Complex Numbers (FP2)

Questions

Q1.

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + 3\mathrm{i}}{1 + \mathrm{i}z}, \quad z \neq \mathrm{i}$$

The transformation T maps the circle $|z| = 1$ in the z-plane onto the line l in the w-plane.

(a) Find a cartesian equation of the line l.

(5)

The circle $|z - a - b\mathrm{i}| = c$ in the z-plane is mapped by T onto the circle $|w| = 5$ in the w-plane.

(b) Find the exact values of the real constants a, b and c.

(6)

(Total for question = 11 marks)

Q2.

Sketch on an Argand diagram the region defined by

$$\left\{z \in \mathbb{C}: -\frac{\pi}{4} < \arg(z+2) < \frac{\pi}{4}\right\} \cap \{z \in \mathbb{C}: -1 < \mathrm{Re}(z) \leqslant 1\}$$

On your sketch

- shade the part of the diagram that is included in the region
- use solid lines to show the parts of the boundary that are included in the region
- use dashed lines to show the parts of the boundary that are not included in the region

(Total for question = 4 marks)

Q3.

A transformation from the *z*-plane to the *w*-plane is given by

$$w = \frac{3iz - 2}{z + i} \qquad z \neq -i$$

(a) Show that the circle *C* with equation $|z + i| = 1$ in the *z*-plane is mapped to a circle *D* in the *w*-plane, giving a Cartesian equation for *D*.

(4)

(b) Sketch *C* and *D* on Argand diagrams.

(2)

(Total for question = 6 marks)

Q4.

A curve *C* in the complex plane is described by the equation

$$|z - 1 - 8i| = 3|z - 1|$$

(a) Show that *C* is a circle, and find its centre and radius.

(4)

(b) Using the answer to part (a), determine whether $z = 3 - 3i$ satisfies the inequality

$$|z - 1 - 8i| \geqslant 3|z - 1|$$

(2)

(c) Shade, on an Argand diagram, the set of points that satisfies both

$$|z - 1 - 8i| \geqslant 3|z - 1| \quad \text{and} \quad 0 \leqslant \arg(z + i) \leqslant \frac{\pi}{4}$$

(4)

(Total for question = 10 marks)

Q5.

The point *P* in the complex plane represents a complex number *z* such that

$$|z+9| = 4|z-12\mathrm{i}|$$

Given that, as *z* varies, the locus of *P* is a circle,

(a) determine the centre and radius of this circle.

(6)

(b) Shade on an Argand diagram the region defined by the set

$$\{z \in \mathbb{C} : |z+9| < 4|z-12\mathrm{i}|\} \cap \left\{z \in \mathbb{C} : -\frac{\pi}{4} < \arg\left(z - \frac{3+44\mathrm{i}}{5}\right) < \frac{\pi}{4}\right\}$$

(4)

(Total for question = 10 marks)

Q6.

A complex number $z = x + \mathrm{i}y$ is represented by the point *P* in an Argand diagram.

Given that

$$|z-3| = 4|z+1|$$

(a) show that the locus of *P* has equation

$$15x^2 + 15y^2 + 38x + 7 = 0$$

(2)

(b) Hence find the maximum value of $|z|$

(3)

(Total for question = 5 marks)

Q7.

The locus of points z satisfies

$$|z + a\mathrm{i}| = 3|z - a|$$

where a is an integer.

The locus is a circle with its centre in the third quadrant and radius $\frac{3}{2}\sqrt{2}$

Determine

(a) the value of a,

(4)

(b) the coordinates of the centre of the circle.

(2)

(Total for question = 6 marks)

Q8.

The locus of points $z = x + \mathrm{i}y$ that satisfy

$$\arg\left(\frac{z - 8 - 5\mathrm{i}}{z - 2 - 5\mathrm{i}}\right) = \frac{\pi}{3}$$

is an arc of a circle C.

(a) On an Argand diagram sketch the locus of z.

(2)

(b) Explain why the centre of C has x coordinate 5

(1)

(c) Determine the radius of C.

(2)

(d) Determine the y coordinate of the centre of C.

(2)

(Total for question = 7 marks)

Q9.

A curve has equation

$$|z+6| = 2|z-6| \quad z \in \mathbb{C}$$

(a) Show that the curve is a circle with equation $x^2 + y^2 - 20x + 36 = 0$

(2)

(b) Sketch the curve on an Argand diagram.

