Continuous Distributions

Questions

Q1.

Fence panels come in two sizes, large and small. The lengths of the large panels are normally distributed with mean 198 cm and standard deviation 5 cm. The lengths of the small panels are normally distributed with mean 74 cm and standard deviation 3 cm.

(a) Find the probability that the total length of a random sample of 3 large panels is greater than the total length of a random sample of 8 small panels.

(6)

One large panel and one small panel are selected at random

(b) Find the probability that the length of the large panel is more than $\frac{8}{3}$ times the length of the small panel.

(5)

Rosa needs 1000 cm of fencing. The large panels cost £80 each and the small panels cost £30 each. Rosa's plan is to buy 5 large panels and measure the total length. If the total length is less than 1000 cm she will then buy one small panel as well.

(c) Calculate whether or not the expected cost of Rosa's plan is cheaper than simply buying 14 small panels.

(6)

(Total for question = 17 marks)

Q2.

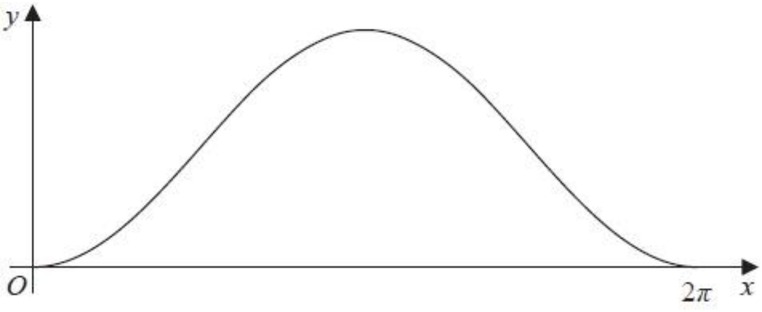


Figure 1

The random variable X has probability density function f (x) and Figure 1 shows a sketch of f (x) where

$$f(x) = \begin{cases} k(1 - \cos x) & 0 \leqslant x \leqslant 2\pi \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \dfrac{1}{2\pi}$

(3)

The random variable $Y \sim N(\mu, \sigma^2)$ and $E(Y) = E(X)$

The probability density function of Y is g (y), where

$$g(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} \qquad -\infty < y < \infty$$

Given that g (μ) = f (μ)

(b) find the exact value of σ

(3)

(c) Calculate the error in using $P\left(\frac{\pi}{2} < Y < \frac{3\pi}{2}\right)$ as an approximation to $P\left(\frac{\pi}{2} < X < \frac{3\pi}{2}\right)$

(4)

(Total for question = 10 marks)

Q3.

A circle, centre *O*, has radius *x* cm, where *x* is an observation from the random variable *X* which has a rectangular distribution on [0, π]

(a) Find the probability that the area of the circle is greater than 10 cm^2

(3)

(b) State, giving a reason, whether the median area of the circle is greater or less than 10 cm^2

(1)

The triangle *OAB* is drawn inside the circle with OA and OB as radii of length *x* cm and angle *AOBx* radians.

(c) Use algebraic integration to find the expected value of the area of triangle OAB.

Give your answer as an exact value.

(7)

(Total for question = 11 marks)

Q4.

The continuous random variable *X* has a probability density function

$$f(x) = \begin{cases} k(x-2) & 2 \leqslant x \leqslant 3 \\ k & 3 < x < 5 \\ k(6-x) & 5 \leqslant x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

where *k* is a positive constant.

(a) Sketch the graph of f (*x*).

(2)

(b) Show that the value of *k* is $\frac{1}{3}$

(2)

(c) Define fully the cumulative distribution function F(*x*).

(7)

(d) Hence find the 90th percentile of the distribution.

(3)

(e) Find P[E(*X*) < *X* < 5.5]

(2)

(Total for question = 16 marks)

Q5.

The lifetime, X, in tens of hours, of a battery is modelled by the probability density function

$$f(x) = \begin{cases} \frac{1}{9}x(4-x) & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

Use algebraic integration to find

(a) E(X)

(4)

(b) P(X > 2.5)

(3)

A radio runs using 2 of these batteries, both of which must be working. Two fully-charged batteries are put into the radio.

(c) Find the probability that the radio will be working after 25 hours of use.

(2)

Given that the radio is working after 16 hours of use,

(d) find the probability that the radio will be working after being used for another 9 hours.

(3)

(Total for question = 12 marks)

Q6.

The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x \leqslant 0 \\ k\left(x^3 - \frac{3}{8}x^4\right) & 0 < x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{2}$

(1)

(b) Showing your working clearly, use calculus to find

(i) E(X)
(ii) the mode of X

(6)

(c) Describe, giving a reason, the skewness of the distribution of *X*

(1)

(Total for question = 8 marks)

Q7.

Lloyd regularly takes a break from work to go to the local cafe. The amount of time Lloyd waits to be served, in minutes, is modelled by the continuous random variable *T*, having probability density function

$$\mathrm{f}(t) = \begin{cases} \dfrac{t}{120} & 4 \leqslant t \leqslant 16 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the cumulative distribution function is given by

$$\mathrm{F}(t) = \begin{cases} 0 & t < 4 \\ \dfrac{t^2}{240} - c & 4 \leqslant t \leqslant 16 \\ 1 & t > 16 \end{cases}$$

where the value of *c* is to be found.

(2)

(b) Find the exact probability that the amount of time Lloyd waits to be served is between 5 and 10 minutes.

(2)

(c) Find the median of *T*.

(2)

(d) Find the value of *k* such that

$$\mathrm{P}(T < k) = \frac{2}{3}\,\mathrm{P}(T > k)$$

giving your answer to 3 significant figures.

(3)

(Total for question = 9 marks)

Q8.

The continuous random variable *X* has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 4 \\ px - k\sqrt{x} & 4 \leqslant x \leqslant 9 \\ 1 & x > 9 \end{cases}$$

where *p* and *k* are constants.

(a) Find the value of *p* and the value of *k*.

(4)

Given that E(*X*) = $\frac{119}{18}$

(b) show that Var(*X*) = 2.05 to 3 significant figures.

(6)

(c) Write down the mode of *X*.

(1)

(d) Find the exact value of the constant a such that $P(X \leqslant a) = \frac{7}{27}$

(3)

(Total for question = 14 marks)

Q9.

The continuous random variable *X* has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 2 \\ 1.25 - \frac{2.5}{x} & 2 \leqslant x \leqslant 10 \\ 1 & x > 10 \end{cases}$$

(a) Find $P(\{X < 5\} \cup \{X > 8\})$

(2)

(b) Find the median of *X*.

(2)

(c) Find $E(X^2)$

(3)

(d) (i) Sketch the probability density function of *X*.

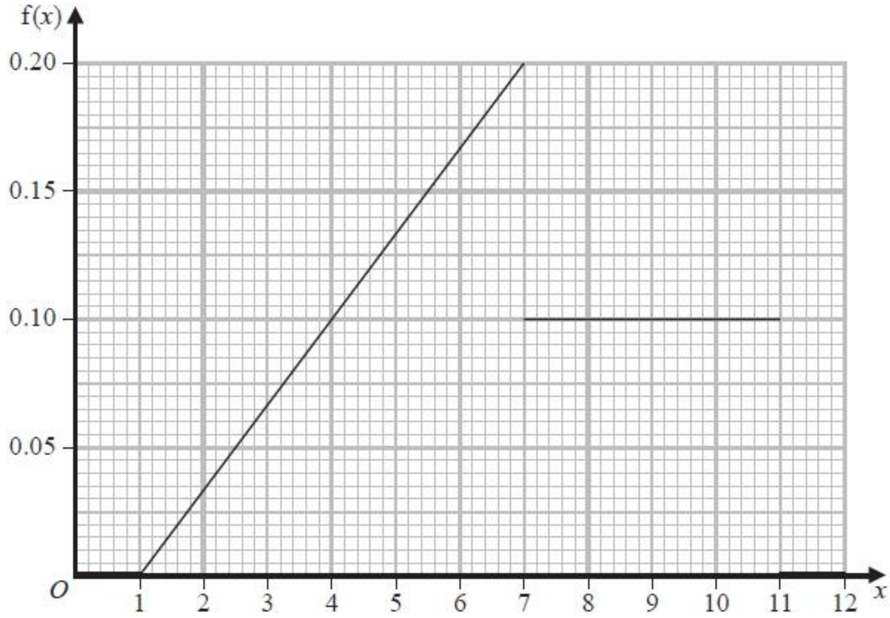
(ii) Describe the skewness of the distribution of *X*.

(3)

(Total for question = 10 marks)

Q10.

The graph shows the probability density function f(x) of the continuous random variable X



(a) Find P(X < 4)

(2)

(b) Specify the cumulative distribution function of X for $7 \leq x \leq 11$

(3)

(Total for question = 5 marks)

Q11.

A random variable X has probability density function given by

$$f(x) = \begin{cases} 0.8 - 6.4x^{-3} & 2 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

The median of X is m

(a) Show that $m^3 - 3.625m^2 + 4 = 0$

(3)

(b) (i) Find *fʹ*(x)

(ii) Explain why the mode of X is 4

(2)

Given that E(X^2) = 10.5 to 3 significant figures,

(c) find Var(X), showing your working clearly.

