Kinematics

Questions

Q1.

A car moves in a straight line along a horizontal road. The car is modelled as a particle. At time t seconds, where $t \geq 0$, the speed of the car is v m s^{-1}

At the instant when $t = 0$, the car passes through the point A with speed 2 m s^{-1}

The acceleration, a m s^{-2}, of the car is modelled by

$$a = \frac{4}{2+v}$$

in the direction of motion of the car.

(a) Use algebraic integration to show that $v = \sqrt{8t+16} - 2$

(6)

At the instant when the car passes through the point B, the speed of the car is 4 m s^{-1}

(b) Use algebraic integration to find the distance AB.

(6)

(Total for question = 12 marks)

Q2.

At time $t = 0$, a toy electric car is at rest at a fixed point O. The car then moves in a horizontal straight line so that at time t seconds ($t > 0$) after leaving O, the velocity of the car is v m s^{-1} and the acceleration of the car is modelled as $(p + qv)$ m s^{-2}, where p and q are constants.

When $t = 0$, the acceleration of the car is 3 m s^{-2}

When $t = T$, the acceleration of the car is $\frac{1}{2}$ m s^{-2} and $v = 4$

(a) Show that

$$8\frac{dv}{dt} = (24 - 5v)$$

(6)

(b) Find the exact value of T, simplifying your answer.

(6)

(Total for question = 12 marks)

Q3.

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = \frac{1}{2}(3e^{2t} - 1) \qquad t \geqslant 0$$

The acceleration of P at time t seconds is a m s^{-2}

(a) Show that $a = 2v + 1$

(2)

(b) Find the acceleration of P when $t = 0$

(1)

(c) Find the exact distance travelled by P in accelerating from a speed of 1 m s^{-1} to a speed of 4 m s^{-1}

(7)

(Total for question = 10 marks)

Q4.

A particle, P, of mass 0.4 kg is moving along the positive x-axis, in the positive x direction under the action of a single force. At time t seconds, $t > 0$, P is x metres from the origin O and the speed of P is v m s^{-1}. The force is acting in the direction of x increasing and has magnitude $\frac{k}{v}$ newtons, where k is a constant.

At $x = 3$, $v = 2$ and at $x = 6$, $v = 2.5$

Show that $v^3 = \frac{61x + 9}{24}$

(6)

The time taken for the speed of P to increase from 2 m s^{-1} to 2.5 m s^{-1} is T seconds.

(b) Use algebraic integration to show that $T = \frac{81}{61}$

(4)

(Total for question = 10 marks)

Q5.

A particle, P, moves on the x-axis. At time t seconds, $t \geq 0$, the velocity of P is v m s^{-1} in the direction of x increasing and the acceleration of P is a m s^{-2} in the direction of x increasing.

When $t = 0$ the particle is at rest at the origin O.

Given that $a = \frac{5}{2}(5 - v)$

(a) show that $v = 5(1 - e^{-2.5t})$

(5)

(b) state the limiting value of v as t increases.

(1)

At the instant when $v = 2.5$, the particle is d metres from O.

(c) Show that $d = 2\ln 2 - 1$

(7)

(Total for question = 13 marks)

Q6.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = (t - 2)(3t - 10), \qquad t \geq 0$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Find the total distance travelled by P in the first 2 seconds of its motion.

(3)

(c) Show that P never returns to O, explaining your reasoning.

(3)

(Total for question = 8 marks)

Q7.

A car of mass 500 kg moves along a straight horizontal road.

The engine of the car produces a constant driving force of 1800 N.

The car accelerates from rest from the fixed point *O* at time $t = 0$ and at time t seconds the car is x metres from *O*, moving with speed v m s^{-1}.

When the speed of the car is v m s^{-1}, the resistance to the motion of the car has magnitude $2v^2$ N.

At time T seconds, the car is at the point *A*, moving with speed 10 m s^{-1}.

