# Elastic Collisions in 1D

## Questions

**Q1.**

Two particles, *P* and *Q*, have masses *m* and *em* respectively. The particles are moving on a smooth horizontal plane in the same direction along the same straight line when they collide directly. The coefficient of restitution between *P* and *Q* is *e*, where $0 < e < 1$

Immediately before the collision the speed of *P* is *u* and the speed of *Q* is *eu*.

(a) Show that the speed of *Q* immediately after the collision is *u*.

**(6)**

(b) Show that the direction of motion of *P* is unchanged by the collision.

**(3)**

The magnitude of the impulse on *Q* in the collision is $\frac{2}{9}mu$

(c) Find the possible values of *e*.

**(4)**

**(Total for question = 13 marks)**

**Q2.**

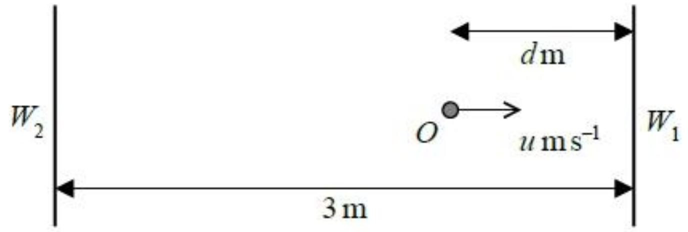



**Figure 1**

Figure 1 represents the plan of part of a smooth horizontal floor, where $W_1$ and $W_2$ are two fixed parallel vertical walls. The walls are 3 metres apart.

A particle lies at rest at a point $O$ on the floor between the two walls, where the point $O$ is $d$ metres, $0 < d \leq 3$, from $W_1$

At time $t = 0$, the particle is projected from $O$ towards $W_1$ with speed $u$ m s$^{-1}$ in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{2}{3}$

The particle returns to $O$ at time $t = T$ seconds, having bounced off each wall once.

(a) Show that $T = \dfrac{45 - 5d}{4u}$

**(6)**

The value of $u$ is fixed, the particle still hits each wall once but the value of $d$ can now vary.

(b) Find the least possible value of $T$, giving your answer in terms of $u$. You must give a reason for your answer.

**(2)**

**(Total for question = 8 marks)**

**Q3.**

Two particles, $P$ and $Q$, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of $P$ is $3m$ and the mass of $Q$ is $4m$.
Immediately before the collision the speed of $P$ is $2u$ and the speed of $Q$ is $u$.
The coefficient of restitution between $P$ and $Q$ is $e$.

(a) Show that the speed of $Q$ immediately after the collision is u e $\frac{u}{7}(9e + 2)$

(6)

After the collision with $P$, particle $Q$ collides directly with a fixed vertical wall and rebounds.
The wall is perpendicular to the direction of motion of $Q$.

The coefficient of restitution between $Q$ and the wall is $\frac{1}{2}$

(b) Find the complete range of possible values of $e$ for which there is a second collision between $P$ and $Q$.

(4)

**(Total for question = 10 marks)**

**Q4.**

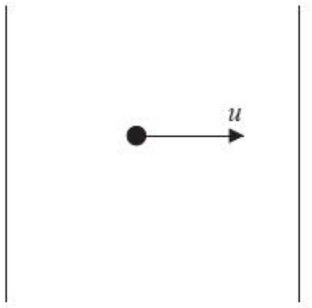



**Figure 2**

A particle of mass *em* is at rest on a smooth horizontal plane between two smooth fixed parallel vertical walls, as shown in the plan view in Figure 2. The particle is projected along the plane with speed *u* towards one of the walls and strikes the wall at right angles. The coefficient of restitution between the particle and each wall is *e* and air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of *m*, *u* and *e*, an expression for the total loss in the kinetic energy of the particle as a result of the first two impacts.

**(3)**

Given that *e* can vary such that $0 < e < 1$ and using the model,

(b) find the value of *e* for which the total loss in the kinetic energy of the particle as a result of the first two impacts is a maximum,

**(4)**

(c) describe the subsequent motion of the particle.

**(2)**

**(Total for question = 9 marks)**

**Q5.**

Two particles, $A$ and $B$, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Particle $A$ has mass $5m$ and particle $B$ has mass $3m$.

The coefficient of restitution between $A$ and $B$ is $e$, where $e > 0$

Immediately **after** the collision the speed of $A$ is $v$ and the speed of $B$ is $2v$.

Given that $A$ and $B$ are moving in the same direction after the collision,

(a) find the set of possible values of $e$.

**(8)**

Given also that the kinetic energy of $A$ immediately after the collision is 16% of the kinetic energy of $A$ immediately before the collision,

(b) find

(i) the value of $e$,
(ii) the magnitude of the impulse received by $A$ in the collision, giving your answer in terms of $m$ and $v$.

**(6)**

**(Total for question = 14 marks)**

**Q6.**

A particle $P$ of mass $3m$ and a particle $Q$ of mass $2m$ are moving along the same straight line on a smooth horizontal plane. The particles are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of $P$ is $u$ and the speed of $Q$ is $2u$.

Immediately after the collision $P$ and $Q$ are moving in opposite directions.

The coefficient of restitution between $P$ and $Q$ is $e$.

(a) Find the range of possible values of $e$, justifying your answer.

**(8)**

Given that $Q$ loses 75% of its kinetic energy as a result of the collision,

(b) find the value of $e$.

**(3)**

**(Total for question = 11 marks)**

**Q7.**

Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g$ = 9.8 ms$^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

A particles $P$ of mass 2$m$ and a particle $Q$ of mass 5$m$ are moving along the same straight line on a smooth horizontal plane.

They are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of $P$ is 2$u$ and the speed of $Q$ is $u$.

The direction of motion of $Q$ is reversed by the collision.

The coefficient of restitution between $P$ and $Q$ is $e$.

(a) Find the range of possible values of $e$.

**(8)**

Given that $e = \frac{1}{3}$

(b) show that the kinetic energy lost in the collision is $\frac{40mu^2}{7}$.

**(5)**

(c) Without doing any further calculation, state how the amount of kinetic energy lost in the collision would

change if $e > \frac{1}{3}$

**(1)**

**(Total for question = 14 marks)**

**Q8.**

Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g$ = 9.8 m s$^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

A small ball of mass 0.1 kg is dropped from a point which is 2.4 m above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of 0.6 m above the floor. The ball is modelled as a particle.

(a) Show that the coefficient of restitution between the ball and the floor is 0.5

**(6)**

(b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

(3)

(c) By considering your answer to (b), describe the subsequent motion of the ball.

(1)

**(Total for question = 10 marks)**

**Q9.**

Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g$ = 9.8 ms$^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

A particle of mass $m$ kg lies on a smooth horizontal surface.

Initially the particle is at rest at a point $O$ between two fixed parallel vertical walls.

The point $O$ is equidistant from the two walls and the walls are 4 m apart.

At time $t$ = 0 the particle is projected from $O$ with speed $u$ m s$^{-1}$ in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{3}{4}$

The magnitude of the impulse on the particle due to the first impact with a wall is $\lambda mu$ N s.

(a) Find the value of $\lambda$.

(3)

The particle returns to $O$, having bounced off each wall once, at time $t$ = 7 seconds.

(b) Find the value of $u$.

(5)

**(Total for question = 8 marks)**

**Q10.**

Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g$ = 9.8 m s$^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

A particle $P$ of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle $Q$ of mass $m$ is moving in the opposite direction to $P$ along the same straight line. The particles collide directly. Immediately before the collision the speed of $P$ is $u$ and the speed of $Q$ is $2u$. The velocities of $P$ and $Q$ immediately after the collision, measured in the direction of motion of $P$ before the collision, are $v$ and $w$ respectively. The coefficient of restitution between $P$ and $Q$ is $e$.