(4)

(Total for question = 9 marks)

Q12.

Korhan and Louise challenge each other to find an estimator for the mean, μ, of the continuous random variable X which has variance σ^2

$X_1, X_2, X_3, \ldots, X_n$ are n independent observations taken from X

Korhan's estimator is given by

$$K = \frac{2}{n(n+1)}\sum_{r=1}^{n} rX_r$$

Louise's estimator is given by

$$L = \frac{X_1 + X_2}{3} + \frac{X_3 + X_4 + \ldots + X_n}{3(n-2)}$$

(a) Show that K and L are both unbiased estimators of μ

(5)

(b) (i) Find Var (K)

(ii) Find Var (L)

(7)

The winner of the challenge is the person who finds the better estimator.

(c) Determine the winner of the challenge for large values of n.
Give reasons for your answer.

(3)

(Total for question = 15 marks)

Q13.

The continuous random variable X has cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 1 \\ 1.5x - 0.25x^2 - 1.25 & 1 \leqslant x \leqslant 3 \\ 1 & x > 3 \end{cases}$$

(a) Find the exact value of the median of X

(2)

(b) Find $P(X < 1.6 \mid X > 1.2)$

(3)

The random variable $Y = \dfrac{1}{X}$

(c) Specify fully the cumulative distribution function of Y

(4)

(d) Hence or otherwise find the mode of Y

(3)

(Total for question = 12 marks)

Q14.

The continuous random variable X is uniformly distributed over the interval $[\alpha, \beta]$

Given that $E(X) = 3.5$ and $P(X > 5) = \frac{2}{5}$

(a) find the value of α and the value of β

(4)

Given that $P(X < c) = \frac{2}{3}$

(b) (i) find the value of c

(ii) find $P(c < X < 9)$

(3)

A rectangle has a perimeter of 200 cm. The length, S cm, of one side of this rectangle is uniformly distributed between 30 cm and 80 cm.

(c) Find the probability that the length of the shorter side of the rectangle is less than 45 cm.

(4)

(Total for question = 11 marks)

Q15.

The continuous random variable X is uniformly distributed over the interval $[0, 4\beta]$, where β is an unknown constant.

Three independent observations, X_1, X_2 and X_3, are taken of X and the following estimators for β are proposed

$$A = \frac{X_1 + X_2}{2}$$

$$B = \frac{X_1 + 2X_2 + 3X_3}{8}$$

$$C = \frac{X_1 + 2X_2 - X_3}{8}$$

(a) Calculate the bias of A, the bias of B and the bias of C

(5)

(b) By calculating the variances, explain which of B or C is the better estimator for β

(4)

(c) Find an unbiased estimator for β

(1)

(Total for question = 10 marks)

Q16.

The random variable X has the continuous uniform distribution over the interval [0.5, 2.5]

Talia selects a number, T, at random from the distribution of X

(a) Find P(T < 1)

(1)

Malik takes Talia's number, T, and calculates his number, M, where $M = \frac{1}{T^2}$

(b) Find the probability that both T and M are less than 2.25

(3)

Raja and Greta play a game many times.
Each time they play they use a number, R, randomly selected from the distribution of X

Raja's score is R

Greta's score is G, where $G = \frac{2}{R^2}$

(c) Determine, giving a reason, who you would expect to have the higher total score.

(5)

(Total for question = 9 marks)

Q17.

A rectangle is to have an area of 40 cm^2

The length of the rectangle, L cm, follows a continuous uniform distribution over the interval [4, 10]

Find the expected value of the perimeter of the rectangle.
Use algebraic integration, rather than your calculator, to evaluate any definite integrals.

(Total for question = 7 marks)

Q18.

The random variable *X* has a continuous uniform distribution over the interval [5, *a*], where *a* is a constant.

Given that Var(*X*) = $\frac{27}{4}$

(a) show that *a* = 14

(3)

The continuous random variable *Y* has probability density function

$$f(y) = \begin{cases} \frac{1}{20}(2y-3) & 2 \leqslant y \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

The random variable $T = 3(X^2 + X) + 2Y$

(b) Show that E(*T*) = $\frac{9857}{30}$

(7)

(Total for question = 10 marks)

Q19.

The three independent random variables *A*, *B* and *C* each have a continuous uniform distribution over the interval [0, 5].

(a) Find the probability that *A*, *B* and *C* are all greater than 3

(3)

The random variable *Y* represents the maximum value of *A*, *B* and *C*.

The cumulative distribution function of *Y* is

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y^3}{125} & 0 \leqslant y \leqslant 5 \\ 1 & y > 5 \end{cases}$$

(b) Using algebraic integration, show that Var (*Y*) = 0.9375

(4)

(c) Find the mode of Y, giving a reason for your answer.

(2)

(d) Describe the skewness of the distribution of Y. Give a reason for your answer.

(1)

(e) Find the value of k such that $P(k < Y < 2k) = 0.189$

(3)

(Total for question = 13 marks)

Q20.

The continuous random variable X is uniformly distributed over the interval $[–3, 5]$.

(a) Sketch the probability density function $f(x)$ of X.

(2)

(b) Find the value of k such that $P(X < 2[k – X]) = 0.25$

(3)

(c) Use algebraic integration to show that $E(X^3) = 17$

(3)

(Total for question = 8 marks)

Q21.

The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{8} & 1 \leqslant x \leqslant 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Write down the name given to this distribution.

(1)

The continuous random variable $Y = 5 - 2X$

(b) Find $P(Y > 0)$

(2)

(c) Find $E(Y)$

(2)

(d) Find $P(Y < 0 \mid X < 7.5)$

(3)

(Total for question = 8 marks)

Q22.

The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 3 \\ c - 4.5x^n & 3 \leqslant x \leqslant 9 \\ 1 & x > 9 \end{cases}$$

where c is a positive constant and n is an integer.

(a) Showing all stages of your working, find the value of c and the value of n

(7)

(b) Find the lower quartile of X

(2)

(Total for question = 9 marks)

Mark Scheme – Continuous Distributions

Q1.

Qu	Scheme	Marks		AO
(a)	$X = L_1 + L_2 + L_3 \sim \text{N}\left(594, \sqrt{75}^2\right)$	M1		3.3
		A1		1.1b
	$Y = S_1 + ... + S_8 \sim \text{N}\left(592, \sqrt{72}^2\right)$	A1		1.1b
	$\text{P}(X > Y) = \text{P}(D > 0)$ where $D \sim \text{N}\left(2, \sqrt{147}^2\right)$	M1		2.1
		A1ft		1.1b
	$= 0.56551...$ awrt **<u>0.566</u>**	A1		3.4
		(6)		
(b)	$W = L - \frac{8}{3}S \Rightarrow W \sim \text{N}\left(\frac{2}{3}, 25 + \frac{64}{9} \times 9\right) = \text{N}\left(\frac{2}{3}, \sqrt{89}^2\right)$	M1		3.3
		M1,A1		2.1,1.1b
	$\text{P}(W > 0) = 0.528168...$ awrt **<u>0.528</u>**	M1A1		3.4,1.1b
		(5)		
(c)	$F = L_1 + ... + L_5 \sim \text{N}\left(990, \sqrt{125}^2\right)$	M1		3.1b
		A1		1.1b
	$\text{P}(F < 1000) = 0.814455...$ (o.e.)	A1		3.4
	E(cost of Rosa's plan) = $430 \times "0.814..." + 400 \times (1 - "0.814...")$	M1		2.1
	$= £\,424.43$	A1		1.1b
	Buying 14 small panels cost $14 \times 30 = £420$ So Rosa's plan is likely to be <u>more expensive</u>	A1		3.2a
		(6)		
		(17 marks)		

	Notes
(a)	1st M1 for an attempt at X or Y – expression or implied by one correct distribution 1st A1 for a correct distribution for X <u>or</u> implied by E(D) = 2 2nd A1 for a correct distribution for Y <u>or</u> implied by Var(D) = 147 2nd M1 for a correct strategy – attempt $X - Y$ and P(D > 0) statement 3rd A1ft for a correct distribution for D ft their X and Y 4th A1 for awrt 0.566
(b)	1st M1 for attempt at a correct model (normal and mean) 2nd M1 for correct expression for variance of their model provided of the form $L - kS$ <u>or</u> $kL - S$ 1st A1 for a fully correct distribution 3rd M1 for a correct probability statement using their distribution 2nd A1 for awrt 0.528
(c)	1st M1 for a correct start to solve the problem attempt at F and correct mean 1st A1 for a correct distribution 2nd A1 for using this model to find P(F < 1000) = awrt 0.814 or P(F > 1000) = awrt 0.186 2nd M1 for a correct strategy to solve the problem i.e. attempt at expected cost ft their prob 3rd A1 for awrt £424 4th A1 for a correct conclusion must have comparison with £420 and reject Rosa's plan [(c) is an extended problem and a 3.1, 3.2 question]

Q2.