(2)

The line l has equation $az^* + a^*z = 0$, where $a \in \mathbb{C}$ and $z \in \mathbb{C}$

Given that the line l is a tangent to the curve and that $\arg a = \theta$

(c) find the possible values of $\tan\theta$

(5)

(Total for question = 9 marks)

Q10.

A transformation from the z-plane to the w-plane is given by

$$w = z^2$$

(a) Show that the line with equation $\text{Im}(z) = 1$ in the z-plane is mapped to a parabola in the w-plane, giving an equation for this parabola.

(4)

(b) Sketch the parabola on an Argand diagram.

(2)

(Total for question = 6 marks)

Q11.

A complex number z is represented by the point P on an Argand diagram.

Given that $\arg\left(\frac{z-6\mathrm{i}}{z-3\mathrm{i}}\right)=\frac{\pi}{3}$

(a) sketch the locus of P as z varies,

(3)

(b) find the exact maximum possible value of $|z|$

(5)

(Total for question = 8 marks)

Q12.

A curve C is described by the equation

$$|z-9+12\mathrm{i}|=2|z|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Sketch C on an Argand diagram.

(2)

Given that w lies on C,

(c) find the largest value of a and the smallest value of b that must satisfy

$$a \le \mathrm{Re}(w) \le b$$

(2)

(Total for question = 8 marks)

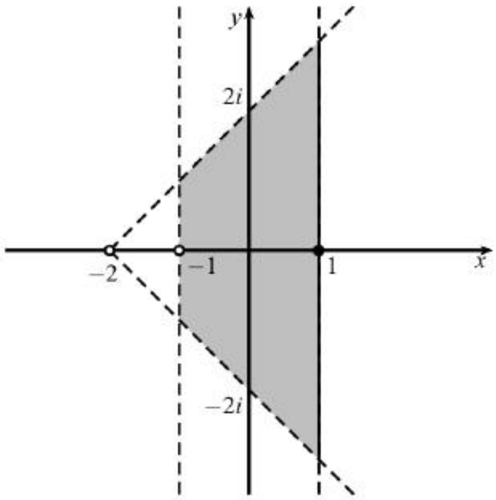
Mark Scheme – Complex Numbers (FP2)

Q1.

Question Number	Scheme	Notes	Marks
	$w = \dfrac{z+3\mathrm{i}}{1+\mathrm{i}z}$		
(a)	$z = \dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}$ oe	M1: Attempt to make z the subject A1: Correct equation	M1A1
	$\lvert z\rvert = 1 \Rightarrow \left\lvert\dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}\right\rvert = 1 \Rightarrow \lvert w-3\mathrm{i}\rvert = \lvert 1-w\mathrm{i}\rvert$ $\therefore\ \lvert u+\mathrm{i}v-3\mathrm{i}\rvert = \lvert(u+\mathrm{i}v)\mathrm{i}-1\rvert$	Uses $\lvert z\rvert = 1$ and introduce "$u + \mathrm{i}v$" (or $x+\mathrm{i}y$) for w	M1
	$u^2+(v-3)^2 = u^2+(v+1)^2$	Correct use of Pythagoras on either side.	M1
	$v = 1$ oe	$v = 1$ or $y = 1$	A1
			(5)
	Alternative 1 for (a)		
	eg $w(1) = \dfrac{1+3\mathrm{i}}{1+\mathrm{i}} = 2+\mathrm{i}$	M1: Maps one point on the circle using the given transformation A1:Correct mapping	M1A1
	eg $w(-\mathrm{i}) = \dfrac{2\mathrm{i}}{2} = \mathrm{i}$	Maps a second point on the circle	M1
	$v = 1$ oe	M1: Forms Cartesian equation using their 2 points A1: $v = 1$ or $y = 1$	M1A1
	Alternative 2 for (a)		
	$z = \dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}$ oe	M1: Attempt to make z the subject A1: Correct equation	M1A1
	$\lvert z\rvert = 1 \Rightarrow \left\lvert\dfrac{w-3\mathrm{i}}{1-\mathrm{i}w}\right\rvert = 1 \Rightarrow \lvert w-3\mathrm{i}\rvert = \lvert 1-w\mathrm{i}\rvert$ $\lvert w-3\mathrm{i}\rvert = \lvert w+\mathrm{i}\rvert = \lvert w-(-\mathrm{i})\rvert$	Uses $\lvert z\rvert = 1$ and changes to form $\lvert w-...\rvert = \lvert w-...\rvert$ or draws a diagram	M1
	Perpendicular bisector of points $(0,3)$ and $(0,-1)$	Uses a correct geometrical approach	M1
	$v = 1$ oe	$v = 1$ or $y = 1$	A1