(a) Find an expression for $v$ in terms of $u$ and $e$.

**(6)**

Given that the direction of motion of $P$ is changed by the collision,

(b) find the range of possible values of $e$.

**(2)**

(c) Show that $w = \frac{u}{4}(1 + 9e)$.

**(2)**

Following the collision with $P$, the particle $Q$ then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of $Q$. The coefficient of restitution between $Q$ and the wall is $f$.

Given that $e = \frac{5}{9}$, and that $P$ and $Q$ collide again in the subsequent motion,

(d) find the range of possible values of $f$.

**(6)**

**(Total for question = 16 marks)**

**Q11.**

A particle $P$ of mass $2m$ kg is moving with speed $2u$ m s$^{-1}$ on a smooth horizontal plane. Particle $P$ collides with a particle $Q$ of mass $3m$ kg which is at rest on the plane. The coefficient of restitution between $P$ and $Q$ is $e$. Immediately after the collision the speed of $Q$ is $v$ m s$^{-1}$

(a) Show that $v = \dfrac{4u(1+e)}{5}$

**(6)**

(b) Show that $\dfrac{4u}{5} \leqslant v \leqslant \dfrac{8u}{5}$

**(2)**

Given that the direction of motion of *P* is reversed by the collision,

(c) find, in terms of *u* and *e*, the speed of *P* immediately after the collision.

**(2)**

After the collision, *Q* hits a wall, that is fixed at right angles to the direction of motion of *Q*, and rebounds.

The coefficient of restitution between *Q* and the wall is $\frac{1}{6}$

Given that *P* and *Q* collide again,

(d) find the full range of possible values of *e*.

**(5)**

**(Total for question = 15 marks)**

**Q12.**

Two particles, *A* and *B*, have masses *m*and 3*m* respectively. The particles are moving in opposite directions along the same straight line on a smooth horizontal plane when they collide directly.

Immediately before they collide, *A* is moving with speed 2*u* and *B* is moving with speed *u*.

The direction of motion of each particle is reversed by the collision.

In the collision, the magnitude of the impulse exerted on *A* by *B* is $\frac{9mu}{2}$

(a) Find the value of the coefficient of restitution between *A* and *B*.

**(7)**

(b) Hence, write down the total loss in kinetic energy due to the collision, giving a reason for your answer.

**(1)**

**(Total for question = 8 marks)**

**Q13.**

Three particles, $P$, $Q$ and $R$, are at rest on a smooth horizontal plane. The particles lie along a straight line with $Q$ between $P$ and $R$. The particles $Q$ and $R$ have masses $m$ and $km$ respectively, where $k$ is a constant.

Particle $Q$ is projected towards $R$ with speed $u$ and the particles collide directly.

The coefficient of restitution between each pair of particles is $e$.

(a) Find, in terms of $e$, the range of values of $k$ for which there is a second collision.

**(9)**

Given that the mass of $P$ is $km$ and that there is a second collision,

(b) write down, in terms of $u$, $k$ and $e$, the speed of $Q$ after this second collision.

**(1)**

**(Total for question = 10 marks)**

**Q14.**

Three particles $A$, $B$ and $C$ are at rest on a smooth horizontal plane. The particles lie along a straight line with $B$ between $A$ and $C$.

Particle $B$ has mass $4m$ and particle $C$ has mass $km$, where $k$ is a positive constant. Particle $B$ is projected with speed $u$ along the plane towards $C$ and they collide directly.

The coefficient of restitution between $B$ and $C$ is $\frac{1}{4}$

(a) Find the range of values of $k$ for which there would be no further collisions.

**(8)**

The magnitude of the impulse on $B$ in the collision between $B$ and $C$ is $3mu$

(b) Find the value of $k$.

**(4)**

**(Total for question = 12 marks)**

**Q15.**

Two particles, $A$ and $B$, of masses $2m$ and $3m$ respectively, are moving on a smooth horizontal plane. The particles are moving in opposite directions towards each other along the same straight line when they collide directly. Immediately before the collision the speed of $A$ is $2u$ and the speed of $B$ is $u$. In the collision the impulse of $A$ on $B$ has magnitude $5mu$.

(a) Find the coefficient of restitution between $A$ and $B$.

**(9)**

(b) Find the total loss in kinetic energy due to the collision.

**(4)**

**(Total for question = 13 marks)**

**Q16.**

Two particles $A$ and $B$, of masses $3m$ and $4m$ respectively, lie at rest on a smooth horizontal surface. Particle $B$ lies between $A$ and a smooth vertical wall which is perpendicular to the line joining $A$ and $B$. Particle $B$ is projected with speed $5u$ in a direction perpendicular to the wall and collides with the wall. The coefficient of restitution between $B$ and the wall is $\frac{3}{5}$.

(a) Find the magnitude of the impulse received by $B$ in the collision with the wall.

**(3)**

After the collision with the wall, $B$ rebounds from the wall and collides directly with $A$. The coefficient of restitution between $A$ and $B$ is $e$.

(b) Show that, immediately after they collide, $A$ and $B$ are both moving in the same direction.

**(7)**

The kinetic energy of $B$ immediately after it collides with $A$ is one quarter of the kinetic energy of $B$ immediately before it collides with $A$.

(c) Find the value of $e$.

**(4)**

**(Total for question = 14 marks)**

**Q17.**

A particle *P* of mass 3*m* is moving in a straight line on a smooth horizontal floor. A particle *Q* of mass 5*m* is moving in the opposite direction to *P* along the same straight line.

The particles collide directly.

Immediately before the collision, the speed of *P* is 2*u* and the speed of *Q* is *u*.
The coefficient of restitution between *P* and *Q* is *e*.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{8}(9e + 1)$

**(6)**

(b) Find the range of values of *e* for which the direction of motion of *P* is not changed as a result of the collision.

**(2)**

When *P* and *Q* collide they are at a distance *d* from a smooth fixed vertical wall, which is perpendicular to their direction of motion. After the collision with *P*, particle *Q* collides directly with the wall and rebounds so that there is a second collision between *P* and *Q*. This second collision takes place at a distance *x* from the wall.

Given that $e = \frac{1}{18}$ and the coefficient of restitution between *Q* and the wall is $\frac{1}{3}$

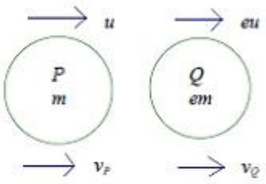
(c) find *x* in terms of *d*.

**(6)**

**(Total for question = 14 marks)**

## <u>Mark Scheme</u> – Elastic Collisions in 1D

**Q1.**

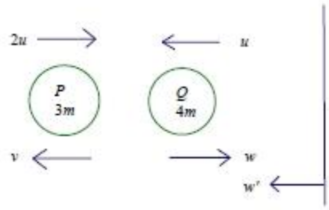
| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) |  | | |
| | Conservation of momentum | M1 | 3.4 |
| | $mu + e^2mu = mv_P + emv_Q$ | A1 | 1.1b |
| | Newton's Impact Law | M1 | 3.4 |
| | $e(u - eu) \quad = -v_P + v_Q$ | A1 | 1.1b |
| | Solve these equations for $v_Q$ | M1 | 3.1a |
| | $v_Q = u$ * | A1* | 1.1b |
| | | **(6)** | |
| (b) | $v_P = u(e^2 - e + 1) \left( = \frac{(e^3+1)u}{e+1} \right)$ | M1 | 1.1b |
| | $= u\left( (e - \frac{1}{2})^2 + \frac{3}{4} \right)$ | A1 | 1.1b |
| | $> 0$ so $P$ continues to move in the same direction * | A1* | 1.1b |
| | | **(3)** | |
| | | **(9)** | |
| (c) | Use impulse-momentum principle | M1 | 3.4 |
| | $I = em(u - eu)$ or $m(-u(e^2 - e + 1) - (-u))$ $\quad (= (e - e^2)mu)$ | A1 | 1.1b |
| | $(e - e^2) = \frac{2}{9}$ and solve | M1 | 1.1b |
| | $e = \frac{1}{3}$ or $\frac{2}{3}$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(13 marks)** |