Qu	Scheme	Marks		AO
(a)	$\int (1-\cos x)\ \mathrm{d}x = [x - \sin x]$	M1		1.1b
	Use of correct limits and $\int \mathrm{f}(x)\ \mathrm{d}x = 1 \Rightarrow 2\pi - 0 - 0\ = 1$	M1		1.1b
	so $k = \frac{1}{2\pi}$ (*)	A1*cso		1.1b
		(3)		
(b)	$\mathrm{E}(X) = \pi$ (symmetry) so $\mu = \pi$ so $\mathrm{f}(\mu) = \frac{1}{2\pi}(1-\cos\pi) = \frac{1}{\pi}$	B1		2.2a
	$\frac{1}{\sigma\sqrt{2\pi}} = "\frac{1}{\pi}"$; so $\sigma = \sqrt{\frac{\pi}{2}}$	M1; A1		1.1b 1.1b
		(3)		
(c)	$\mathrm{P}\left(\frac{\pi}{2} < X < \frac{3\pi}{2}\right) = \frac{1}{2\pi}[x - \sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = \frac{1}{2\pi}\left[\left(\frac{3\pi}{2} - -1\right) - \left(\frac{\pi}{2} - 1\right)\right]$	M1		3.4
	$= \frac{2+\pi}{2\pi}$ (= 0.81830…)	A1		1.1b
	$\mathrm{P}\left(\frac{\pi}{2} < Y < \frac{3\pi}{2}\right) = 0.7899\ldots$	B1		1.1b
	So error is 0.81830... − 0.7899... = 0.0284	A1		1.1b
		(4)		
		(10 marks)		

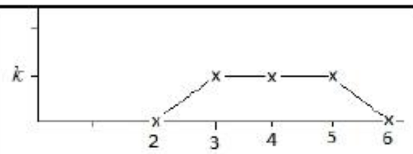
	Notes
(a)	1st M1 attempt to integrate (1 – cos*x*) – one correct term 2nd M1 for use of correct limits and correct method for *k* A1* cso use of $\int \mathrm{f}(x)\ \mathrm{d}x = 1$ seen and no incorrect working seen
(b)	B1 for correctly deducing the value of f(μ) M1 for a correct equation for σ – ft their value for f(μ) [condone for sight of correct g(μ)] A1 for $\sqrt{\frac{\pi}{2}}$ or exact equivalent
(c)	M1 for a correct attempt to find prob – some correct integration and use of limits 1st A1 for a correct answer (exact or 0.818.. or better) B1 for a correct probability from their calculator i.e. 0.7899 or better accept 0.79 2nd A1 for 0.0284 or better

Q3.

Qu	Scheme	Marks	AO
(a)	$\text{P}(\pi X^2 > 10) \;\Rightarrow\; \text{P}\left(X > \sqrt{\frac{10}{\pi}}\right)$	M1	3.1a
	$= \dfrac{\pi - \sqrt{\frac{10}{\pi}}}{\pi}$	M1	2.1
	$= 0.43209\ldots$ = awrt **0.432**	A1	1.1b
		(3)	
(b)	P(area > median) = 0.5; since (a) < 0.5 therefore **median < 10**	B1	2.2a
		(1)	
(c)	Area of triangle = $0.5x^2\sin x$	M1	3.1a
	$\text{E(area)} = \int_{[0]}^{[\pi]} \frac{1}{\pi}\frac{1}{2}x^2 \sin x \, dx$	M1	1.1b
	$= \frac{1}{2\pi}\int_{[0]}^{[\pi]} x^2 \, d(-\cos x) = \frac{1}{2\pi}\left\{\left[-x^2\cos x\right]_{[0]}^{[\pi]} - \int_{[0]}^{[\pi]} -2x\cos x \, dx\right\}$	M1	2.1
	$= \left\{\left[\frac{-x^2\cos x}{2\pi}\right]_{[0]}^{[\pi]}\right\} + \left[\frac{x\sin x}{\pi}\right]_{[0]}^{[\pi]} - \frac{1}{\pi}\int_{[0]}^{[\pi]} \sin x \, dx$	M1	1.1b
		A1	1.1b
	$= \frac{\pi^2}{2\pi} - 0 + 0 - 0 + \left(-\frac{1}{\pi}\right) - \left(\frac{1}{\pi}\right) =, \underline{\frac{\pi}{2} - \frac{2}{\pi}}$	M1	1.1b
		A1	1.1b
		(7)	
		(11 marks)	

Qu	Notes	
(a)	1st M1	reduce the problem to a probability about X
	2nd M1	for use of the uniform distribution (a correct expression ft their value 1.784..)
	A1	for awrt 0.432
(b)	B1	for statement that median < 10 supported by argument about answer to (a) being < 0.5
ALT		Median area is given by $\pi \times \left(\frac{\pi}{2}\right)^2 = 7.751\ldots < 10$ so median < 10
(c)	1st M1	for a correct expression for area in terms of x
	2nd M1	for realisation that need to use E(g(X)) formula and a correct expression (ignore limits)
	3rd M1	for attempt to use integration by parts
	4th M1	for a 2nd use of integration by parts
	1st A1	for correct integration (ignore limits)
	5th M1	for clear use of the correct limits
	2nd A1	for $\frac{\pi}{2} - \frac{2}{\pi}$
		[(c) is an extended problem and also involves work from pure for the integration]

Q4.

Question Number	Scheme		Marks
(a)		**B1** correct shape with the end points on the x-axis **B1** correct shape with k, 2,3,5,6 marked on in the correct places. Allow $^1/_3$ for k	**B1** **B1**
			(2)
(b)	$\frac{1}{2}\times k+2\times k+\frac{1}{2}\times k=1$	**M1** An attempt to find area using any correct method and putting equal to 1	**M1**
	$3k=1$		
	$k=\frac{1}{3}$*	**A1 cso**. AG Method must be shown and there must be no incorrect working. Need to have these 3 lines as a minimum.	**A1 cso**
	alternative		**(2)**
	$\int_2^3 k(x-2)\mathrm{d}x+\int_3^5 k\mathrm{d}x+\int_5^6 k(6-x)\mathrm{d}x=1$ $\left[\frac{kx^2}{2}-2kx\right]_2^3+[kx]_3^5+k\left[6x-\frac{x^2}{2}\right]_5^6=1$	**M1** Correct integration to find the whole area, put = 1 and an attempt to integrate, ignore limits for attempt $x^n\to x^{n+1}$	**M1**
	$\left(-\frac{3}{2}k+2k\right)+\ 5k-3k\ +\left(18k-\frac{35}{2}k\right)=1$		
	$3k=1$		
	$k=\frac{1}{3}$	**A1 cso** Method must be shown – at least one step between integration and $k = 1/3$ and there must be no incorrect working.	**A1 cso**
	SC For using verification they could get **M1 A0** if there are no errors		
(c)	$\mathrm{F}(x)=\begin{cases}0 & x<2\\ \frac{x^2}{6}-\frac{2x}{3}+\frac{2}{3} & 2\le x\le 3\\ \frac{x}{3}-\frac{5}{6} & 3<x<5\\ 2x-\frac{x^2}{6}-5 & 5\le x\le 6\\ 1 & x>6\end{cases}$	**Alternative** $\mathrm{F}(x)=\begin{cases}0 & x<2\\ \frac{1}{6}(x-2)^2 & 2\le x\le 3\\ \frac{x}{3}-\frac{5}{6} & 3<x<5\\ 1-\frac{1}{6}(6-x)^2 & 5\le x\le 6\\ 1 & x>6\end{cases}$	**M1A1** **M1A1** **M1A1** **B1**
			(7)

1st M1 For $2 \le x \le 3$, $\int_2^x \frac{1}{3}(t-2)\mathrm{d}t = \left[\frac{t^2}{6} - \frac{2t}{3}\right]_2^x$ and attempt to subst 2 and x **Or** $\mathrm{F}(x) = \frac{x^2}{6} - \frac{2x}{3} + C$ and using $\mathrm{F}(2) = 0$ **1st A1** for the second row in the above F(x) **oe**. Condone < instead of ≤ and vice versa
2nd M1 For $3 < x < 5$, $\int_3^x \frac{1}{3}\mathrm{d}t + "\frac{1}{6}" = \left[\frac{t}{3}\right]_3^x + "\frac{1}{6}"$ and attempt to subst 3 and x. Allow F(3) instead of $"\frac{1}{6}"$ **or** $\mathrm{F}(x) = \frac{x}{3} + C$ and using $\mathrm{F}(3) = \frac{1}{6}$ or $\mathrm{F}(5) = \frac{5}{6}$ **2nd A1** for the third row in the above F(x) **oe**. Condone ≤ instead of < and vice versa
3rd M1 For $5 \le x \le 6$, $\int_5^x 2 - \frac{t}{3}\mathrm{d}t + "\frac{5}{6}" = \left[2t - \frac{t^2}{6}\right]_5^x + "\frac{5}{6}"$ and subst 5 and x. Allow F(5) instead of $"\frac{5}{6}"$ **or** $\mathrm{F}(x) = 2x - \frac{x^2}{6} + C$ and using $\mathrm{F}(6) = 1$ **3rd A1** for the fourth row in the above F(x) **oe**. Condone < instead of and vice versa
B1 For both Top line of F(x) ie 0 $x < 2$ and Bottom line of F(x) ie 1 $x > 6$ Condone ≤ instead of < and vice versa. Allow one of the lines to have otherwise as its range