(a) Show that $T = \frac{25}{6} \ln 2$

(6)

(b) Show that the distance from *O* to *A* is $125 \ln \frac{9}{8}$ m.

(5)

(Total for question = 11 marks)

Q8.

A particle *P* of mass m kg is initially held at rest at the point *O* on a smooth plane which is inclined at 30° to the horizontal. The particle is released from rest and slides down the plane against a force of magnitude $\frac{1}{2}mx^2$ newtons acting towards *O*, where x metres is the distance of *P* from *O*.

(a) Find the speed of *P* when $x = 3$

(7)

(b) Find the distance *P* has moved when it first comes to instantaneous rest.

(2)

(Total for question = 9 marks)

Mark Scheme - Kinematics

Q1.

Question	Scheme	Marks	AOs
(a)	$a = \dfrac{4}{2+v} \Rightarrow \int (2+v)\,\mathrm{d}v = \int 4\,\mathrm{d}t$	M1	2.1
	$\dfrac{(2+v)^2}{2} = 4t + C_1$	M1	1.1b
		A1	1.1b
	$t=0,\ v=2 \Rightarrow C_1 = 8$	M1	3.4
	$\dfrac{(2+v)^2}{2} = 4t + 8$	A1	1.1b
	$(2+v)^2 = 8t+16,\quad v = \sqrt{8t+16} - 2$ *	A1*	2.2a
		(6)	
(a) alt	$a = \dfrac{4}{2+v} \Rightarrow \int (2+v)\,\mathrm{d}v = \int 4\,\mathrm{d}t$	M1	2.1
	$2v + \dfrac{v^2}{2} = 4t + C_2$	M1	1.1b
		A1	1.1b
	$t=0,\ v=2 \Rightarrow C_2 = 6$	M1	3.4
	$2v + \dfrac{v^2}{2} = 4t + 6$	A1	1.1b
	$4v + v^2 = 8t + 12,\quad (v+2)^2 = 8t+16$ $\Rightarrow v = \sqrt{8t+16} - 2$ *	A1*	2.2a
		(6)	
(b)	$v = 4 \Rightarrow 36 = 8t + 16 \Rightarrow t = 2.5$	B1	1.1b
	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{8t+16} - 2$	M1	3.3
	$x = k(8t+16)^{\frac{3}{2}} - 2t + C$	M1	1.1b
	$x = \dfrac{1}{12}(8t+16)^{\frac{3}{2}} - 2t + C$	A1	1.1b
	$t=0,\ x=0 \Rightarrow \dfrac{64}{12} + C = 0,\ C = -\dfrac{16}{3}$	M1	3.4
	$AB = \dfrac{1}{12}(36)^{\frac{3}{2}} - 5 - \dfrac{16}{3} = \dfrac{23}{3}$ (m)	A1	1.1b
		(6)	
			(12 marks)

Notes		
(a)	M1	Form differential equation in v and t and prepare to integrate.
	M1	Integrate to obtain $k(2+v)^2$ or equivalent
	A1	Correct integration. Condone missing constant of integration.
	M1	Use the model to find the value of constant of integration.
	A1	Correct solution in any form
	A1*	Obtain **given solution** from correct working. Allow use of quadratic formula.
(a) alt	M1	Form differential equation in v and t and prepare to integrate.
	M1	Integrate to obtain $k(2+v)^2$ or equivalent
	A1	Correct integration. Condone missing constant of integration.
	M1	Use the model to find the value of constant of integration.
	A1	Correct solution in any form
	A1*	Obtain **given solution** from correct working.
(b)	B1	Use the result from (a) to find t when $v = 4$: seen or implied
	M1	Form differential equation in x and t
	M1	Integrate to obtain terms of the correct form. Condone missing constant of integration.
	A1	Correct integration. Condone missing constant of integration.
	M1	Use boundary conditions in the model to find constant of integration, or as limits on a definite integral. Note this is an independent M mark. M0 if they use $t = 4$
	A1	Correct answer only. 7.7 (m) or better

Q2.