| Notes: | | |
|---|---|---|
| a | M1 | Correct no. of terms, allow consistent cancelled $m$'s $\left(u+e^2u=v_P+ev_Q\right)$ |
| | A1 | Correct unsimplified equation |
| | M1 | Correct no. of terms, with $e$ on correct side |
| | A1 | Correct unsimplified equation |
| | M1 | Solve for $v_Q$ |
| | A1* | cao |
| b | M1 | Solve for $v_P$ |
| | M1 | Completing the square or any other appropriate method |
| | A1* | Correct conclusion correctly reached |
| c | M1 | Correct no. of terms, dimensionally correct. Must be subtracting. Needs to be in terms of $e$ and $u$. |
| | A1 | Correct unsimplified expressiom (allow -ve answer at this stage) |
| | M1 | Solving an appropriate quadratic equation |
| | A1 | Two correct answers |

**Q2.**

| Question | Scheme | Marks | AOs | Notes |
|---|---|---|---|---|
| a | Speed after first impact $=\frac{2}{3}u$ | B1 | 3.4 | Correct use of impact law, seen or implied. Allow +/- |
| | Speed after second impact $=\frac{4}{9}u$ | B1 | 3.4 | Correct use of impact law a second time, seen or implied. Allow +/- |
| | Correct method for total time | M1 | 2.1 | Use of $t=\frac{d}{v}$ or equivalent for at least 2 of the 3 parts added |
| | $T=\frac{d}{u}+\frac{3}{\frac{2}{3}u}+\frac{3-d}{\frac{4}{9}u}$ | A1ft<br>A1ft | 1.1b<br>1.1b | Unsimplified expression for $T$ with all 3 terms and at most one error. Follow their speeds.<br>Correct unsimplified expression for $T$. Follow their speeds |
| | $=\frac{4d+18+27-9d}{4u}=\frac{45-5d}{4u}$ * | A1* | 2.2a | Obtain **given answer** from correct working |
| | | (6) | | |
| b | • Least $T$ when $d$ is maximum<br>• Furthest distance at highest speed<br>• Highest average speed<br>• Sketch graph of function | B1 | 2.4 | Correct reasoning |
| | i.e. $d=3$, least $T=\frac{30}{4u}=\frac{15}{2u}$ | B1 | 2.2a | Correct answer only. Any equivalent form. $\left(\frac{7.5}{u}\right)$ |
| | | (2) | | |
| | | (8 marks) | | |

**Q3.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| a |  | | |
| | Using CLM: | M1 | 3.4 |
| | $6mu-4mu=-3mv+4mw \quad (2u=-3v+4w)$ | A1 | 1.1b |
| | Use of impact law | M1 | 3.1a |
| | $w+v=e\times 3u$ | A1 | 1.1b |
| | Complete method to find $w$ | M1 | 2.1 |
| | $\begin{cases} 3w+3v=9eu \\ -3v+4w=2u \end{cases} \Rightarrow 7w=9eu+2u, \quad w=\frac{u}{7}(9e+2) \quad *$ | A1* | 2.2a |
| | | **(6)** | |
| b | $w'=\frac{1}{2}\times\frac{u}{7}(9e+2) \quad \left(=\frac{u}{14}(9e+2)\right)$ | B1 | 1.1b |
| | $v=\frac{u}{7}(12e-2)$ | B1 | 1.1b |
| | For a second collision: $w'>v$ | M1 | 3.3 |
| | $9e+2>2(12e-2), \quad 0<e<\frac{2}{5}$ | A1 | 1.1b |
| | | **(4)** | |
| | | **(Total 10 marks)** | |

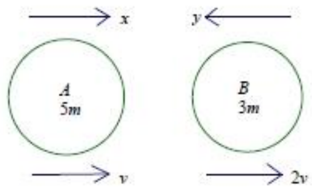
| **Notes** | |
|---|---|
| (a) M1 | Use of CLM. Need all terms. Must be dimensionally correct. Condone sign errors. Accept consistent cancelling of $m$ |
| A1 | Correct unsimplified equation for CLM. They can have $v$ in either direction |
| M1 | Correct use of the impact law (used the right way round) Condone sign errors in finding speed of approach and speed of separation. |
| A1 | Correct unsimplified equation. Signs consistent with equation for CLM. |
| M1 | Complete method to find $w$ e.g. by forming simultaneous equations using CLM and Impact Law and solving. This requires both of the preceding M marks |
| A1* | Obtain **given answer** from correct working. Accept with $2 + 9e$ in place of $9e + 2$ |
| | Check that the answer does follow from the working. |
| (b) B1 | Speed of $Q$ after impact with the wall. Any equivalent form. Correct speed can be implied by a correct negative velocity. |
| B1 | Speed of $P$ after impact with $Q$. Accept $\pm$. Any equivalent form in $u$ and $e$ (seen or implied) |
| M1 | Form correct inequality using their $v$ and $w$'. A correct inequality has $P$ and $Q$ both moving away from the wall |
| A1 | Correct interval only. Accept unsimplified fraction. Need both ends of the interval. Must be strict inequality at both ends. |

**Q4.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Speeds after 1st and 2nd impacts: $eu$ and $e^2u$ | B1 | 3.4 |
| | KE Loss, $K = \frac{1}{2}emu^2 - \frac{1}{2}em(e^2u)^2$ (difference in KE's) | M1 | 3.3 |
| | $\frac{1}{2}mu^2(e - e^5)$ | A1 | 1.1b |
| | | **(3)** | |
| (b) | Differentiate wrt $e$ | M1 | 2.1 |
| | $\frac{\mathrm{d}K}{\mathrm{d}e} = \frac{1}{2}mu^2(1 - 5e^4)$ | A1 | 1.1b |
| | Equate to zero and solve for $e$ | M1 | 3.1a |
| | $e^4 = \frac{1}{5} \Rightarrow e = 0.67$ or better | A1 | 1.1b |
| | | **(4)** | |
| (c) | Particle continues to bounce off each wall (indefinitely). | B1 | 2.4 |
| | Speed of particle decreases oe | B1 | 2.4 |
| | | **(2)** | |
| | | | **(9 marks)** |

| **Notes:** | | |
|---|---|---|
| a | B1 | Need both for the mark |
| | M1 | Allow terms reversed |
| | A1 | cao |
| b | M1 | Clear attempt to differentiate their KE loss, in terms of $e$, wrt $e$, with powers decreasing by 1 |
| | A1 | Correct derivative |
| | | If working from $\frac{1}{2}mu^2(1-e^4)$ allow M1A0 for a correct argument leading to $e=0$ |
| | M1 | Clear attempt to equate to zero |
| | A1 | cao |
| c | B1 | Any clear equivalent statement |
| | B1 | Any clear equivalent statement. Allow speed tends to 0. |