(d)	$2x - \frac{x^2}{6} - 5 = 0.9$	**1st M1** using their cdf for $5 \le x \le 6 = 0.9$	**M1**
	$\frac{x^2}{6} - 2x + 5.9 = 0$ $x = \frac{2 \pm \sqrt{4 - 4 \times \frac{1}{6} \times 5.9}}{\frac{1}{3}}$	**2nd M1** using either the quadratic formula or completing the square or factorising or any correct method to solve their **3 term quadratic** which must have been correctly rearranged. If they write the formula down then allow a slip. If no formula written down then it must be correct for their equation. May be implied by awrt 5.23 or 6.77	**M1**
	$x =$ awrt 5.23	**A1** awrt 5.23 – (allow $\frac{30-\sqrt{15}}{5}$). If they have 6.77… this must be eliminated	**A1**
			(3)
(e)	$\mathrm{E}(X) = 4$		
	$\mathrm{F}(5.5) - \mathrm{F}(4) = \frac{11}{24}$	**M1** for writing or attempting to find $\mathrm{F}(5.5) - \mathrm{F}(4)$ or $\mathrm{P}(X \le 5.5) - \mathrm{P}(x \le 4)$ or $\mathrm{P}(X < 5.5) - \mathrm{P}(x < 4)$ or $\mathrm{F}(5.5) - 0.5$ or $\int_4^5 k\,\mathrm{d}x + \int_5^{5.5} k(6-x)\,\mathrm{d}x$ with correct limits and $x^n \to x^{n+1}$. May be implied by a correct answer.	**M1**
		A1 $\frac{11}{24}$ **oe** or awrt 0.458	**A1**
			(2)
			(Total 16)

Q5.

Question Number	Scheme		Marks
(a)	$E(X)=\frac{1}{9}\int_1^4 (4x^2-x^3)\ dx$	**1st M1** Using $\int xf(x)\ dx$, multiplying out and at least one of $x^2 \to x^3$ or $x^3 \to x^4$ ignore limits	**M1**
	$=\frac{1}{9}\left[\frac{4x^3}{3}-\frac{x^4}{4}\right]_1^4$	**1st A1** correct integration, ignore limits	**A1**
	$=\frac{1}{9}\left[\frac{4\times4^3}{3}-\frac{4^4}{4}\right]-\frac{1}{9}\left[\frac{4}{3}-\frac{1}{4}\right]$	**2nd M1d** subst in correct limits (allow 1 sign error)	**M1d**
	$=\frac{9}{4}$ or 2.25	**2nd A1** cao allow equivalent fractions	**A1**
			(4)
(b)	$P(X>2.5)=\frac{1}{9}\int_{2.5}^4 x(4-x)\ dx$	**M1** for using $\frac{1}{9}\int_{2.5}^4 x(4-x)\ dx$ or $1-\frac{1}{9}\int_1^{2.5} x(4-x)\ dx$ correct limits needed at some point Or $1-\left(\frac{2}{9}x^2-\frac{1}{27}x^3-\frac{5}{27}\right)$ and attempt to subst 2.5	**M1**
	$=\frac{1}{9}\left[2x^2-\frac{x^3}{3}\right]_{2.5}^4$	**1st A1** correct integration with correct limits at some point	**A1**
	$=\frac{3}{8}$ **oe** or 0.375	**2nd A1** allow equivalent fractions	**A1**
			(3)
(c)	P(both batteries working after 25 hours) $=(0.375)^2$	**M1** (their part(b))^2 or writing $(P(X>2.5))^2$	**M1**
	$=0.140625$ or $\frac{9}{64}$	**A1** awrt 0.141	**A1**
			(2)
(d)	$P(X>1.6)=\frac{1}{9}\int_{1.6}^4 x(4-x)dx$ $=\frac{96}{125}$ or 0.768	**B1** 0.768 or awrt 0.77 or 0.5898…or awrt 0.59. These may be seen in the conditional probability or implied by a correct final answer	**B1**
	P(works for 25 hours|worked for 16 hours) $=\frac{0.140625}{(0.768)^2}$	**M1** $\frac{\text{their part(c)}}{prob}$ or $\frac{(\text{their}(b))^2}{prob}$ and numerator < denominator	**M1**
	$=0.2384\ldots$	**A1** awrt 0.238	**A1**
	NB if use one battery rather than 2 they could get B1 M0 A0		
			(3)
			(Total 12)

Q6.

Question	Scheme	Marks	AOs
(a)	$k\left(2^3-\frac{3}{8}2^4\right)=1$	B1*	1.1b
	$2k=1$		
	$k=\frac{1}{2}$ *		
	Or $\frac{1}{2}\left(2^3-\frac{3}{8}2^4\right)=1 \therefore k=\frac{1}{2}$ *	(B1*)	
		(1)	
(b)	$\mathrm{f}(x)=k\left[3x^2-\frac{3}{2}x^3\right]$	M1	2.1
	(i) $\int_0^2 x\mathrm{f}(x)\,\mathrm{d}x=k\int_0^2\left(3x^3-\frac{3}{2}x^4\right)\mathrm{d}x$	M1d	1.1b
	$=\left[\frac{3x^4}{8}-\frac{3x^5}{20}\right]_0^2$		
	$=\frac{6}{5}$ or 1.2	A1	1.1b
	(ii) $3x-\frac{9x^2}{4}=0$	M1d	3.1a
	$x\left(3-\frac{9x}{4}\right)=0$	M1d	1.1b
	$x=0$ or $\frac{4}{3}$ $\quad\therefore$ mode $=\frac{4}{3}$	A1	1.1b
		(6)	
(c)	Mode > mean implies it is negative skew	B1ft	2.4
		(1)	
		(8 marks)	

Notes:		
(a)	**B1***	substituting $x=2$ into F(x) and equating to 1 leading to $k=\frac{1}{2}$ with no errors. Minimum subst seen is $k(8-6)=1$ or $0.5(8-6)=1$
(b)	**M1**	Realising they need to find the pdf and attempting to differentiate $k\left[x^3-\frac{3}{8}x^4\right]$ at least 1 correct term
(i)	**M1d**	dep on 1st M1 Attempting to find $\int_0^2 x(\text{their f}(x))\,\mathrm{d}x$ At least one correct term ft their pdf
	A1	$\frac{6}{5}$ or 1.2 oe NB 1.2 with no working gains M0M0A0
(ii)	**M1d**	dep on 1st M1 for realising they need to differentiate their pdf. At least one correct term but ft their pdf
	M1d	Dep on 3rd M1. correct method for solving their differential of their pdf = 0 pdf must be of the form ax^2+bx
	A1	$\therefore$ mode $=\frac{4}{3}$ only. They must eliminate 0
(c)	**B1ft**	ft their mode and mean or a correct sketch.

Q7.

Question	Scheme	Marks	AOs
(a)	$\int \frac{t}{120}\mathrm{d}t = \frac{t^2}{240}$ and use of F(4) = 0 or F(16) = 1 or limits of t and 4 or attempt at area of trapezium allow 1 mistake. $\frac{1}{2}\times(t-4)\left(\frac{4}{120}+\frac{t}{120}\right)$	M1	2.1
	$=\frac{t^2}{240}-\frac{1}{15}$	A1	1.1b
		(2)	
(b)	$\mathrm{F}(10)-\mathrm{F}(5)=\frac{100}{240}-"c"-\frac{25}{240}+"c"$	M1	1.1b
	$=\frac{5}{16}$	A1	1.1b
		(2)	
(c)	$\frac{m^2}{240}-\frac{1}{15}=0.5$	M1	1.1b
	$m=11.66....$ awrt 11.7	A1	1.1b
		(2)	
(d)	$\mathrm{F}(k)=\frac{2}{3}(1-\mathrm{F}(k))$ or $\int_4^k \frac{t}{120}\mathrm{d}t=\frac{2}{3}\int_k^{16}\frac{t}{120}\mathrm{d}t$	M1	3.1a
	$\frac{k^2}{240}-"\frac{1}{15}"=\frac{2}{3}\left(1-\left(\frac{k^2}{240}-"\frac{1}{15}"\right)\right)$ or $\frac{k^2}{240}-\frac{1}{15}=\frac{2}{3}\times\left(\frac{16}{15}-\frac{k^2}{240}\right)$	dM1	1.1b
	$\frac{k^2}{144}=\frac{7}{9}$		
	$k=\sqrt{112}$ or awrt 10.6	A1	1.1b
	Alternative		
	Let $\mathrm{P}(T<k)=p$ then $p=\frac{2}{3}(1-p)\ \therefore p=\frac{2}{5}$	(M1)	
	$\frac{k^2}{240}-\frac{1}{15}=\frac{2}{5}$	(dM1)	
	$k=\sqrt{112}$ or awrt 10.6	(A1)	
		(3)	

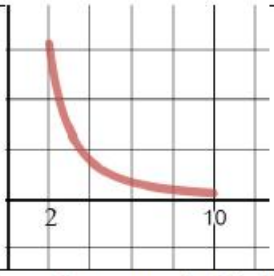
Notes
(a) M1: for attempting to integrate and a correct method
A1: $=\frac{t^2}{240}-\frac{1}{15}$ or $=\frac{t^2}{240}-0.0667$
(b) M1: writing or using F(10) – F(5)
A1: awrt $\frac{5}{16}$ or 0.3125 or exact equivalent
(c) M1: setting their F(t) = 0.5
A1: awrt 11.7 or $2\sqrt{34}$ or exact equivalent
(d) M1: Setting up a correct equation to solve the mathematical problem or setting up correct equation to find p and an attempt to solve
dM1: attempted to integrate and limits substituted or using "Their F(k)" = "their p"
A1: $\sqrt{112}$ or awrt 10.6

Q8.