Question	Scheme	Marks	AOs
(a)	Use of initial condition to find p	M1	3.1b
	$t = 0, v = 0,$ acceleration $= 3 \Rightarrow p = 3$	A1	1.1b
	Use $v = 4$, acceleration $= \frac{1}{2}$	M1	1.1b
	$q = -\frac{5}{8}$	A1	1.1b
	Use acceleration $= \frac{\mathrm{d}v}{\mathrm{d}t}$ and rearrange	M1	1.1b
	$8\frac{\mathrm{d}v}{\mathrm{d}t} = (24 - 5v)$ *	A1*	2.2a
		(6)	
(b)	Separate the variables and integrate	M1	3.1b
	$8\int \frac{\mathrm{d}v}{(24-5v)} = \int \mathrm{d}t$	A1	1.1b
	$-\frac{8}{5}\ln(24 - 5v) = t + C$	A1	1.1b
	Use $t = 0, v = 0$ to give $C = -\frac{8}{5}\ln 24$	M1	1.1b
	Substitute $v = 4$ and find and simplify T	M1	1.1b
	$T = \frac{8}{5}\ln 6$	A1	1.1b
		(6)	
		(12 marks)	

Notes
(a) **M1:** Use initial conditions **A1:** cao **M1:** Use second condition **A1:** cao **M1:** Use appropriate derivative and rearrange **A1*:** Correct given answer
(b) **M1:** Separate the variables and integrate **A1:** Correct integration (C not required) **M1:** Use of limits or initial conditions to find C **M1:** Use $v = 4$ to find T **A1:** Correct answer (single log)

Q3.

Question	Scheme	Marks	AOs
(a)	$a = \frac{dv}{dt} = \frac{1}{2} \times 6e^{2t}$	M1	1.1b
	$= 2v + 1$ *	A1*	1.1b
		(2)	
(b)	3 ($m\,s^{-2}$)	B1	1.1b
		(1)	
(c)	$\frac{dx}{dt} = \frac{1}{2}(3e^{2t} - 1)$ and integrate	M1	3.3
	$x = \frac{1}{2}(\frac{3}{2}e^{2t} - t)(+C)$	A1	1.1b
	Put either $\frac{1}{2}(3e^{2t} - 1) = 1$ or 4 and solve for t	M1	2.1
	$t = 0$	A1	1.1b
	$t = \frac{1}{2}\ln 3$ (0.549306…)	A1	1.1b
	Substitute their t values into their x expression and subtract	M1	3.1a
	$\frac{3}{2} - \frac{1}{4}\ln 3$ (m)	A1	1.1b
		(7)	
			(10 marks)

Notes:		
a	M1	Need to see evidence of attempt to differentiate v wrt t, not just a statement of intent.
	A1*	Given answer correctly obtained
b	B1	cao
c	M1	Set up differential equation and attempt to solve
	A1	Condone missing C
	M1	Use at least one of the given speeds to find a t value
	A1	cao
	A1	0.55 or better
	M1	Substitute their t values to find x values and showing subtracting. Need to see evidence. M0 if using 1 and 4.
	A1	cao

Q4.

Question	Scheme	Marks	AOs
(a)	Strategy to find v^3 in terms of x	M1	3.1a
	Differential equation in v and x: $0.4v\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{k}{v}$	M1	2.1
	$\Rightarrow 0.4v^2\frac{\mathrm{d}v}{\mathrm{d}x}=k\ ,\quad \frac{0.4}{3}v^3=kx+C$	A1	1.1b
	$x=3,\ v=2\quad \frac{3.2}{3}=3k+C$ $x=6,\ v=2.5\quad \frac{25}{12}=6k+C$	M1	2.1
	$\Rightarrow 3k=\frac{25}{12}-\frac{3.2}{3}\ ,\quad k=\frac{61}{180}\ ,\quad C=\frac{1}{20}$	A1	1.1b
	$v^3=\frac{3}{0.4}\left(\frac{61x}{180}+\frac{1}{20}\right)=\frac{61x+9}{24}$ *	A1*	2.2a
		(6)	