**Q5.**

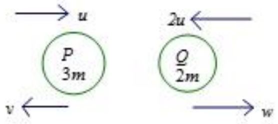
| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) |  | | |
| | Use of CLM | M1 | 3.1a |
| | $5mv + 6mv(= 11mv) = 5mx - 3my \qquad (11v = 5x - 3y)$ | A1 | 1.1b |
| | Use of impact law | M1 | 3.1a |
| | $v = e(x + y)$ | A1 | 1.1b |
| | $\begin{cases} 11ev = 5ex - 3ey \\ 3v = 3ex - 3ey \end{cases} \Rightarrow x = \frac{v}{8e}(11e + 3)$ | M1 | 3.1a |
| | $y = \frac{v}{8e}(5 - 11e)$ | A1 | 1.1b |
| | $e > 0 \quad (\Rightarrow \quad x > 0) \quad \Rightarrow \quad 5 - 11e > 0$ | M1 | 3.4 |
| | $\Rightarrow 0 < e < \frac{5}{11}$ | A1 | 2.2a |
| | | (8) | |

| | | | |
|---|---|---|---|
| (b) | Form equation for KE | M1 | 2.1 |
| | $\frac{1}{2} \times 5m \times v^2 = \frac{16}{100} \times \frac{1}{2} \times 5m \times \frac{v^2}{64e^2}(11e + 3)^2$ | A1ft | 1.1b |
| | $(4(11e + 3) = (\pm)80e) \qquad e = \frac{1}{3}$ | A1 | 1.1b |
| | Impulse $= -5m(v - x)$ | M1 | 3.1a |
| | $= -5m\left(v - \frac{11v}{8} - \frac{3v}{8e}\right)$<br>Or: $3m\left(2v + \frac{5v}{8e} - \frac{11v}{8}\right)$ | A1ft | 1.1b |
| | Magnitude $= \frac{15}{2}mv$ | A1 | 2.2a |
| | | (6) | |
| Alt(b) | Form equation for KE | M1 | 2.1 |
| | $\frac{1}{2} \times 5m \times v^2 = \frac{16}{100} \times \frac{1}{2} \times 5m \times x^2$ | A1 | 1.1b |
| | $\Rightarrow x = \frac{5v}{2}, y = \frac{v}{2} \quad \Rightarrow e = \frac{1}{3}$ | A1 | 1.1b |

| | | | |
|---|---|---|---|
| | Impulse $= -5m(v-x)$ | M1 | 3.1a |
| | $= -5m\left(v - \frac{5v}{2}\right)$<br>Or: $3m\left(2v + \frac{v}{2}\right)$ | A1 | 1,16 |
| | Magnitude $= \frac{15}{2}mv$ | A1 | 2.2a |
| | | (6) | |
| | | | **(14 marks)** |

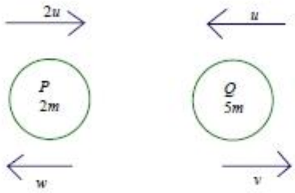
| | |
|---|---|
| **Notes:** | |
| (a)M1 | All terms required. Dimensionally correct. Condone sign errors |
| A1 | Correct unsimplified equation |
| M1 | Used correctly. Condone sign errors |
| A1 | Correct unsimplified equation |
| M1 | Use their correctly formed equations to solve for $v$ or $w$ or a multiple of $v$ or $w$ |
| A1 | Both velocities correct |
| M1 | Use their velocities (in general form – not by considering one specific value) to form inequality for both moving in the same direction. |
| A1 | Correct only. |
| (b)M1 | Dimensionally correct. Condone 16% on wrong side Allow $M$ or $5m$ |
| A1ft | Or equivalent. Correct unsimplified equation. Follow their $x$ Allow $M$ or $5m$ |
| A1 | Correct answer only Allow $M$ or $5m$ |
| M1 | Correct use of $I = mv - mu$. Must be subtracting. |
| A1ft | Accept $\pm$ Follow their $x$, $y$, $e$ |
| A1 | Correct only. Must be positive. |

**Q6.**

| Question | Scheme | Marks | AOs | Notes |
|---|---|---|---|---|
| (a) |  | | | |
| | Use of CLM | M1 | 3.1a | Use of CLM. All terms required. Must be dimensionally correct. Condone sign errors |
| | $3mu - 4mu = 2mw - 3mv$ $(-u = -3v + 2w)$ | A1 | 1.1b | Correct unsimplified equation |
| | Use of impact law | M1 | 3.4 | Use of impact law. Must be dimensionally correct and used correctly. Condone sign errors |
| | $w + v = 3ue$ | A1 | 1.1b | Correct unsimplified equation Signs consistent with CLM equation |
| | Correct strategy to form equation in $w$ and find critical value of $e \in (0,1)$ $(5w = u(9e-1))$ | M1 | 3.1a | Correct overall strategy to find the critical value of $e$ in $(0,1)$ in $e$ eg by using CLM and impact law to form equation or inequality in $w$ and solve for $e$. |
| | $w > 0: e > \frac{1}{9}$ | A1 | 1.1b | One inequality for $e$ correct Condone $e \geq \frac{1}{9}$ |
| | Complete strategy to justify the range of values of $e(5v = u(1+6e))$ $v > 0$: true for all $e$ | M1 | 3.1a | Correct strategy to find the range of possible value of $e$. i.e find second speed and form second inequality |
| | Therefore $\frac{1}{9} < e \leq 1$ | A1 | 2.2a | Correct final conclusion |
| | | **(8)** | | |

| Question | Scheme | Marks | AOs | Notes |
|---|---|---|---|---|
| (b) | Final KE = 25% of initial KE | M1 | 3.1a | Use KE to form equation in $e$. 25% should be used correctly Condone if mass cancelled throughout |
| | $\frac{1}{2} \times 2m \times \frac{u^2(9e-1)^2}{25} = \frac{1}{4} \times \frac{1}{2} \times 2m \times 4u^2$ $\left(\text{or } w = \frac{1}{2} \times 2u\right)$ | A1ft | 1.1b | Correct unsimplified equation – follow their $w$ |
| | $\Rightarrow (9e-1)^2 = 25,\ e = \frac{2}{3}$ only | A1 | 1.1b | Or equivalent. Correct conclusion ISW after correct answer. |
| | | **(3)** | | |
| | | **(11marks)** | | |

**Q7.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) |  | | |
| | Complete overall strategy to find $v$ | M1 | 3.1a |
| | Use of CLM | M1 | 3.1a |
| | $2m\times 2u - 5m\times u = 5m\times v - 2m\times w$ , $(-u = 5v - 2w)$ | A1 | 1.1b |
| | Use of Impact law | M1 | 3.1a |
| | $v + w = e(2u + u)$ | A1 | 1.1b |
| | Solve for $v$: $-u = 5v - 2w$<br>$6eu = 2v + 2w$ | | |
| | $7v = u(6e-1)$ $\left(v = \frac{u}{7}(6e-1)\right)$ | A1 | 1.1b |
| | Direction of $Q$ reversed: $v > 0$ | M1 | 3.4 |
| | $\Rightarrow 1 \ge e > \frac{1}{6}$ | A1 | 1.1b |
| | | (8) | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (b) | $e = \frac{1}{3} \Rightarrow v = \frac{u}{7}$ , $w = \frac{6u}{7}$ | B1 | 2.1 |
| | Equation for KE lost | M1 | 2.1 |
| | $\frac{1}{2}\times 2m\left(4u^2 - \frac{36u^2}{49}\right) + \frac{1}{2}\times 5m\left(u^2 - \frac{u^2}{49}\right)$ | A1<br>A1 | 1.1b<br>1.1b |
| | $\frac{1}{2}mu^2\left(8 - \frac{72}{49} + 5 - \frac{5}{49}\right) = \frac{40mu^2}{7}$ * | A1* | 2.2a |
| | | (5) | |
| (c) | Increase $e \Rightarrow$ more elastic $\Rightarrow$ less energy lost | B1 | 2.2a |
| | | (1) | |
| | | | (14 marks) |

| Question continued | |
|---|---|
| **Notes:** | |
| (a) | |
| M1: | Complete strategy to form sufficient equations in $v$ and $w$ and solve for $v$. |
| M1: | Use CLM to form equation in $v$ and $w$. |
| | Needs all 4 terms & dimensionally correct |
| A1: | Correct unsimplified equation |
| M1: | Use NEL as a model to form a second equation in $v$ and $w$. Must be used the right way round |
| A1: | Correct unsimplified equation |
| A1: | for $v$ or $7v$ correct |
| M1: | Use the model to form a correct inequality for their $v$ |
| A1: | Both limits required |
| (b) | |
| B1: | or equivalent statements |
| M1: | terms of correct structure combined correctly |
| A1: | Fully correct unsimplified A1A1 |
| | One error on unsimplified expression A1A0 |
| A1*: | cso. plus a 'statement' that the required result has been achieved |
| (c) | |
| B1: | "less energy lost" or equivalent |