	Alternative 3 for (a)		
	Let $z = x + \mathrm{i}y$, $\lvert z\rvert = 1 \Rightarrow x^2 + y^2 = 1$		
	$w = \frac{z+3\mathrm{i}}{1+\mathrm{i}z} = \frac{x+\mathrm{i}y+3\mathrm{i}}{1+\mathrm{i}(x+\mathrm{i}y)} = \frac{x+\mathrm{i}(y+3)}{(1-y)+\mathrm{i}x}$		
	$w = \frac{x+\mathrm{i}(y+3)}{(1-y)+\mathrm{i}x} \times \frac{(1-y)-\mathrm{i}x}{(1-y)-\mathrm{i}x}$	Substitute $z = x + \mathrm{i}y$ and multiply numerator and denominator by complex conjugate of their denominator	M1
	$w = \frac{x(1-y)-\mathrm{i}x^2+\mathrm{i}(y+3)(1-y)-\mathrm{i}^2x(y+3)}{(1-y)^2-\mathrm{i}x(1-y)+\mathrm{i}x(1-y)-\mathrm{i}^2x^2}$		
	$w = \frac{\left[x(1-y)+x(y+3)\right]+\mathrm{i}\left[-x^2+(y+3)(1-y)\right]}{(1-y)^2+x^2}$	M1: Multiply out and collect real and imaginary parts in numerator. Denominator must be real. A1: all correct	M1 A1
	$w = \frac{\left[x-xy+xy+3x\right]+\mathrm{i}\left[-x^2+y-y^2+3-3y\right]}{1-2y+y^2+x^2}$		
	$w = \frac{[4x]+\mathrm{i}[-1+3-2y]}{2-2y}$	Applies $x^2 + y^2 = 1$	M1
	$w = \frac{4x+\mathrm{i}[2-2y]}{2-2y} = \frac{4x}{2-2y} + \mathrm{i}$		
	$y = 1$	$y = 1$ or $v = 1$	A1
(b)	$\lvert w\rvert = 5 \Rightarrow \left\lvert\frac{z+3\mathrm{i}}{1+\mathrm{i}z}\right\rvert = 5 \Rightarrow \lvert z+3\mathrm{i}\rvert = 5\lvert 1+\mathrm{i}z\rvert$ $\therefore\ \lvert x+\mathrm{i}y+3\mathrm{i}\rvert = 5\lvert (x+\mathrm{i}y)\mathrm{i}+1\rvert$	Uses $\lvert w\rvert = 5$ and introduce "$x + \mathrm{i}y$"	M1
	$x^2 + (y+3)^2 = 25\left(x^2 + (1-y)^2\right)$	M1: Correct use of Pythagoras Allow 25 or 5 A1: Correct equation	M1A1
	$x^2 + y^2 - \frac{7}{3}y + \frac{2}{3} = 0$		
	$x^2 + \left(y - \frac{7}{6}\right)^2 = \frac{25}{36}$	Attempt circle form or attempt r^2 from the line above.	M1
	$a = 0,\ b = \frac{7}{6},\ c = \frac{5}{6}$	A1: 2 correct A1: All correct	A1, A1
			(6)
			Total 11
	Or, for the last 3 marks:		
	$\left\lvert z - 0 - \frac{7}{6}\mathrm{i}\right\rvert = \frac{5}{6}$		M1A1A1
	If 0 not shown score M1A1A0		
	No need to list a, b, c separately if answer in this form.		

Q2.

Question	Scheme		Marks	AOs
		Sector from ±2 or ±2i	M1	1.1b
		Strip about either axis	M1	1.1b
		Correct strip and sector	A1	3.1a
		Fully correct	A1	2.5
			(4)	
			(4 marks)	

Notes:

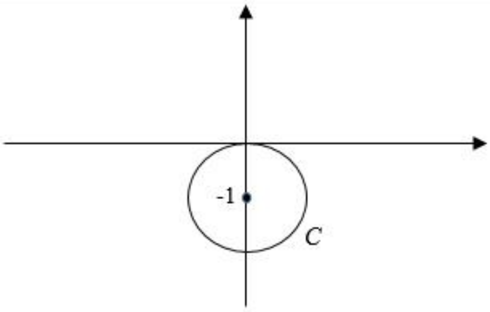
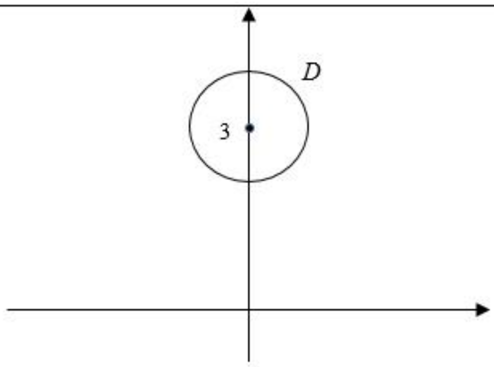
M1: Draws or identifies a sector, or relevant part thereof, spanning from either ±2 or ±2i.