(b)	$\frac{5k}{2v}=\frac{\mathrm{d}v}{\mathrm{d}t}=\left(\frac{61}{72v}\right)$	M1	2.5
	$\int 2v\mathrm{d}v=\int 5k\mathrm{d}t\ \Rightarrow\ v^2=5kt+C'\quad \left(36v^2=61t+C'\right)$	M1	2.1
	$\left[v^2\right]_2^{2.5}=\left[5kt\right]_0^T$ $\left(61T=36\left(2.5^2-2^2\right)\right)$	M1	1.1b
	$T=\frac{180}{61}\left(\frac{9}{20}\right)=\frac{81}{61}$ *	A1*	2.2a
		(4)	
(b) alt	$\frac{\mathrm{d}x}{\mathrm{d}t}=\sqrt[3]{\frac{61x+9}{24}}$	M1	2.5
	$\int(61x+9)^{-\frac{1}{3}}\mathrm{d}x=\int\frac{1}{\sqrt[3]{24}}\mathrm{d}t\ ,\quad \frac{3}{2\times 61}(61x+9)^{\frac{2}{3}}=\frac{t}{\sqrt[3]{24}}+C''$	M1	2.1
	$T=2\times\sqrt[3]{3}\times\frac{3}{2\times 61}\left(375^{\frac{2}{3}}-192^{\frac{2}{3}}\right)=\frac{3\times\sqrt[3]{3}}{61}\left(\left(5\sqrt[3]{3}\right)^2-\left(4\sqrt[3]{3}\right)^2\right)$	M1	1.1b
	$T=\frac{9}{61}(25-16)=\frac{81}{61}$ *	A1*	2.2a
			(10 marks)

Question	Marks	Marking Guidance
(a)	M1	Complete strategy e.g. use of $F = ma$ with appropriate form for a, and use boundary conditions to confirm given result
	M1	Separate variables and integrate. Usual rules for integration. Condone missing C
	A1	Correct integration. Accept equivalent forms. Condone missing C.
	M1	Use boundary conditions to form 2 equations in 2 unknowns and solve for k or C
	A1	Obtain correct values for the constants
	A1*	Obtain **given answer** from correct working
	(6)	
(b)	M1	Select correct form for derivative and form a correct differential equation in v and t – follow their k
	M1	Separate and integrate. Condone with no $+C'$ – follow their k
	M1	Evaluate definite integral of the form $pv^2 = qt + C'$ or use limits to find value of constant of integration – follow their k
	A1*	Obtain **given answer** from correct working
	(4)	
(b) alt	M1	Select correct form for derivative and form a correct differential equation in x and t
	M1	Separate and integrate. Condone with no $+C''$
	M1	Evaluate definite integral of the form $pt = q\left(61x+9\right)^{\frac{2}{3}} + C''$ or use limits to find value of constant of integration
	A1*	Obtain **given answer** from correct working
		NB: Both parts have given answers, so check very carefully

Q5.