Qu	Answer	Marks	AO
(a)	$F(4) = 0 \Rightarrow 4p - 2k = 0$ <u>or</u> $F(9) = 1 \Rightarrow 9p - 3k = 1$	M1	2.1
	Both	A1	1.1b
	Solving e.g. sub $k = 2p \Rightarrow 9p - 6p = 1$	M1	1.1b
	$\underline{p = \frac{1}{3} \quad k = \frac{2}{3}}$	A1	1.1b
		(4)	
(b)	Find f(x): $f(x) = F'(x) = "\frac{1}{3}" - "\frac{2}{3}" \times \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{3}\left(1 - x^{-\frac{1}{2}}\right)$	M1	3.1a
	$E(X^2) = \frac{1}{3}\int_4^9 x^2(1 - x^{-\frac{1}{2}})\,dx$ <u>or</u> $\frac{1}{3}\int_4^9 (x^2 - x^{\frac{3}{2}})\,dx$	M1	2.1
	$= \frac{1}{3}\left[\frac{x^3}{3} - \frac{2x^{\frac{5}{2}}}{5}\right]_4^9$ <u>or</u> $\frac{1}{3}\left[\left(\frac{9^3}{3} - \frac{2 \times 3^5}{5}\right) - \left(\frac{64}{3} - \frac{2 \times 2^5}{5}\right)\right] = \frac{2059}{45}$	M1 A1	1.1b 1.1b
	$\text{Var}(X) = "\frac{2059}{45}" - \left(\frac{119}{18}\right)^2$	M1	1.1b
	$= 2.048765\ldots = 2.05$ (3sf) (*)	A1*	1.1b
		(6)	
(c)	$\left[f(x) = \frac{1}{3}\left(1 - \frac{1}{\sqrt{x}}\right)\right.$ so max is when x is greatest so] mode = <u>9</u>	B1	2.2a
		(1)	
(d)	$F(a) = \frac{7}{27} \Rightarrow a - 2\sqrt{a} - \frac{7}{9} = 0$	M1	3.1a
	$y^2 - 2y - \frac{7}{9} = 0 \Rightarrow (y-1)^2 = \frac{16}{9}$	M1	2.1
	$y = \frac{7}{3}$ or $\left(-\frac{1}{3}\text{ not valid}\right)$ so $\underline{a = \frac{49}{9}}$	A1	3.2a
		(3)	
		(14 marks)	

	Notes
(a)	1st M1 for selecting a correct approach and getting 1 correct equation 1st A1 for 2 correct equations in p and k 2nd M1 for solving two simultaneous equations (based on F(x))– reducing to a linear eqn in 1 var 2nd A1 for <u>both</u> correct values
(b)	1st M1 for realising need to find f(x) first and attempt to differentiate F(x) – some correct 2nd M1 for attempting $\int x^2 f(x)\,dx$ ft their f(x) provided different from F(x) 3rd M1 for some correct integration using their f(x) or a numerical expression for $E(X^2)$ 1st A1 for a correct value for $E(X^2)$ exact fraction or at least 45.755…. 4th M1 for a correct method for Var(X) ft their $E(X^2)$ 2nd A1* for a fully correct solution
(c)	B1 for 9
(d)	1st M1 for realising the need to use F(x) and forming a correct equation in a (ft their p and k) 2nd M1 for recognising equation as a quadratic and trying to solve A1 for exact working and selection of the appropriate value

Q9.

Question	Scheme	Marks	AOs
(a)	F(5) + (1 − F(8)) $\left[\frac{3}{4}+\left(1-\frac{15}{16}\right)\right]$	M1	2.1
	$=\frac{13}{16}$	A1	1.1b
		(2)	
(b)	F(*m*) = 0.5 $\left[1.25-\frac{2.5}{m}=0.5\right]$	M1	1.1b
	$m=\frac{10}{3}$	A1	1.1b
		(2)	
(c)	$f(x)[=\frac{d}{dx}(F(x))]=2.5x^{-2}$	M1	2.1
	$E(X^2)[=\int_2^{10} x^2 f(x)\,dx]=\int_2^{10} 2.5\,dx$	M1	1.1b
	= 20	A1	1.1b
		(3)	
(d)(i)		M1	1.1b
(ii)	2 and 10 correctly labelled on horizontal axis	A1	2.1
	Therefore positive skew.	A1	2.2b
		(3)	
			(10 marks)

Notes	
(a)	**M1:** Equivalent correct probability statement, e.g. [1 − (F(8) − F(5))] **A1:** $\frac{13}{16}$ oe
(b)	**M1:** Use of F(*m*) = 0.5 **A1:** $\frac{10}{3}$ oe
(c)	**M1:** Realising that f(*x*) is required and attempting to differentiate F(*x*) **M1:** Use of $\int_2^{10} x^2 f(x)\,dx$ **A1:** 20 cao
(d)(i) and (ii)	**M1:** Correct shape **A1:** Correct labels **A1:** Positive skew provided M1 scored

Q10.

Question	Scheme			Marks	AOs
(a)	$\text{P}(X < 4) = \frac{(4-1)\times 0.1}{2}$ or $\int_1^4 \frac{1}{30}(x-1)\,\text{d}x$			M1	2.1
	$= 0.15$			A1	1.1b
				(2)	
(b)	$\frac{(7-1)\times 0.2}{2} + \ldots$ $\ldots 0.1(x-7)$	$\int_1^7 \frac{1}{30}(x-1)\,\text{d}x + \ldots$ $\ldots \int_7^x 0.1\,\text{d}t$	$\int 0.1\,\text{d}x$ $\text{F}(7) = 0.1x + c$ or $\text{F}(11) = 0.1x + c$	M1 M1	3.1a 1.1b
	$\text{F}(x) = 0.1x - 0.1$ $\quad [\text{for } 7 \le x \le 11]$			A1	1.1b
				(3)	
					(5 marks)

Notes

(a)	**M1**: Use of area of triangle or integration with limits to find required area Condone $\frac{4\times 0.1}{2}$ or $\int_1^4 \frac{1}{30}(x)\,\text{d}x$ for M1 **A1**: 0.15 oe
(b)	**M1**: Complete method for finding the cdf including area from $1 \le x < 7$ Condone one slip for the area from $1 \le x < 7$ **M1**: Attempt at area from $7 \le x \le 11$ **A1**: $0.1x - 0.1$

Q11.

Question	Scheme	Marks	AOs
(a)	$\int_2^m (0.8-6.4x^{-3})\mathrm{d}x = 0.5$	M1	2.1
	$\left[0.8x+3.2x^{-2}\right]_2^m = 0.5$	M1	1.1b
	$0.8m+\frac{3.2}{m^2}-\left(0.8(2)+\frac{3.2}{2^2}\right)=0.5 \rightarrow 0.8m+\frac{3.2}{m^2}-2.9=0 \rightarrow$ $m^3-3.625m^2+4=0$ *	A1*cso	1.1b
		(3)	
(b)(i)	$\mathrm{f}'(x)=19.2x^{-4}$	B1	1.1b
(ii)	Since $\mathrm{f}'(x)>0$, $\mathrm{f}(x)$ is increasing (the pdf has its maximum value at the upper end of the interval), the mode is 4	B1	2.4
		(2)	
(c)	$\mathrm{E}(X)=\int_2^4 x(0.8-6.4x^{-3})\mathrm{d}x$	M1	1.1b
	$\mathrm{E}(X)=\left[0.4x^2+6.4x^{-1}\right]_2^4$ $=0.4(4^2)+6.4(4^{-1})-\left(0.4(2^2)+6.4(2^{-1})\right)[=3.2]$	M1	1.1b
	$\mathrm{Var}(X)=10.5-'3.2'^2$	M1	1.1b
	$\mathrm{Var}(X)=0.26$	A1	1.1b
		(4)	
			(9 marks)

Notes	
(a)	**M1:** Integral = 0.5 (ignore limits) **M1:** Integration with limits **A1*cso:** Given answer with at least one line of intermediate working.
(b)(i) (ii)	**B1:** $19.2x^{-4}$ **B1:** Correct reasoning and conclusion (allow equivalent correct reasoning e.g. no turning points with a sketch of $\mathrm{f}(x)$). Do not allow unsupported comments on their own to score e.g. '$x = 4$ is the highest point on $\mathrm{f}(x)$'
(c)	**M1:** Multiplying out $x\mathrm{f}(x)$ and attempt to integrate **M1:** correct use of limits (implied by 3.2oe) **M1:** Use of $\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ **A1:** 0.26

Q12.