**Q8.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Using the model and $v^2 = u^2 + 2as$ to find $v$ | M1 | 3.4 |
| | $v^2 = 2as = 2g \leftarrow 2.4 = 4.8g \Rightarrow v = \sqrt{(4.8g)}$ | A1 | 1.1b |
| | Using the model and $v^2 = u^2 + 2as$ to find $u$ | M1 | 3.4 |
| | $0^2 = u^2 - 2g \leftarrow 0.6 \Rightarrow u = \sqrt{(1.2g)}$ | A1 | 1.1b |
| | Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR | M1 | 3.1b |
| | $e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ * | A1 * | 1.1b |
| | | **(6)** | |
| (b) | Using the model and $e$ = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$ | M1 | 3.4 |
| | Using the model and $v^2 = u^2 + 2as$ | M1 | 3.4 |
| | $0^2 = 0.25\,(1.2g) - 2gh \Rightarrow h = 0.15$ (m) | A1 | 1.1b |
| | | **(3)** | |
| (c) | Ball continues to bounce with the height of each bounce being a quarter of the previous one. | B1 | 2.2b |
| | | **(1)** | |
| | | | **(10 marks)** |

| Notes: | |
|---|---|
| (a) | |
| M1: | for a complete method to find $v$ |
| A1: | for a correct value (may be numerical) |
| M1: | for a complete method to find $u$ |
| A1: | for a correct value (may be numerical) |
| M1: | for finding both $v$ and $u$ and use of Newton's Law of Restitution |
| A1*: | for the given answer |
| (b) | |
| M1: | for use of Newton's Law of Restitution to find rebound speed |
| M1: | for a complete method to find $h$ |
| A1: | for 0.15 (m) oe |
| (c) | |
| B1: | for a clear description including reference to a quarter |

**Q9.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Use NEL to find the speed of particle after the first impact $= eu = \frac{3}{4}u$ | B1 | 3.4 |
| | Impulse $= \lambda mu = mv - mu = \pm\left[\frac{3}{4}mu - (-mu)\right]$ | M1 | 3.1b |
| | $\lambda = \frac{7}{4}$ | A1 | 1.1b |
| | | (3) | |
| (b) | Use NEL to find the speed of the particle after the second impact $= \frac{3}{4} \times \frac{3}{4}u = \frac{9}{16}u$ | B1 | 3.4 |
| | Use of $s = vt$ to find total time | M1 | 3.1b |
| | $7 = \frac{2}{u} + \frac{4}{\frac{3}{4}u} + \frac{2}{\frac{9}{16}u}\left(= \frac{2}{u} + \frac{16}{3u} + \frac{32}{9u}\right)$ | A1 | 1.1b |
| | Solve for $u$: $63u = 18 + 48 + 32$ | M1 | 1.1b |
| | $u = \frac{98}{63} = \frac{14}{9}\left(= 1.\dot{5}\right)$ | A1 | 1.1b |
| | | (5) | |
| | | | **(8 marks)** |

| **Notes:** | |
|---|---|
| (a) | |
| **B1:** | Using Newton's experimental law as a model to find the speed after the first impact |
| **M1:** | Must be a difference of two terms, taking account of the change in direction of motion. |
| **A1:** | cao |
| (b) | |
| **B1:** | Using NEL as a model to find the speed after the second impact. |
| **M1:** | Needs to be used for at least one stage of the journey |
| **A1:** | or equivalent |
| **M1:** | Solve their linear equation for $u$ |
| **A1:** | Accept 1.56 or better |

**Q10.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Use of conservation of momentum | M1 | 3.1a |
| | $3mu - 2mu = 3mv + mw$ | A1 | 1.1b |
| | Use of NLR | M1 | 3.1a |
| | $3ue = -v + w$ | A1 | 1.1b |
| | Using a correct strategy to solve the problem by setting up two equations (need both) in $u$ and $v$ and solving for $v$ | M1 | 3.1b |
| | $v = \frac{u}{4}(1 - 3e)$ | A1 | 1.1b |
| | | (6) | |
| (b) | $\frac{u}{4}(1 - 3e) < 0$ | M1 | 3.1b |
| | $\frac{1}{3} < e \leq 1$ | A1 | 1.1b |
| | | (2) | |
| (c) | Solving for $w$ | M1 | 2.1 |
| | $w = \frac{u}{4}(1 + 9e)$ * | A1 * | 1.1b |
| | | (2) | |
| (d) | Substitute $e = \frac{5}{9}$ | M1 | 1.1b |
| | $v = \frac{-u}{6}, w = \frac{3u}{2}$ | A1 | 1.1b |
| | Use NLR for impact with wall, $x = fw$ | M1 | 1.1b |
| | Further collision if $x > -v$ | M1 | 3.4 |
| | $f\frac{3u}{2} > \frac{u}{6}$ | A1 | 1.1b |
| | $1 \geq f > \frac{1}{9}$ | A1 | 1.1b |
| | | (6) | |
| | | | **(16 marks)** |

| Notes: | |
|---|---|
| (a) | |
| M1: | for use of CLM, with correct no. of terms, condone sign errors |
| A1: | for a correct equation |
| M1: | for use of Newton's Law of Restitution, with $e$ on the correct side |
| A1: | for a correct equation |
| M1: | for setting *two* equations and solving their equations for $v$ |
| A1: | for a correct expression for $v$ |

| | |
|---|---|
| (b) | |
| M1: | for use of an appropriate inequality |
| A1: | for a complete range of values of $e$ |
| (c) | |
| M1: | for solving their equations for $w$ |
| A1: | for the given answer |
| (d) | |
| M1: | for substituting $e = 5/9$ into their $v$ and $w$ |
| A1: | for correct expressions for $v$ and $w$ |
| M1: | for use of Newton's Law of Restitution, with $e$ on the correct side |
| M1: | for use of appropriate inequality |
| A1: | for a correct inequality |
| A1: | for a correct range |

**Q11.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $2u \rightarrow$    $0$ <br> $P\ (2m)$    $Q(3m)$ <br> $w \leftarrow$    $\rightarrow v$ | | |
| | Use of CLM | M1 | 3.4 |
| | $2m \times 2u = -2mw + 3mv$ | A1 | 1.1b |
| | Use of NEL | M1 | 3.4 |
| | $2ue = w + v$ | A1 | 1.1b |
| | Solve for $v$ | D M1 | 1.1b |
| | $v = \dfrac{4u(1+e)}{5}$ * | A1* | 2.2a |
| | | (6) | |
| (b) | Since $0 \le e \le 1$, $\dfrac{4u(1+0)}{5} \le v \le \dfrac{4u(1+1)}{5}$ | M1 | 3.1a |
| | i.e. $\dfrac{4u}{5} \le v \le \dfrac{8u}{5}$ * | A1* | 2.2a |
| | | (2) | |

| | | | |
|---|---|---|---|
| (c) | Solve for $w$ | M1 | 1.1b |
| | $w = \frac{2u(3e-2)}{5}$ oe $(\mathrm{m\,s^{-1}})$ or $\left\lvert\frac{2u(2-3e)}{5}\right\rvert$ oe | A1 | 1.1b |
| | | **(2)** | |
| (d) | Speed of $Q$ after hitting the wall $= \frac{1}{6}v$ $(\mathrm{m\,s^{-1}})$ | M1 | 3.4 |
| | For a further collision between $P$ and $Q$, $\frac{1}{6}v > w$ | M1 | 3.1a |
| | Substitute for $v$ and $w$ and solve for $e$ | M1 | 1.1b |
| | $e < \frac{7}{8}$ | A1 | 1.1b |
| | $\frac{2}{3} < e < \frac{7}{8}$ | A1 | 1.1b |
| | | **(5)** | |
| | | | **(15 marks)** |