Question	Scheme	Marks	AOs
(a)	$\int \frac{1}{4}\mathrm{d}t = \int \frac{1}{50-10v}\mathrm{d}v$	M1	3.1a
	$\frac{1}{4}t = -\frac{1}{10}\ln(50-10v)(+C)$	A1	1.1b
	$\frac{1}{4}t = -\frac{1}{10}\ln(50-10v)+\frac{1}{10}\ln 50$	M1	1.1b
	$-\frac{5t}{2} = \ln\left(\frac{5-v}{5}\right)$	M1	1.1b
	$v = 5\left(1-e^{-2.5t}\right)$ *	A1*	2.1
		(5)	
(b)	limiting value is 5	B1	2.2a
		(1)	
(c)	Equation in x and t: $\frac{\mathrm{d}x}{\mathrm{d}t} = 5\left(1-\mathrm{e}^{-2.5t}\right)$	M1	1.1a
	$\Rightarrow \int 1\mathrm{d}x = \int 5\left(1-\mathrm{e}^{-2.5t}\right)\mathrm{d}t$	M1	1.1b
	$x = 5t + 2e^{-2.5t}(+C)$	A1	1.1b
	Use $v = 2.5$ and $v = 5\left(1-e^{-2.5t}\right)$ to find value of t	M1	3.1a
	$1-\frac{2.5}{5} = \mathrm{e}^{-2.5t} \Rightarrow t = \frac{2}{5}\ln 2$	A1	1.1b
	$[x]_0^d = \left[5t+2e^{-2.5t}\right]_0^{\frac{2}{5}\ln 2}$	M1	2.1
	$d = 2\ln 2 - 1$ *	A1*	1.1b
		(7)	
			(13 marks)

Notes
(a) **M1**: Strategy to find v and attempt the integration **A1**: Correct integration **M1**: Use boundary conditions as limits or evaluate constant of integration in an expression involving $\lambda\ln(a+bv)$ and μt **M1**: Remove logarithm to express v in terms of t **A1***: Obtain given answer from correct working
(b) **B1**: Correct answer from correct working
(c) **M1**: Set up equation of motion in terms of x and t **M1**: Separate variables and attempt integration of both sides **A1**: Any equivalent form. Condone if $+C$ not seen **M1**: Use $v = 2.5$ to find limit for t **A1**: Any equivalent exact form. (0.277) **M1**: Use boundary conditions as limits or evaluate constant of integration in an expression involving λt and $\mu e^{-2.5t}$ **A1***: Sufficient correct working to justify given answer

Q6.

Question	Scheme	Marks	AOs
(a)	Multiply out and differentiate wrt t	M1	1.1b
	$v = 3t^2 - 16t + 20 \Rightarrow a = 6t - 16$	A1	1.1b
		(2)	
(b)	Multiply out and integrate wrt t	M1	1.1b
	$s = \int 3t^2 - 16t + 20 \mathrm{d}t = t^3 - 8t^2 + 20t (+C)$	A1	1.1b
	$t = 0, s = 0 \Rightarrow C = 0$ $t = 2, s = 8 - 32 + 40 = 16$	A1	1.1b
		(3)	
(c)	$s = 0 \Rightarrow t^3 - 8t^2 + 20t = 0$ and $t \neq 0 \Rightarrow t^2 - 8t + 20 = 0$	M1	2.1
	Explanation to show that $t^2 - 8t + 20 > 0$ for all t.	M1	2.4
	So $s = 0$ has no non-zero solutions, so s is never zero again, so never returns to O *	A1*	3.2a
		(3)	
			(8 marks)

Notes:

(a)
M1: for multiplying out and differentiating (powers decreasing by 1)
A1: for a correct expression for a

(b)
M1: for multiplying out and integrating (powers increasing by 1)
A1: for a correct expression for s with or without C
A1: for $C = 0$ and correct final answer

(c)
M1: for equating their s to 0 and producing a quadratic
M1: for clear explanation that $t^2 - 8t + 20 > 0$ for all t (e.g. completing the square or another complete method)
A1*: for a correct conclusion in context

Q7.