M1: Either a vertical or horizontal strip drawn or identified, with lines either imaginary or real axis.

A1: Correct sector and strip – so starting sector from −2 on negative real axis, with vertical strip about imaginary axis approximately halfway on real axis between start of sector and *O*. Sector at ±45° (do not worry about the boundary lines for this mark). The strip must be roughly even spaced either side of the imaginary axis, and the sector roughly symmetric in the real axis (allow small tolerance, but A0 if clearly not symmetric)

A1: Inside strip and sector shaded, with correct boundary lines.

Q3.

Question	Scheme	Marks	AOs
(a)	$w=\dfrac{3\mathrm{i}z-2}{z+\mathrm{i}} \Rightarrow z=\dfrac{2+w\mathrm{i}}{3\mathrm{i}-w}$	M1	2.1
	$\lvert z+\mathrm{i}\rvert=1 \Rightarrow \left\lvert\dfrac{2+w\mathrm{i}}{3\mathrm{i}-w}+\mathrm{i}\right\rvert=1$	M1	2.1
	$\left\lvert\dfrac{2+w\mathrm{i}-3-\mathrm{i}w}{3\mathrm{i}-w}\right\rvert=1 \Rightarrow \left\lvert\dfrac{1}{w-3\mathrm{i}}\right\rvert=1$	A1	1.1b
	$\lvert w-3\mathrm{i}\rvert=1 \Rightarrow u^2+(v-3)^2=1$	A1	2.2a
		(4)	
(b)		B1	1.1b
		B1ft	1.1b
		(2)	
			(6 marks)

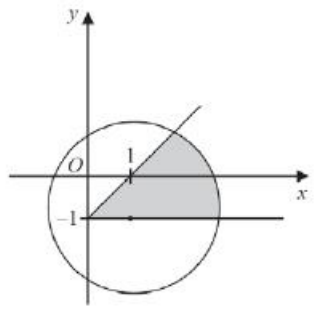
Notes
(a) M1: Attempts to solve the problem by attempting to make z the subject of the formula M1: Uses the given locus to obtain an equation in w A1: Obtains a correct simplified equation in terms of w A1: Deduces the correct form of the equation (allow x, y or u, v etc.) (b) B1: The circle C correctly positioned, passing through the origin coordinates of centre labelled. Accept as coordinates or marked on axes. B1ft: Their D correctly positioned with the centre correctly labelled (accept as coordinates or marked on axes). Accept both drawn on the same diagram. Allow S.C. B1B0 for two circles in correct respective positions but with no labelling.

(a) **ALT 1**	$w=\dfrac{3\mathrm{i}z-2}{z+\mathrm{i}} \Rightarrow w(z+\mathrm{i})=3\mathrm{i}(z+\mathrm{i})+3-2$ Attempts to isolate $z+\mathrm{i}$ terms	M1	2.1
	$(z+\mathrm{i})(w-3\mathrm{i})=1 \Rightarrow \lvert(z+\mathrm{i})(w-3\mathrm{i})\rvert=1 \Rightarrow \lvert(w-3\mathrm{i})\rvert=1$ Gathers $z+\mathrm{i}$ terms and applies $\lvert(z+\mathrm{i})\rvert=1$	M1	2.1
	As main scheme	A1	1.1b
	As main scheme	A1	2.2a
(a) **ALT 2**	$w=\dfrac{3\mathrm{i}z-2}{z+\mathrm{i}} \Rightarrow z=\dfrac{2+w\mathrm{i}}{3\mathrm{i}-w}$ as main scheme	M1	2.1
	$x+y\mathrm{i}=\dfrac{2-v+u\mathrm{i}}{-u-(v-3)\mathrm{i}}\times\dfrac{-u+(v-3)\mathrm{i}}{-u+(v-3)\mathrm{i}}=\ldots=\dfrac{u-(u^2+v^2-5v+6)\mathrm{i}}{u^2+(v-3)^2}$ $\Rightarrow\left(\dfrac{u}{u^2+(v-3)^2}\right)^2+\left(\dfrac{-(u^2+v^2-5v+6)}{u^2+(v-3)^2}+1\right)^2=1$ Applies Cartesian coordinates to both sides, extracts x and y terms and attempts to apply $x^2+(y+1)^2=1$	M1	2.1
	$\left(\dfrac{u}{u^2+(v-3)^2}\right)^2+\left(\dfrac{3-v}{u^2+(v-3)^2}\right)^2=1$ Correct expression with $y+1$ term combined and simplified.	A1	1.1b
	$\Rightarrow u^2+(v-3)^2=1$	A1	2.2a