**Notes:**

| | | |
|---|---|---|
| a | M1 | Correct no. of terms, condone sign errors, allow consistently cancelled $m$'s or extra $g$'s or common factors throughout |
| | A1 | Correct equation; they may have $w$ instead of $-w$ |
| | M1 | Correct no. of terms, condone sign errors. M0 if $e$ on the wrong side of the equation |
| | A1 | Correct equation; they may have $w$ instead of $-w$ |
| | DM 1 | Solve for $v$, dependent on previous two marks |
| | A1* | Correct answer correctly obtained |
| b | M1 | Use of $0 \le e \le 1$ in the given answer; allow use of $e = 0$ and $e = 1$ to obtain the min and max expressions<br>M1A0 for 'verification'. |
| | A1* | Correct answer correctly obtained (including use of max and min) |
| c | M1 | Solve for their $w$ |
| | A1 | cao |
| d | M1 | Speed so must see a positive quantity<br>M0 if $\frac{1}{6}$ is on the wrong side of the equation |
| | M1 | Correct inequality for their $w$ (allow even if their $w$ is dimensionally incorrect) |
| | M1 | Independent M mark but must have an inequality in $v$ and $w$:<br>Substitute for $v$, using given answer, and $w$ and solve for $e$ |
| | A1 | Correct upper bound for $e$ |
| | A1 | cao |

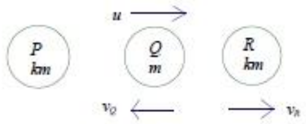
**Q12.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $2u \rightarrow$ $\quad \leftarrow u$<br>$\frac{9mu}{2} \longleftarrow m \quad 3m \longrightarrow \frac{9mu}{2}$<br>$v \leftarrow$ $\quad \rightarrow w$ | | |
| | Use of Impulse-momentum principle for $A$ or $B$ | M1 | 3.4 |
| | $A$: $\frac{9mu}{2} = m(v - -2u)$ **or** $B$: $\frac{9mu}{2} = 3m(w - -u)$ | A1 | 1.1b |
| | Use of Impulse-momentum principle for $B$ or $A$ or CLM | M1 | 3.4 |
| | $\frac{9mu}{2} = 3m(w - -u)$ **or** $\frac{9mu}{2} = m(v - -2u)$ **or**<br>$2mu - 3mu = -mv + 3mw$ | A1 | 1.1b |
| | $v = \frac{5u}{2}$ and $w = \frac{u}{2}$ | A1 | 1.1b |
| | $e = \dfrac{\frac{5u}{2} + \frac{u}{2}}{2u + u}$ | M1 | 3.1a |
| | $e = 1$ | A1cso | 1.1b |
| | **ALTERNATIVE:** | | |
| | NEL is written down before $v$ and $w$ are found: $v + w = 3ue$ | 3rd M1 | |
| | Use of Impulse-momentum principle for $A$ or $B$ | 1st M1 | |
| | $A$: $\frac{9mu}{2} = m(v - -2u)$ **or** $B$: $\frac{9mu}{2} = 3m(w - -u)$ | 1st A1 | |
| | Use of Impulse-momentum principle for $B$ or $A$ or CLM | 2nd M1 | |
| | $\frac{9mu}{2} = 3m(w - -u)$ **or** $\frac{9mu}{2} = m(v - -2u)$ **or**<br>$2mu - 3mu = -mv + 3mw$ | 2nd A1 | |
| | An equation (not an identity) in $u$ and $e$ only is produced | 3rd A1 | |
| | $e = 1$ | A1cso | |
| | | (7) | |

| | | | |
|---|---|---|---|
| (b) | Perfectly elastic (or the coefficient of restitution is 1) so no loss in kinetic energy.<br>Allow a direct evaluation of the KE loss i.e.<br>$\frac{1}{2}m(2u)^2 + \frac{1}{2} \times 3mu^2 - \left(\frac{1}{2}m\left(\frac{5u}{2}\right)^2 + \frac{1}{2} \times 3m\left(\frac{u}{2}\right)^2\right) = 0$<br>B0 if incorrect extras | DB1 | 2.4 |
| | | (1) | |
| | | **(8 marks)** | |

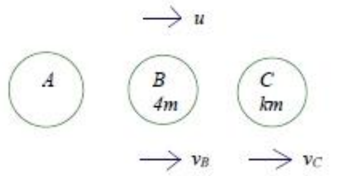
| | | |
|---|---|---|
| **Notes:** | | |
| **N.B. Ignore diagrams if it helps the candidate.**<br>**Equations need to be consistent, where appropriate, to earn A marks.** | | |
| a | M1 | Use of Impulse-momentum principle for $A$ or $B$, condone sign errors but M0 if dimensionally incorrect e.g. if $m$ missing |
| | A1 | Correct unsimplified equation |
| | M1 | Use of Impulse-momentum principle for other particle or CLM, condone sign errors but M0 if dimensionally incorrect e.g. if $m$ missing from impulse<br>For CLM, allow consistent missing $m$'s or extra $g$'s. |
| | A1 | Correct unsimplified equation |
| | A1 | Cao for both. Allow one or both negative if correct for their symbols. |
| | M1 | Use of NEL to obtain $e = \ldots$, condone sign errors in numerator but must be terms in $u$ only AND must be $(2u + u)$ in denominator.<br>M0 if inverted |
| | A1 | cso |
| b | DB1 | Dependent on $e = 1$ **correctly** obtained in (a)<br>A correct statement e.g. zero, 0 etc and a correct reason |

**Q13.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| |  | | |
| | Use of conservation of momentum | M1 | 3.1a |
| | $mu = -mv_Q + kmv_R$ | A1 | 1.1b |
| | Use of NLR | M1 | 3.4 |
| | $eu = v_Q + v_R$ | A1 | 1.1b |
| | Using correct strategy to solve problem by finding $v_Q$ | M1 | 3.1a |
| | $v_Q = \frac{u(ke-1)}{k+1}$ or $v_Q = \frac{v_R(ke-1)}{1+e}$ | A1 | 1.1b |
| | For second collision, $v_Q > 0$ | M1 | 3.1a |
| | $\frac{u(ke-1)}{k+1} > 0$ | M1 | 1.1b |
| | $k > \frac{1}{e}$ | A1 | 1.1b |
| | | **(9)** | |
| **(b)** | $\frac{u(ke-1)^2}{(k+1)^2}$ | B1 | 2.2a |
| | | **(1)** | |
| | | | **(10 marks)** |

