1 A1	3.1a 1.1b
	$\mathrm{P}(\bar{X} > 2.5) = \mathrm{P}\left(Z > \frac{2.5 - 2.3}{\sqrt{0.023}}\right)$	M1	3.4
	$= \mathrm{P}(Z > 1.318..)$		
	$= 0.09632\ldots$	A1	1.1b
		(4)	
			(4 marks)
	M1: For realising the need to use the CLT to set $\bar{X} \approx$ normal with correct mean. May be implied by using the correct normal distribution. **A1:** For fully correct normal stated or used **M1:** Use of the normal model to find $\mathrm{P}(\bar{X} > 2.5)$. Can be awarded for $\frac{2.5 - 2.3}{\sqrt{0.023}}$ or awrt 1.32 **A1:** awrt 0.0963		