(a) **ALT 3**	$u+\mathrm{i}v=\dfrac{3\mathrm{i}(x+\mathrm{i}y)-2}{x+\mathrm{i}y+\mathrm{i}}\times\dfrac{x-(y+1)\mathrm{i}}{x-(y+1)\mathrm{i}}=\dfrac{\mathrm{f}(x,y)+\mathrm{g}(x,y)\mathrm{i}}{x^2+(y+1)^2}$ Applies Cartesian coordinates to expression and use complex conjugate of denominator to reach Cartesian form.	M1	2.1
	$x^2+(y+1)^2=1 \Rightarrow u+\mathrm{i}v=x+(3x^2+3y^2+5y+2)\mathrm{i}$ $\Rightarrow u=x$ and $v=3x^2+3(y+1)^2-y-1=a+by$ Uses $x^2+(y+1)^2=1$ in their equation and extract u and v as linear terms in x and y	M1	2.1
	$u=x$ and $v=2-y$ Correct u and v	A1	1.1b
	$\Rightarrow u^2+(2-v+1)^2=1 \Rightarrow u^2+(v-3)^2=1$ Uses $x^2+(y+1)^2=1$ again to find correct equation.	A1	2.2a

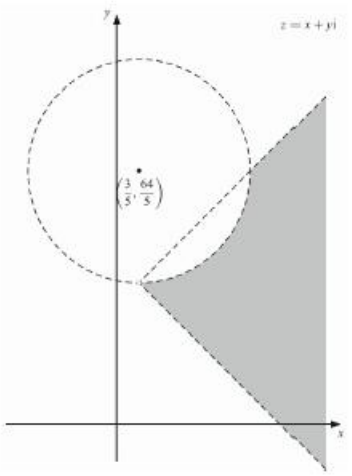
Note that there may be attempts via identifying images of points on a diameter. If seen, send to review.

Q4.

Question	Scheme	Marks	AOs
(a)	$(x-1)^2+(y-8)^2=9\left[(x-1)^2+y^2\right]$ Or $\sqrt{(x-1)^2+(y-8)^2}=3\sqrt{(x-1)^2+y^2}$	M1	2.1
	$8x^2-16x+8y^2+16y-56=0$	A1	1.1b
	$x^2-2x+y^2+2y-7=0$ so $(x-1)^2+(y+1)^2=9$ and finds the centre and radius	M1	1.1b
	Therefore, a circle with centre $(1, -1)$ and radius $= 3$	A1	2.2a
		(4)	
(b)	Distance $=\sqrt{(3-1)^2+(-3--1)^2}=...$ or finds $\left(d^2=\right)(3-1)^2+(-3--1)^2=...$	M1	1.1b
	Distance $=\sqrt{8}=2.828<3$ $\therefore$ $z=3-3\text{i}$ **satisfies the inequality** Or $8<9$ $\therefore$ $z=3-3\text{i}$ **satisfies the inequality**	A1	2.2a
		(2)	
(c)	 Circle with their centre and radius	M1	1.1b
	Circle with centre in the fourth quadrant	A1	1.1b
	Half line drawn from $(0, -1)$ and passes through the x-axis within the circle	M1	1.1b
	Correct region shaded	A1	2.2a
		(4)	
			(10 marks)

Notes
(a) **M1:** Obtains an equation in terms of x and y using the given information. Condone $(x-1)^2+(y-8)^2=3\left[(x-1)^2+y^2\right]$ for this mark. **A1:** Expands and simplifies the algebra, collecting terms and obtains a correct equation. **M1:** Completes the square for their equation to find the centre and radius. **A1**: Deduces that it is a circle (may be seen anywhere in their solution) with centre $(1, -1)$ and radius $= 3$
(b) **M1:** Finds the distance between $(3,-3)$ and their centre or d^2 (note: correct centre is $(1,-1)$ **A1:** Compares distance with 3 or compares d^2 with 9 and deduces that the inequality is satisfied – must be using correct centre and radius.
(c) **M1:** Circle for their centre and radius. **A1:** Correct circle with centre in the fourth quadrant and passing through all four quadrants. Condone dotted circle. **M1:** Half line drawn from $(0,-1)$ and passing the x-axis within the circle. Condone dotted line. **A1:** Correct region shaded with both half-line and circle correct and not dotted. <u>**Special case:**</u> M1A1M1A0 if no coordinates stated throughout and it is clear that the half-line intersects the coordinate axes level with the correct centre of the circle.