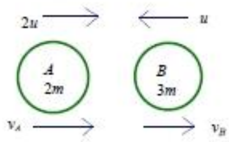
| **Notes** | | |
|---|---|---|
| **(a)** | M1 | Correct no. of terms and dimensionally correct but condone sign errors |
| | A1 | Correct equation |
| | M1 | Use of NLR with $e$ on the correct side |
| | A1 | Correct equation (any equivalent form)<br>Signs consistent with CLM equation |
| | M1 | Solving for $v_Q$ - complete correct strategy (i.e. correct use of CLM and of NLR) |
| | A1 | Correct expression for their $v_Q$<br>Can be implied by a correct multiple of $v_Q$ |
| | M1 | Use of appropriate condition for their $v_Q$ |
| | M1 | Complete correct strategy to find values for $k$ (i.e. set up and solve inequality) |
| | A1 | cso |
| **(b)** | B1 | Or equivalent cao |

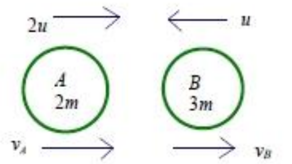
**Q14.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| a |  | | |
| | Use of CLM | M1 | 3.1a |
| | $4mu = 4mv_B + kmv_C$ | A1 | 1.1b |
| | Use of NLR | M1 | 3.1a |
| | $\frac{1}{4}u = -v_B + v_C$ | A1 | 1.1b |
| | Solve for $v_B$ | M1 | 1.1b |
| | $v_B = \frac{u(16-k)}{4(k+4)}$ $\quad \left(v_C = \frac{5u}{k+4}\right)$ | A1 | 1.1b |
| | Use of $v_B \geq 0$ and solve for $k$ | M1 | 3.4 |
| | $(0 <) k \leq 16$ | A1 | 1.1b |
| | Alternative for last 4 marks | | |
| | Solve for $v_B$ in terms of $v_C$ only | M1 | |
| | $v_B = \frac{(16-k)v_C}{20}$ | A1 | |
| | Use of $v_B \geq 0$ and $v_C > 0$ to solve for $k$ | M1 | |
| | $(0 <) k \leq 16$ | A1 | |
| | | (8) | |
| b | Impulse-momentum equation | M1 | 3.1a |
| | $-3mu = 4m(v_B - u)$ $\quad \left(v_B = \frac{u}{4}\right)$ $\quad$ or $3mu = kmv_C$ | A1 | 1.1b |
| | Complete method to solve for $k$ | M1 | 1.1b |
| | $k = 6$ | A1 | 2.2a |
| | | (4) | |
| | | **(12 marks)** | |

| Notes | | |
|---|---|---|
| a | M1 | Correct no. of terms, condone extra $g$ s, sign errors |
| | A1 | Correct equation |
| | M1 | $e$ must be on correct side |
| | A1 | Correct equation |
| | M1 | Complete method to solve for $v_B$ (or a multiple of $v_B$ ) |
| | A1 | Correct expression for *their* $v_B$ or a multiple of *their* $v_B$ |
| | M1 | Use of appropriate inequality, allow strict inequality for method mark |
| | A1 | Cao LHS not needed, but if there it must be correct. |
| b | M1 | Correct no. of terms, condone sign errors, but must be subtracting momentum terms |
| | A1 | Correct equation |
| | M1 | Eliminate and solve for $k$ |
| | A1 | $k = 6$ |

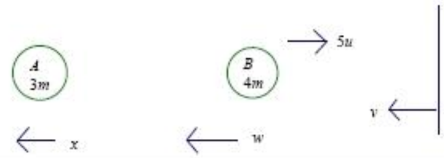
**Q15.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Using the Impulse-momentum principle for $B$ | M1 | 3.1a |
| | $5mu == 3m(v_B - -u)$ | A1 | 1.1b |
| | $v_B = \frac{2u}{3}$ | A1 | 1.1b |
| | Use of conservation of momentum | M1 | 3.1a |
| | $4mu - 3mu = 2mv_A + 3mv_B \left( = 2mv_A + 3m.\frac{2u}{3} \right)$ | A1ft | 1.1b |
| | $v_A = -\frac{u}{2}$ | A1 | 1.1b |
| | Use of NLR | M1 | 3.4 |
| | $e = \frac{v_B - v_A}{2u + u} \left( = \frac{\frac{u}{2} + \frac{2u}{3}}{2u + u} \right)$ | A1ft | 1.1b |
| | $e = \frac{7}{18} = 0.39$ or better | A1 | 1.1b |
| |  | | |
| | | **(9)** | |
| (b) | KE Loss = Initial KE − Final KE | M1 | 2.1 |
| | $= \frac{1}{2}.2m(2u)^2 + \frac{1}{2}.3mu^2 - \left( \frac{1}{2}.2m\left(-\frac{u}{2}\right)^2 + \frac{1}{2}.3m\left(\frac{2u}{3}\right)^2 \right)$ | A1ft<br>A1ft | 1.1b<br>1.1b |
| | $= \frac{55mu^2}{12}$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(13 marks)** |

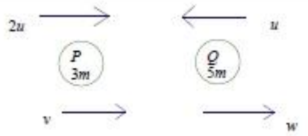
| | | Notes |
|---|---|---|
| (a) | M1 | Correct no. of terms and dimensionally correct but condone sign errors but must be a difference of momenta |
| | A1 | Correct unsimplified equation |
| | A1 | Correct appropriate velocity |
| | M1 | Use of CLM with correct no. of terms and dimensionally correct but condone sign errors **Alternative**: Use Impulse - momentum for $A$ |
| | A1ft | Correct unsimplified CLM equation<br>**Or:** $-5mu = 2m(v_A - 2u)$ |
| | A1 | Correct speed |
| | M1 | Use of NLR with $e$ on the correct side |
| | A1ft | Correct unsimplified equation |
| | A1 | Correct answer |
| **ALT** | | <br>Could find $v_A$ before $v_B$ :<br>M1A1A1 for first velocity, M1A1A1 for second<br>M1A1A1 for $e$ found correctly |
| | | Candidates are approaching this in many different ways.<br>They need<br>- two of momentum impulse equation for each particle and CLM<br>- impact law<br>M1A1 for each correct equation (in the order seen)<br>Of the remaining 3 A marks,<br>A1 for a correct expression for $v_A$ or $v_B$<br>A1 for a correct expression in $e$<br>A1 for the correct answer |

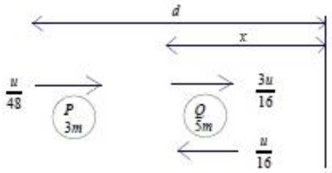
| | | |
|---|---|---|
| e.g | M1A1 | CLM: $4mu - 3mu = 2mv_A + 3mv_B$ |
| | M1A1 | Impact: $v_B - v_A = 3ue$ |
| | A1 | $v_B = \frac{u}{5}(1+6e)$ *or* $v_A = \frac{u}{5}(1-9e)$ |
| | M1A1 | $5mu = 3m(v_B - (-u))$ $\left(= 3m\left(\frac{u}{5}(1+6e)+u\right)\right)$<br>Or $-5mu = 2m(v_A - 2u)$ $\left(= 2m\left(\frac{u}{5}(1-9e)-2u\right)\right)$ |
| | A1 | $5 = 3\left(\frac{1}{5}(1+6e)+1\right)$ or $-5 = 2\left(\frac{1}{5}(1-9e)-2\right)$ |
| | A1 | $e = \frac{7}{18} = 0.39$ or better |
| **(b)** | M1 | Correct no. of terms and must be a difference.<br>Must be dimensionally correct at the point when they state their expression for the loss (change) in KE |
| | A1ft | Unsimplified expression in $u$ with at most 1 error, ft on their speeds from (a) |
| | A1ft | Correct unsimplified expression in $u$. (These first 3 marks can be scored for a correct loss or gain in KE), ft on their speeds from (a) |
| | A1 | cso Accept $4.58mu^2$ or $4.6mu^2$ |