Question	Scheme	Marks	AOs
(a)	$\mathrm{E}(K)=\frac{2\mathrm{E}(X)}{n(n+1)}+\frac{4\mathrm{E}(X)}{n(n+1)}+\frac{6\mathrm{E}(X)}{n(n+1)}+\ldots+\frac{2n\mathrm{E}(X)}{n(n+1)}$ oe	M1	3.1a
	$=\frac{1}{n(n+1)}\left(\sum_{r=1}^{n}2r\right)\mathrm{E}(X)=\frac{1}{n(n+1)}\left(\frac{(2+2n)n}{2}\right)\mathrm{E}(X)(=\mathrm{E}(X))$	M1	2.1
	$=\mu$ (therefore K is an unbiased estimator of μ)	A1cso	1.1b
	$\mathrm{E}(L)=\frac{2\mathrm{E}(X)}{3}+\frac{(n-2)\mathrm{E}(X)}{3(n-2)}=\frac{2\mathrm{E}(X)}{3}+\frac{1\mathrm{E}(X)}{3}(=\mathrm{E}(X))$	M1	1.1b
	$=\mu$ (therefore L is an unbiased estimator of μ)	A1cso	1.1b
		(5)	
(b)(i)	$\mathrm{Var}(K)=\mathrm{Var}\left(\frac{2X}{n(n+1)}\right)+\mathrm{Var}\left(\frac{4X}{n(n+1)}\right)+\ldots+\mathrm{Var}\left(\frac{2n(X)}{n(n+1)}\right)$	M1	3.1a
	$=\frac{2^2}{n^2(n+1)^2}\mathrm{Var}(X)+\frac{4^2}{n^2(n+1)^2}\mathrm{Var}(X)+\ldots+\frac{(2n)^2}{n^2(n+1)^2}\mathrm{Var}(X)$	M1	1.1b
	$=\frac{\sum_{r=1}^{n}(2r)^2}{n^2(n+1)^2}\mathrm{Var}(X)=\frac{4\sum_{r=1}^{n}r^2}{n^2(n+1)^2}\mathrm{Var}(X)=\frac{\frac{4}{6}(n)(n+1)(2n+1)}{n^2(n+1)^2}\sigma^2$	M1	2.1
	$=\frac{2(2n+1)}{3n(n+1)}\sigma^2$	A1	1.1b
(ii)	$\mathrm{Var}(L)=\mathrm{Var}\left(\frac{X_1+X_2}{3}\right)+\mathrm{Var}\left(\frac{X_3+X_4+\ldots+X_n}{3(n-2)}\right)$	M1	1.1b
	$=\frac{2}{9}\mathrm{Var}(X)+\frac{(n-2)}{9(n-2)^2}\mathrm{Var}(X)$	M1	2.1
	$=\frac{2n-3}{9(n-2)}\sigma^2$	A1	1.1b
		(7)	
(c)	For large values of n $\quad \mathrm{Var}(K)\to 0 \quad \mathrm{Var}(L)\to\frac{2}{9}(\sigma^2)$	M1	2.1
	(Since both are unbiased,) the better estimator is the one with the smaller variance or $0<\frac{2}{9}(\sigma^2)$	M1	2.4
	Therefore K is the better estimator and Korhan wins the challenge.	A1	2.2a
		(3)	
			(15 marks)

Notes	
(a)	**M1**: Using independence to set up expression for E(K) **M1**: Use of $\sum_{r=1}^{n} r \left(= \frac{n(n+1)}{2}\right)$ **A1cso**: Correct conclusion from correct working **M1**: Using independence to set up expression for E(L) **A1cso**: Correct conclusion from correct working
(b)(i)	**M1**: Using independence to set up expression for Var(K) **M1**: Use of $\text{Var}(aX) = a^2\text{Var}(X)$ Implied by $\frac{2^2}{n^2(n+1)^2}$
(ii)	**M1**: Use of $\sum_{r=1}^{n} r^2$ **A1**: Correct equivalent expression for Var(K) oe **M1**: Using independence to set up expression for Var(L) **M1**: Use of $\text{Var}(aX) = a^2\text{Var}(X)$ and understanding $\text{Var}(X_1 + X_2) = 2\text{Var}(X)$ Implied by either correct term **A1**: Correct equivalent expression for Var(L) oe
(c)	**M1**: Finding correct limits as n gets larger for each expression allow ft Note: Solving $\frac{2n-3}{9(n-2)} = \frac{2(2n+1)}{3n(n+1)} \to n = -0.53,\ 2.46,\ 4.57$ so allow comments relating to $n \geq 5\,(4.57)$ **M1**: Correct explanation **A1**: Deducing that Korhan is the winner (dependent upon both M marks)

Q13.

Question	Scheme	Marks	AOs
(a)	$1.5m - 0.25m^2 - 1.25 = 0.5 \quad (\rightarrow 0.25m^2 - 1.5m + 1.75 = 0)$	M1	1.1b
	$m = 3 - \sqrt{2} \qquad$ (reject $m = 3 + \sqrt{2}$)	A1	2.2a
		(2)	
(b)	$P(X < 1.6 \mid X > 1.2) = \dfrac{P(1.2 < X < 1.6)}{P(X > 1.2)}$	M1	3.1a
	$\dfrac{F(1.6) - F(1.2)}{1 - F(1.2)}$	M1	1.1b
	$= \dfrac{32}{81}$	A1	1.1b
		(3)	
(c)	$P(Y \leqslant y) = P\left(\dfrac{1}{X} \leqslant y\right)$		
	$= P\left(X \geqslant \frac{1}{y}\right) = 1 - F(\frac{1}{y})$	M1	3.1a
	$= 1 - (\frac{1.5}{y} - 0.25(\frac{1}{y})^2 - 1.25)$	M1	1.1b
	$F(y) = \begin{cases} 0 & y < \frac{1}{3} \\ 2.25 - \frac{1.5}{y} + 0.25(\frac{1}{y})^2 & \frac{1}{3} \le y \le 1 \\ 1 & y > 1 \end{cases}$	A1 A1	2.1 1.1b
		(4)	
(d)	$f(y) = \dfrac{d}{dy}(F(y)) = 1.5y^{-2} - 0.5y^{-3} \rightarrow f'(y) = -3y^{-3} + 1.5y^{-4}$	M1	3.1a
	$f'(y) = -3y^{-3} + 1.5y^{-4} = 0$	depM1	1.1b
	$1.5y^{-4} = 3y^{-3} \rightarrow \frac{1.5}{y^4} = \frac{3}{y^3}$		
	[Mode of Y =] 0.5 (since $f''(0.5) = -48 < 0$)	A1	1.1b
		(3)	
			(12 marks)

Notes	
(a)	**M1:** Correct equation for m **A1:** $m = 3 - \sqrt{2}$ only (isw if exact answer is given then rounded)
(b)	**M1:** Determining the two probabilities required to find the probability **M1:** Correct ratio of probabilities **A1:** allow awrt 0.395
(c)	**M1:** Realising that $P\left(X \geqslant \frac{1}{y}\right)$ is necessary **M1:** Correct use of $F(x)$ **A1:** Determining correct limits (allow $\le$ for $<$ etc.) **A1:** All lines of cdf correct (ignoring limits) must be in terms of y SC: If 0 scored, then [$F(y)$ =] $\frac{1.5}{y} - 0.25(\frac{1}{y})^2 - 1.25 \quad \frac{1}{3} \le y \le 1$ scores M0M0A1A0
(d)	**M1:** Realising that the cdf must be differentiated twice **depM1:** (dep on previous M1) Equating their $f'(y) = 0$ with attempt to solve **A1:** 0.5 cao

Q14.