Q5.

Question	Scheme	Marks	AOs
(a)	$z = x + \mathrm{i}y \Rightarrow \lvert x + 9 + \mathrm{i}y \rvert = 4\lvert x + (y-12)\mathrm{i} \rvert$	M1	1.1b
	$\Rightarrow (x+9)^2 + y^2 = 16\left(x^2 + (y-12)^2\right)$	M1 A1	1.1b 1.1b
	$\Rightarrow 15x^2 + 15y^2 - 18x - 384y = 81 - 16\times 12^2$ $\Rightarrow x^2 + y^2 - \frac{6}{5}x - \frac{128}{5}y = -\frac{741}{5}$		
	$\Rightarrow \left(x - \frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 + \left(y - \frac{64}{5}\right)^2 - \left(\frac{64}{5}\right)^2 = -\frac{741}{5}$ $\left(\Rightarrow \left(x - \frac{3}{5}\right)^2 + \left(y - \frac{64}{5}\right)^2 = 16\right)$	M1	2.1
	centre $\frac{3}{5} + \frac{64}{5}\mathrm{i}$ or radius 4	A1	2.2a
	centre $\frac{3}{5} + \frac{64}{5}\mathrm{i}$ and radius 4	A1	2.2a
		(6)	
(b)	 Circle, with centre in correct quadrant for their answer to (a)	M1	1.1b
	Pair of rays at roughly 45° to horizontal, with source in first quadrant OR on the circle.	M1	1.1b
	Correct circle and rays, circle with centre in first quadrant and spanning only quadrant 1 and 2 and pair of rays at roughly 45° to horizontal, meeting at the bottom point of the circle	A1	3.1a
	Region between rays and outside circle shaded	B1ft	3.1a
		(4)	
			(10 marks)

Notes:
(a) **M1:** Applies $z = x + \mathrm{i}y$ to the given equation. Use of other letters, eg $z = u + \mathrm{i}v$ is fine. **M1:** Squares and uses modulus to achieve $(x+a)^2 + y^2 = K\left(x^2 + (y+b)^2\right)$ **A1:** Correct equation, need not be expanded. Award when first seen. **M1:** Expands, gathers terms and completes the square.

A1: Either centre or radius correct. Accept coordinates for centre. **A1:** Correct centre and radius. Accept coordinates for centre.
(b) **M1:** Sketches their circle on an Argand diagram. Look for the centre being in the correct quadrant for their answer to (a). **M1:** Pair of rays added to the sketch, at angles $\frac{\pi}{4}$ above and below the horizontal with vertex in the first quadrant OR somewhere on the circle. Need not stem from base of circle for this mark if it stems from the first quadrant, but if not in the first quadrant it must stem from the circle. **A1:** Circle (or arc) in correct position, centre in first quadrant that would span quadrants 1 and 2, with pair of rays at roughly 45° to horizontal, meeting at the bottom point of the circle. **B1ft:** Area outside the circle and between the rays (minor sector) shaded provided the rays span approximately a 90° sector. NB Only the region is asked for, so allow the marks above if only the relevant part of the circle is shown.

Q6.

Question	Scheme	Marks	AOs
(a)	$(x-3)^2+y^2=16\left((x+1)^2+y^2\right)$	M1	1.1b
	$x^2-6x+9+y^2=16x^2+32x+16+16y^2$ $15x^2+15y^2+38x+7=0*$	A1*	2.1
		(2)	
(b)	$15x^2+15y^2+38x+7=15\left(x\pm\frac{19}{15}\right)^2-...+15y^2+7=0$	M1	2.1
	Centre is $\left(-\frac{"19"}{5},0\right)$ and radius is $\sqrt{\left(\frac{"19"}{15}\right)^2-\frac{7}{15}}\left(=\frac{16}{15}\right)$	M1	2.2a
	$\max\lvert z\rvert=\frac{16}{15}+\frac{19}{15}=\frac{7}{3}$	A1	3.1a
		(3)	
			(5 marks)