**Q16.**

| Q. | Scheme | Marks | Notes |
|---|---|---|---|
| a |  | | |
| | Impact with wall: $v=\frac{3}{5}\times 5u=3u$ | B1 | or $-3u$ |
| | Impulse $\pm 4m\left(3u-(-5u)\right)$ | M1 | M0 if clearly using $mv+mu$, otherwise bod |
| | Magnitude $32mu$ (Ns) | A1 | |
| | | (3) | |
| b | CLM: $3mx+4mw=4m\times 3u$ | M1 | Need all 4 terms. Condone sign errors.<br>Use of $5u$ is M0 |
| | | A1ft | follow their $3u$ |
| | Impact: $x-w=e\times 3u$ | M1 | Used the right way round. Use of $5u$ is M0 |
| | | A1ft | follow their $3u$<br>signs consistent with CLM equation |
| | $3m(w+3eu)+4mw=7mw+9emu=12mu$ | | |
| | $7w=u(12-9e)$ | DM1 | Solve for $w$ or $kw$.<br>Dependent on two preceding M marks |
| | Use of $e\le 1$ in their $w$: $7w\ge 3u$ | M1 | Condone use of $<$ |
| | Hence $w>0$ and $A$ and $B$ are moving in the same direction | A1<br>(7) | Complete argument leading to ***given answer*** |
| c | KE of $B$ before collision<br>$=\frac{1}{2}\times 4m\times(3u)^2\left(=18mu^2\right)$ | B1 | follow their $3u$. seen or implied |
| | $\Rightarrow\frac{1}{2}\times 4m\left(\frac{u}{7}(12-9e)\right)^2=\frac{1}{4}\left(\frac{1}{2}\times 4m\times 9u^2\right)$ | M1 | Follow their $w$. $\frac{1}{4}$ on the right side. |
| | $4(12-9e)^2=49\times 9,\ (4-3e)^2=\frac{49}{4}$ | A1 | Correct equation in $m$, $u$ and $e$ |
| | $e=\frac{1}{6}$ | A1<br>(4) | |
| c alt | KE of $B$ before collision<br>$=\frac{1}{2}\times 4m\times(3u)^2\left(=18mu^2\right)$ | B1 | follow their $3u$ |
| | $\Rightarrow\frac{1}{2}4mw^2=\frac{1}{4}\times\frac{1}{2}\times 4m(3u)^2 \quad \left(w=\frac{1}{2}\times 3u\right)$ | M1 | $\frac{1}{4}$ on the right side. |
| | $\frac{3}{7}(4-3e)=\frac{1}{2}\times 3$ | A1 | Correct equation in $m$, $u$ and $e$ from correct work only |
| | $e=\frac{1}{6}$ | A1<br>(4) | 0.17 or better from correct work only |
| | | **[14]** | |

**Q17.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | Complete strategy to find speed of $Q$ | M1 | 3.1b |
| |  | | |
| | Use of CLM | M1 | 3.1a |
| | $6mu - 5mu(= mu) = 3mv + 5mw$ | A1 | 1.1b |
| | Use of impact law | M1 | 3.1a |
| | $w - v = 3ue$ | A1 | 1.1b |
| | $\left.\begin{array}{l} 3v+5w=u \\ 3w-3v=9ue \end{array}\right\} \Rightarrow 8w = u + 9ue\ ,\quad w = \frac{u}{8}(9e+1)$ * | A1* | 2.1 |
| | | (6) | |
| (b) | $v = w - 3ue = \frac{u}{8}(1-15e)$ and $v > 0$ | M1 | 3.1b |
| | $\Rightarrow\ (0 \le)e < \frac{1}{15}$ | A1 | 1.1b |
| | | (2) | |

| | | | |
|---|---|---|---|
| (c) | Complete strategy to find time for $Q$ to get to second collision | M1 | 3.1a |
| | Speed of $Q$ after impact with wall $= \frac{u}{16}$ | B1 | 1.1b |
| |  | | |
| | Time for Q: $\frac{16d}{3u} + \frac{16x}{u}$ follow their $\frac{u}{16}$ and $\frac{16d}{3u}$ | A1ft | 1.1b |
| | Complete strategy to find time for $P$ to get to second collision $= \frac{48(d-x)}{u}$ | B1ft | 1.1b |
| | Use both at the same place at the same | M1 | 2.1 |
| | $x = \frac{128d}{192} = \frac{2d}{3}$ | A1 | 1.1b |
| | | (6) | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (c) alt | Complete strategy to find position of second collision | M1 | 3.1a |
| | Speed of $Q$ after impact with wall $= \frac{u}{16}$ | B1 | 1.1b |
| | Distance apart when Q strikes the wall $= \frac{8d}{9}$ | B1ft | 1.1b |
| | Gap closing at $\frac{u}{16} + \frac{u}{48}$ | A1ft | 1.1b |
| | $t = \frac{\frac{8d}{9}}{\frac{u}{16}+\frac{u}{48}} \quad \left(= \frac{32d}{3u}\right)$ | M1 | 2.1 |
| | $x = \frac{u}{16} \times \frac{32d}{3u} = \frac{2d}{3}$ | A1 | 1.1b |
| | | (6) | |
| (c) alt | Complete strategy to find position of second collision | M1 | 3.1a |
| | Speed of $Q$ after impact with wall $= \frac{u}{16}$ | B1 | 1.1b |
| | Distance apart when $Q$ strikes the wall $= \frac{8d}{9}$ | B1ft | 1.1b |
| | Ratio of speeds: $v_Q : v_P = 3:1$ | A1ft | 1.1b |
| | Distance travelled by $Q$ $= \frac{3}{4} \times \frac{8d}{9}$ | M1 | 2.1 |
| | $x = \frac{2d}{3}$ | A1 | 1.1b |
| | | (6) | |
| | | | (14 marks) |

| Notes |
|---|
| (a) **M1:** Complete strategy e.g. use of CLM, impact law and solution of simultaneous equations.<br>**M1:** CLM equation. Requires all terms and dimensionally correct. Condone sign errors.<br>**A1:** Correct unsimplified equation<br>**M1:** Impact law. Condone sign error. Must be used the right way round.<br>**A1:** Correct unsimplified equation<br>Signs consistent with CLM equation.<br>**A1*:** Obtain **given answer** from correct working |
| (b) **M1:** Find speed of $P$ and form correct inequality consistent with their directions.<br>**A1:** Correct solution. Need not mention the lower limit. |
| (c) **M1:** Complete strategy e.g. find time to wall and back again<br>**B1:** Correct use of impact law<br>**A1ft:** Correct unsimplified equation using $\text{time} = \frac{\text{distance}}{\text{speed}}$ and following their $\frac{u}{16}$ and $\frac{16d}{3u}$<br>**B1ft:** Correct use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ Follow their $\frac{u}{48}$<br>**M1:** find $x$ by putting both particles in the same place at the same time. Must be valid expressions for the times.<br>**A1:** Correct answer or exact equivalent |
| (c) alt **M1:** e.g. by considering distances and relative velocities<br>**B1:** Correct use of impact law<br>**B1ft:** Follow their $\frac{u}{48}$ and $\frac{3u}{16}$<br>**A1ft:** Follow their $\frac{u}{16}$ and $\frac{u}{48}$<br>**M1:** Correct use of $\text{time} = \frac{\text{distance}}{\text{speed}}$<br>**A1:** Correct answer |
| (c) alt **M1:** e.g. by considering distances and relative velocities<br>**B1:** Correct use of impact law<br>**B1ft:** Follow their $\frac{u}{48}$ and $\frac{3u}{16}$<br>**A1ft:** Follow their $\frac{u}{16}$ and $\frac{u}{48}$<br>**M1:** Correct use of ratio to find $x$<br>**A1:** Correct answer |