Question Number	Scheme		Marks
(a)	$\left[E(X)=\frac{\alpha+\beta}{2}=3.5\right], \Rightarrow \alpha+\beta=7$	**B1** Correct equation. Need not be simplified	**B1**
	$\left[P(X>5)=\frac{\beta-5}{\beta-\alpha}=\frac{2}{5}\right]$, $\Rightarrow 5(\beta-5)=2(\beta-\alpha)$	**M1** a second correct equation, Using simultaneous equations and eliminating α or β to gain a value of α and β.	**M1**
	$\alpha=-4$	1st **A1** for -4	**A1**
	$\beta=11$	2nd **A1** for 11	**A1**
		NB Award full marks for $\alpha=-4, \beta=11$	
			(4)
(b)(i)	$\frac{c+4}{15}=\frac{2}{3}$		
	$[c=]\,6$	**B1** for 6	**B1**
(ii)	$P(6<X<9)=\frac{1}{15}\times(3)$	**M1** $\frac{1}{\beta-\alpha}\times(9-c)$ or $[F(9)-F(c)]=\frac{13}{15}-\frac{2}{3}$ SC if $9>$ "their β" award for $1-\frac{2}{3}$	**M1**
	$=0.2$	**A1cso** 0.2 **oe**	**A1cso**
			(3)
(c)	$[P(S<45)]=\frac{3}{10}$	**B1** $\frac{3}{10}$ seen – it does not need to be associated with $P(S<45)]$	**B1**
	$[P(S>55)]=\frac{1}{2}$	**B1** $\frac{1}{2}$ seen– it does not need to be associated with $P(S>55)]$	**B1**
	$\text{total}=\frac{3}{10}+\frac{1}{2}=\frac{4}{5}$	**M1** for adding their two areas and the total <1. Do not allow $2\times$ a single area **A1** $\frac{4}{5}$ **oe** NB Award full marks for $\frac{4}{5}$	**M1A1**
			(4)
			(Total 11)

Q15.

Question	Scheme	Marks	AOs
(a)	$[E(X) = 2\beta]$ $E(A) = E\left(\frac{X_1 + X_2}{2}\right) = \frac{1}{2}[E(X) + E(X)]$ $E(B) = E\left(\frac{X_1 + 2X_2 + 3X_3}{8}\right) = \frac{1}{8}[E(X) + 2E(X) + 3E(X)]$ $E(C) = E\left(\frac{X_1 + 2X_2 - X_3}{8}\right) = \frac{1}{8}[E(X) + 2E(X) - E(X)]$	M1	3.1a
	Bias for A = $E(A) - \beta = 2\beta - \beta = \beta$ Bias for B = $E(B) - \beta = 1.5\beta - \beta = 0.5\beta$ Bias for $C = E(C) - \beta = 0.5\beta - \beta = -0.5\beta$	M1 A1 A1 A1	2.1 1.1b 1.1b 1.1b
		(5)	
(b)	$[\text{Var}(X) = \frac{4}{3}\beta^2]$ Better estimator would have the smallest bias and the least variance. B and C have equal bias, so we select the estimator with the smallest variance $\text{Var}(B) = \text{Var}\left(\frac{X_1 + 2X_2 + 3X_3}{8}\right)$ $= \frac{1}{64}[\text{Var}(X) + 4\text{Var}(X) + 9\text{Var}(X)]$ $\text{Var}(C) = \text{Var}\left(\frac{X_1 + 2X_2 - X_3}{8}\right)$ $= \frac{1}{64}[\text{Var}(X) + 4\text{Var}(X) + \text{Var}(X)]$	M1	2.1
	$\text{Var}(B) = \frac{7}{32}\text{Var}(X)[= \frac{7}{24}\beta^2]$ $\text{Var}(C) = \frac{3}{32}\text{Var}(X)[= \frac{1}{8}\beta^2]$	A1 A1	1.1b 1.1b
	(Since both have same bias and) $\text{Var}(C) < \text{Var}(B)$ therefore C is the better estimator.	B1ft	2.2a
		(4)	
(c)	Any unbiased estimator, e.g. $\frac{X_1 + X_2 + X_3}{6}$	B1	3.5c
		(1)	
			(10 marks)

Notes	
(a)	**M1:** Using independence to calculate the $E(A)$, $E(B)$ or $E(C)$ **M1:** Use of bias = $E(X) - \beta$ **A1:** Correct bias for A **A1:** Correct bias for B **A1:** Correct bias for C [allow $+0.5\beta$]
(b)	**M1:** Realising that variances need to be compared and attempt at linear combination of variances for B or C **A1:** Correct $\text{Var}(B)$ **A1:** Correct $\text{Var}(C)$ **A1ft:** Correct comparison and deduction that C a better estimator than A and B.
(c)	**B1:** Allow any unbiased estimator, e.g. $\frac{X_1}{2}$

Q16.

Question	Scheme	Marks	AOs
(a)	$\left[\text{P}(T<1)=\frac{1-0.5}{2.5-0.5}\right]=\frac{1}{4}$	B1	3.4
		(1)	
(b)	$\text{P}\left(\{T<2.25\}\cap\left\{\frac{1}{T^2}<2.25\right\}\right)=\text{P}\left(\{T<2.25\}\cap\{T^2>\frac{4}{9}\}\right)$	M1	2.1
	$\text{P}(\frac{2}{3}<T<2.25)=\frac{2.25-\frac{2}{3}}{2.5-0.5}$	M1	1.1b
	$=\frac{19}{24}$	A1	1.1b
		(3)	
(c)	$\text{E}(R)=1.5$	B1	1.1b
	$\text{E}\left(\frac{2}{R^2}\right)=\int_{0.5}^{2.5}\left(\frac{1}{2.5-0.5}\right)\frac{2}{r^2}\,\text{d}r$	M1	3.1b
	$\left[-\frac{1}{r}\right]_{0.5}^{2.5}$	dM1	1.1b
	$=1.6$	A1	1.1b
	Greta is the expected winner since she has the higher expected value $(1.6>1.5)$	A1	2.2b
		(5)	
			(9 marks)

Notes

(a)	**B1:** 0.25 oe
(b)	**M1:** Determining the conditions for both numbers to be smaller than 2.25 **M1:** Use of uniform distribution for their region for T **A1:** allow awrt 0.792
(c)	**B1:** 1.5 **M1:** Attempt to set up an integral for Greta's expectation **dM1:** (dep on previous M1) for integration of expectation **A1:** 1.6 **A1:** Greta with correct supporting reason and all previous marks scored in (c) **SC:** Use of $R=\frac{2}{R^2}\to R=\sqrt[3]{2}\to 1.5>\sqrt[3]{2}(=1.25...)$ therefore Raja is more likely to win a single game, scores B1M0M0A0A1.

Q17.

Question	Scheme	Marks	AOs
	$P = 2(L + \frac{40}{L})$	M1	3.1a
	$f(l) = \frac{1}{6}$	B1	1.1b
	$E(P) = E(2(L + \frac{40}{L})) = \int_4^{10} \frac{1}{6}(\text{"}2l + \frac{80}{l}\text{"})\,dl$	M1	2.1
	$= \left[\frac{1}{6}l^2 + \frac{40}{3}\ln\|l\|\right]_4^{10}$	M1A1	1.1b 1.1b
	$= (\frac{1}{6}(100) + \frac{40}{3}\ln 10) - (\frac{1}{6}(16) + \frac{40}{3}\ln 4))$	M1	1.1b
	$= 26.217...$ awrt 26.2	A1	1.1b
			(7 marks)

Notes

M1: Finding an expression for the perimeter in terms of L
B1: Correct distribution for L (may be implied by $E(L) = 7$)
M1: Setting up integral for expectation of perimeter

(allow 2 separate integrals e.g. $2(7) + \int_4^{10} \frac{1}{6}(\frac{80}{l})\,dl$)

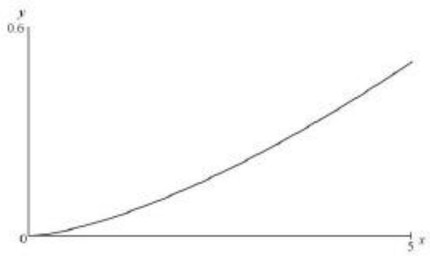
M1: Attempt to integrate an expression for expectation of perimeter (allow two separate expressions)
A1: Correct integration
depM1: (dep on previous M1) Use of correct limits 10 and 4
A1: awrt 26.2 Note: exact value is $14 + \dfrac{40\ln(2.5)}{3}$

Q18.

Question	Scheme	Marks	AOs
(a)	$\frac{1}{12}(a-5)^2=\frac{27}{4}$	M1	3.1a
	$(a-5)^2=81$		
	$a-5=9$ **or** $a-5=-9$	A1	1.1b
	$\therefore$ since $a>5$ $a=14$*	A1cso*	2.2a
		(3)	
(b)	Correct method for $E(Y)$, $E(X)$ and $E(X^2)$ **or** $E(Y)$ and $E(X^2+X)$	M1	3.1a
	$E(Y)=\int_2^6 \frac{1}{20}y(2y-3)\,dy$	M1	1.1b
	$=\frac{68}{15}$	A1	1.1b
	$E(X)=\frac{5+14}{2}$ or 9.5 **and** $\frac{27}{4}=E(X^2)-9.5^2$ or $\int_5^{14}\frac{x^2}{9}dx$ **or** $\int_5^{14}\left(\frac{x^3}{9}+\frac{x^2}{9}\right)dx$ or $3\int_5^{14}\left(\frac{x^3}{9}+\frac{x^2}{9}\right)dx$	M1	1.1b
	$E(X^2)=97$ and $E(X)=9.5$ **or** $E(X^2+X)=106.5$ **or** $3E(X^2+X)=319.5$	A1	1.1b
	$E(T)=3\times\text{"97"}+3\times\text{"9.5"}+2\times\frac{68}{15}$ oe	M1	1.1b
	$E(T)=\frac{9857}{30}$ *	A1*cso	2.1
		(7)	
			(10 marks)

Notes
(a) **M1:** translating a problem in mathematical contexts into a correct equation. Allow $\frac{a^3-125}{3(a-5)}-\left(\frac{a+5}{2}\right)^2=\frac{27}{4}$
A1: for $a-5=9$ or $a-5=-9$ or $a^2-10a-56=0$ or $a^3-15a^2-6a+280=0$
A1cso*: concluding it is 14 giving a reason why -4 is rejected
(b) **M1:** For a complete method to solve the problem
M1: For an attempt at $E(Y)$
A1: $=\frac{68}{15}$ or awrt 4.53
M1: For an attempt at $E(X)$ and $E(X^2)$ or $E(X^2+X)$ or $3E(X^2+X)$ Some sort of working must be seen for $E(X^2)$ eg $\frac{27}{4}=E(X^2)-E(X)^2$. Allow $Var(X)=E(X^2)-E(X)^2$ leading to $=E(X^2)$
A1: 319.5
M1: Method for finding $E(T)$ ft their values
A1*cso: Fully correct solution no errors, must have $E(T)=\frac{9857}{30}$ *

Q19.