Notes
(a) M1: Obtains an equation in terms of x and y using the given information. Allow if the 4 is not squared, but i^2 must have been dealt with correctly (ie positive y^2 terms). Condone invisible brackets for the M mark. A1*: Expands and simplifies and obtains a circle equation correctly. Accept terms in different order but must include =0. No errors seen, so bracketing errors in solution are A0. (b) M1: Completes the square on the x term achieving $A\left(x \pm \frac{19}{15}\right)^2 - B$, or uses other appropriate method in order to attempt the radius and/or centre of the circle. Award if correct x coordinate of centre or radius is found. M1: Deduces both centre and radius for their completed square form, either seen used in work clearly as centre and radius, stated or labelled on a diagram, not just embedded within the equation. This is implied by the correct calculation being carried out for their centre and radius. A1cso: Realises the need to add distance of centre from origin to radius to achieve the correct answer. Must come from correct work. Note that completing the square as $\left(x - \frac{19}{15}\right)^2 - \ldots$ can score a maximum M1M1A0

Q7.

Question	Scheme	Marks	AOs
(a)	$(x)^2+(y+a)^2=9[(x-a)^2+y^2]$ or $\sqrt{(x)^2+(y+a)^2}=3\sqrt{(x-a)^2+y^2}$	M1	2.1
	$8x^2-18ax+8y^2-2ay+8a^2\{=0\}$ o.e.	A1	1.1b
	$x^2-\frac{9}{4}ax+y^2-\frac{1}{4}ay+a^2=0$ $\Rightarrow\left(x-\frac{9}{8}a\right)^2-\left(\frac{9}{8}a\right)^2+\left(y-\frac{1}{8}a\right)^2-\left(\frac{1}{8}a\right)^2+a^2=0$ $\Rightarrow r^2=\left(\frac{9}{8}a\right)^2+\left(\frac{1}{8}a\right)^2-a^2=\left(\frac{3}{2}\sqrt{2}\right)^2\Rightarrow a=\ldots$	M1	1.1b
	$a=-4$ cso	A1	2.2a
		(4)	
(b)	$\left(x-\frac{9}{8}(-4)\right)^2+\left(y-\frac{1}{8}(-4)\right)^2=\ldots$ $(x-\alpha)^2+(y-\beta)^2=\ldots$ implies centre$(\alpha,\ \beta)$	M1	1.1b
	centre $\left(-\frac{9}{2},\ -\frac{1}{2}\right)$	A1	2.2a
		(2)	
		(6 marks)	

Notes:

(a)

M1: Obtains an equation in terms of x and y using the given information. Condone $(x)^2+(y+a)^2=3[(x-a)^2+y^2]$ for this mark

A1: Expands and simplifies the algebra, collects terms and obtains a correct simplified equation. Condone missing = 0

M1: Completes the square for their equation using $x^2+Ax=\left(x+\frac{A}{2}\right)^2+\ldots$Sets their radius squared $=\left(\frac{3}{2}\sqrt{2}\right)^2=\frac{18}{4}=\frac{9}{2}$or radius $=\left(\frac{3}{2}\sqrt{2}\right)$and finds a value for a.

Note the correct values are $r^2=\frac{9}{32}a^2$, $r=\frac{3a\sqrt{2}}{8}$

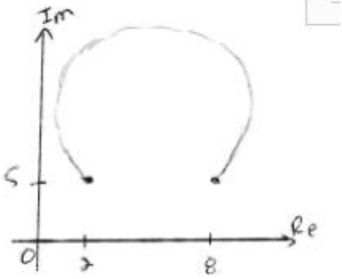
A1: Deduces that $a=-4$cso

(b)

M1: Substitutes their value for a into their equation and finds their centre.

A1: Deduces the correct centre.

Q8.

Question	Scheme		Marks	AOs
(a)		The major arc of a circle drawn anywhere	B1	1.1b
		The end points of their arc (2, 5) and (8, 5) and the arc drawn above the coordinates	B1	1.1b
			(2)	
(b)	The centre lies on the **perpendicular bisector/midpoint/equidistant** of **2 and 8**		B1	2.4
			(1)	