Question	Scheme	Marks	AOs
(a)	Equation of motion $1800-2v^2=500a$ (when seen)	B1	2.1
	Select form for a: $=500\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	2.5
	$\int\dfrac{2}{500}\mathrm{d}t=\int\dfrac{1}{900-v^2}\mathrm{d}v=\dfrac{1}{60}\int\dfrac{1}{30+v}+\dfrac{1}{30-v}\mathrm{d}v$	M1	2.1
	$\dfrac{t}{250}=\dfrac{1}{60}\ln(30+v)-\dfrac{1}{60}\ln(30-v)\ (+C)$	A1	1.1b
	$T=\dfrac{25}{6}\ln\left(\dfrac{30+10}{30-10}\right)=\dfrac{25}{6}\ln 2$ *	M1 A1*	2.1 2.2a
		(6)	
(b)	Equation of motion: $500v\dfrac{\mathrm{d}v}{\mathrm{d}x}=1800-2v^2$	M1	2.5
	$\int\dfrac{500v}{1800-2v^2}\mathrm{d}v=\int 1\mathrm{d}x$	M1	2.1
	$-125\ln\left(1800-2v^2\right)=x\ (+C)$	A1	1.1b
	Use boundary conditions: $x=-125\ln 1600+125\ln 1800$	M1	2.1
	$x=125\ln\dfrac{9}{8}$ (m) *	A1*	2.2a
		(5)	
		(11 marks)	

Notes:

(a)

B1: all three terms & dimensionally correct

M1: use of correct form for acceleration to give equation in v, t only

M1: Separate variables and integrate

A1: Condone missing C

M1: Use boundary conditions correctly

A1*: Show sufficient working to justify **given answer** and a 'statement' that the required result has been achieved

(b)

M1: Correct form of acceleration in the equation of motion to give equation in v, x only

M1: Separate variables and integrate.

A1: Condone missing C

M1: Extract and use boundary conditions

A1*: Show sufficient working to justify **given answer** and a 'statement' that the required result has been achieved

Q8.

Question Number	Scheme	Marks
(a)	$mv\frac{dv}{dx} = mg\sin 30 - \frac{1}{2}mx^2$	M1A1A1
	$\frac{1}{2}v^2 = xg\sin 30 - \frac{1}{6}x^3 \;(+c)$	dM1A1ft
	$x = 3 \;\; \frac{1}{2}v^2 = 3g\sin 30 - \frac{9}{2}$	dM1
	$(v = 4.5166...)$	
	$v = 4.5 \;\text{ or } 4.52 \;\left(\text{ms}^{-1}\right)$	A1cso (7)
(b)	$v = 0 \Rightarrow x^2 = 6g\sin 30 \;\; (x \neq 0)$	
	$x = 5.4 \text{ or } 5.42 \;(\text{m})$	M1A1 (2)
		[9]

(a)M1 Attempt NL2 parallel to the plane. Acceleration must be $v\frac{dv}{dx}$ and weight must be resolved. (Variable force not resolved.) m may be cancelled. Integrating a to obtain $\frac{1}{2}v^2$ gains this mark by implication.

A1 A1 Deduct 1 mark for each error in the equation. Both signs incorrect on RHS is one error.

dM1 Attempt the integration (wrt x) of both sides of the equation. Depends on the first M mark.
A1ft Correct integration with or without the constant. Follow through their integrand.
dM1 Substitute $x = 3$ in their integrated equation. Depends on both previous M marks.
A1cso Correct value of v. Must be 2 or 3 sf. **CSO**: Evidence of a constant of integration must be seen. C included and then crossed out or disappearing is sufficient evidence.

Definite integration:
M1A1A1 as above
dM1A1ft For the integration - ignore any limits shown
dM1 Use of correct limits. No sub need be shown for 0.
A1 Correct value of v. Must be 2 or 3 sf. **CSO**: Evidence of a zero lower limit must be seen.

By work-energy:
F is variable, so if no integral seen score 0/7

. $\frac{1}{2}v^2(-0) = xg\sin 30 - \int \frac{1}{2}x^2 dx$ .. M1A1A1

$\frac{1}{2}v^2(-0) = xg\sin 30 - \frac{1}{6}x^3$ M1A1

For the final A mark, evidence of initial KE being 0 must be seen.

(b)
M1 Substitute $v = 0$ in their equation for v^2 (from (a)) and obtain a numerical value of x
A1 Correct value of x. Must be 2 or 3 sf. Do not penalise missing constant here.