Question	Scheme	Marks	AOs
(a)	$P(A > 3) = \frac{2}{5}$	B1	1.1b
	$\left(\frac{2}{5}\right)^3 = \frac{8}{125}$	M1 A1	1.1a 1.1b
		(3)	
(b)	$f(y) = \frac{3y^2}{125}$	M1	2.1
	$E(Y) = \int_0^5 \frac{3y^3}{125}\,dy$ $= \left[\frac{3y^4}{500}\right]_0^5 \quad \left[= \frac{15}{4}\right]$	M1	1.1b
	$\text{Var}(Y) = \int_0^5 \left(\frac{3y^4}{125}\right) dy - \left(\frac{15}{4}\right)^2$	M1	1.1b
	$= 0.9375$*	A1*cso	1.1b
		(4)	
(c)	Mode = 5	B1	1.2
	 Or reason based on $\frac{df(y)}{dy} > 0$	B1	2.4

Question	Scheme	Marks	AOs
(d)	From a sketch or mode > mean therefore it has negative skew	B1ft	2.4
		(1)	
(e)	$\frac{(2k)^3}{125} - \frac{k^3}{125} = 0.189$	M1	3.1a
	$\frac{7k^3}{125} = 0.189$	A1	1.1b
	$k = 1.5$	A1	1.1b
		(3)	
			(13 marks)

Notes	
(a)	B1: $\frac{2}{5}$ o.e. may be implied by a correct answer M1: $\left(\text{"their}\left(\frac{2}{5}\right)\text{"}\right)^3$ may be implied by a correct answer A1: $\frac{8}{125}$ o.e.
(b)	M1: realising that firstly need to find pdf f(y) and attempt to differentiate F(y) M1: Continuing the argument with an attempt to integrate $y\times$"their f(y)" $y^n \to y^{n+1}$ M1: Integrating $y^2\times$"their f(y)" - ["their E(Y)"]^2 $y^n \to y^{n+1}$ A1*: Complete correct solution no errors.
(c)	B1: 5 only B1: Explain their reason by either an accurate sketch or $\frac{\text{df}(y)}{\text{d}y} > 0$ therefore an increasing function oe
(d)	B1ft: Explaining the reason for their answer. Follow through their part(b) or mean from(d) and mode from(c). A correct sketch of "their f(y)" – may be seen anywhere in question or ft their mean and mode plus a correct conclusion NB: Watch for gaming. A student who writes both negative skew with a reason and positive skew with a reason. Please send these to your Team Leader
(e)	M1: Attempting to translate the problem into an equation using $2k$ and k. Allow if the brackets are missing e.g. $\frac{2k^3}{125}-\frac{k^3}{125}$. No need for the 0.189 A1: A correct equation in any form A1: a correct answer only.

Q20.

Question	Scheme	Marks	AOs
(a)		B1 (shape) B1 (labels)	1.1b 1.1b
		(2)	
(b)	$\mathrm{P}(X < 2(k - X)) = \mathrm{P}(X < \frac{2}{3}k)$	M1	3.1a
	$\dfrac{\frac{2}{3}k-(-3)}{5-(-3)} = 0.25$	M1	1.1b
	$k = -\dfrac{3}{2}$	A1	1.1b
		(3)	
(c)	$\mathrm{E}(X^3) = \int_{-3}^{5} \dfrac{1}{5-(-3)} x^3 \mathrm{d}x$	M1	2.1
	$= \left[\dfrac{1}{32}x^4\right]_{-3}^{5} = \frac{1}{32}(5^4 - (-3)^4)$	dM1	1.1b
	$= 17$*	A1*cso	1.1b
		(3)	
			(8 marks)

Notes		
(a)	1st B1 for correct shape 2nd B1 for correct labels	
(b)	1st M1 for simplifying to $\mathrm{P}(X < \frac{2}{3}k)$ 2nd M1 for equating probability expression to 0.25 A1 for $-\dfrac{3}{2}$	1st M1 for understanding $2[k - x] = -1$ and $x = -1$ 2nd M1 for substitution and attempt to solve A1 for $-\dfrac{3}{2}$
(c)	1st M1 for integrating $x^3\,\mathrm{f}(x)$ 2nd M1 for use of correct limits (dependent on previous M1) A1*cso for fully correct solution leading to the given answer with no errors seen	

Q21.

Question	Scheme		Marks	AOs
(a)	(Continuous) uniform or rectangular		B1	1.2
			(1)	
(b)	[$P(Y>0) = P(5-2X>0) =$] $P(X<2.5)$	$f(y) = \begin{cases}\frac{1}{16} & -13 \le y \le 3\end{cases}$	M1	1.1b
	$\frac{2.5-1}{8} = \underline{\frac{3}{16}}$	$\frac{3-0}{16} = \underline{\frac{3}{16}}$	A1	1.1b
			(2)	
(c)	$E(Y) = 5 - 2E(X)$ [$=5-2(\frac{1+9}{2})$]	$E(Y) = \frac{-13+3}{2}$	M1	1.1b
	$= \underline{-5}$		A1	1.1b
			(2)	
(d)	$[P(Y<0) \mid (X<7.5)] = \frac{P(2.5<X<7.5)}{P(X<7.5)}$		M1	3.1a
	$= \frac{\frac{7.5-2.5}{8}}{\frac{7.5-1}{8}} \left[= \frac{0.625}{0.8125}\right]$		M1	1.1b
	$= \underline{\frac{10}{13}}$		A1	1.1b
			(3)	
				(8 marks)

Notes	
(a)	B1 for (Continuous) uniform or rectangular Discrete uniform is B0
(b)	M1 for using the distribution of X to obtain $P(X<2.5)$ <u>or</u> for finding the distribution of Y in the range $-13 \le y \le 3$ A1 for $\frac{3}{16}$ or 0.1875
(c)	M1 for use of $E(aX+b)$ <u>or</u> for use of $\frac{a+b}{2}$ from the distribution of Y A1 for -5
(d)	1st M1 for a correct ratio expression 2nd M1 for a correct numerical expression A1 for $\frac{10}{13}$ SC: If M0M0A0 scored, then a correct numerator or correct denominator scores M0M1A0

Q22.

Question	Scheme		Marks	AOs
(a)	$F(3)=0$ or $F(9)=1$		M1	3.1a
	$c-4.5(3^n)=0$ and $c-4.5(9^n)=1$		A1	1.1b
	Eliminating c $1+4.5(9^n)=4.5(3^n)$	Eliminating n $\frac{\log(\frac{c-1}{4.5})}{\log(9)}=\frac{\log(\frac{c}{4.5})}{\log(3)}$	M1	1.1b
	Forming a 3 term quadratic $4.5(3^{2n})-4.5(3^n)+1=0$	Forming a 3 term quadratic $4.5c^2-20.25c+20.25=0$	M1	3.1a
	Solving 3TQ leading to a value for n $3^n=\frac{1}{3}\rightarrow n=\ldots$ or $3^n=\frac{2}{3}\rightarrow n=\ldots$	Solving 3TQ leading to a value for c $c=1.5$ or $c=3$	M1	1.1b
	$n=-1$ only (reject other solution as n is an integer)		A1	2.3
	$c=1.5$ only		A1ft	1.1b
			(7)	
(b)	[Let q = lower quartile] "1.5" $-4.5(q^{"-1"})=0.25$		M1	1.1b
	$q=3.6$		A1ft	1.1b
			(2)	
				(9 marks)

Notes	
(a)	1st M1 for use of either $F(3)=0$ or $F(9)=1$ 1st A1 for two correct equations in c and n 2nd M1 for eliminating one variable 3rd M1 for realising that a quadratic formula is required to solve 4th M1 for solving the quadratic formula leading to at least one value of c or n 2nd A1 for $n=-1$ only 3rd A1ft for $c=1.5$ only (ft their n in a correct equation and dep on 1st M1)
(b)	M1 for use of $F(q)=0.25$ A1ft for $q=3.6$ (allow follow through on their integer value of n)