Question	Scheme	Marks	AOs
(c)	A complete method to find the radius of the circle $$sin\left(\frac{\pi}{3}\right) = \frac{3}{r} \Rightarrow r = ...$$ Or $$6^2 = r^2 + r^2 - 2 \times r \times r \times cos\left(\frac{2\pi}{3}\right) \Rightarrow r = ...$$ Or $$h = \frac{3}{tan\left(\frac{\pi}{3}\right)} = \sqrt{3} \Rightarrow r = \sqrt{(\sqrt{3})^2 + 3^2} = ...$$ or $$tan[(arg(x + yi - (8 + 5i)) - arg(x + yi - (2 + 5i))] = tan\left(\frac{\pi}{3}\right)$$ $$\frac{tan[arg(x + yi - (8 + 5i))] - tan[arg(x + yi - (2 + 5i))]}{1 + tan[arg(x + yi - (8 + 5i))]\, tan[arg(x + yi - (2 + 5i))]} = \sqrt{3}$$ $$\frac{\frac{y-5}{x-8} - \frac{y-8}{x-2}}{1 + \frac{y-5}{x-8} \times \frac{y-8}{x-2}} = \sqrt{3}$$ Leading to an equation of a circle by completing the square $(x - a)^2 + (y - b)^2 = r^2$ leading to $r = \ldots$	M1	3.1a
	$$r = \frac{6}{\sqrt{3}} \text{ or } 2\sqrt{3} \text{ o.e.}$$	A1	1.1b
		(2)	
(d)	$$y = 5 + h$$ where $h = \frac{3}{tan\left(\frac{\pi}{3}\right)}$ or $h = '2\sqrt{3}'\, cos\left(\frac{\pi}{3}\right)$ or $h = \sqrt{('2\sqrt{3}')^2 - 3^2}$	M1	3.1a

	$y = 5+\sqrt{3}$	A1	2.2a
		(2)	
			(7 Marks)

Notes:

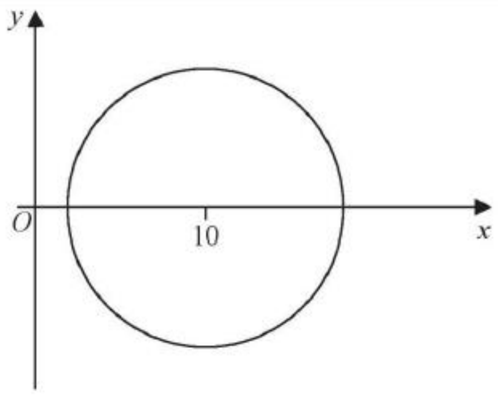
(a)
B1: Major arc drawn anywhere
B1: Correct end points for their arc drawn above the end points, condone written as complex numbers

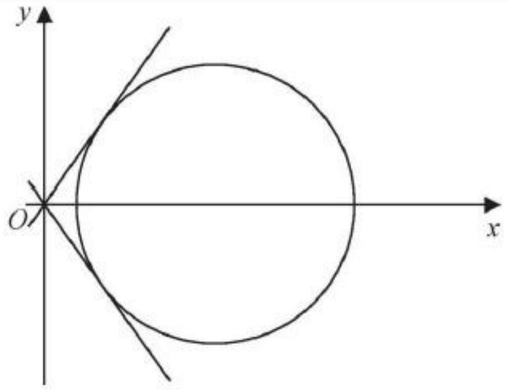
(b)
B1: States perpendicular bisector or midpoint of 2 and 8. Condone "in between 2 and 8" if they write $\frac{2+8}{2} = 5$
Note: $\frac{2+8}{2} = 5$on its own is B0
In between 2 and 8 on its own is B0

(c)
M1: A complete method to find the radius of the circle
A1: Correct radius

(d)
M1: Any correct complete strategy. If they attempt to find the height (even if incorrect method) in part (c) then $y = 5 +$ their height
A1: Correct answer

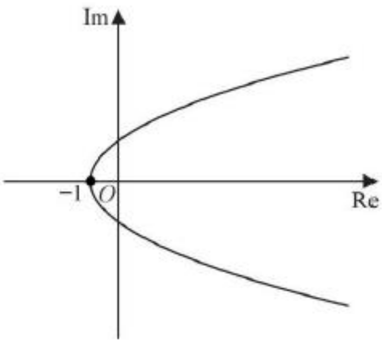
Q9.

Question	Scheme	Marks	AOs
(a)	$(x+6)^2+y^2 = 4[(x-6)^2+y^2]$	M1	2.1
	$x^2+y^2-20x+36=0$ which is the equation of a circle*	A1*	2.2a
		(2)	
(b)		M1	1.1b
		A1	1.1b
		(2)	

(c)	Let $a = c + id$ and $a^* = c - id$ then $(c + id)(x - iy) + (c - id)(x + iy) = 0$	M1	3.1a
	So $y = -\dfrac{c}{d}x$	A1	1.1b
	 The gradients of the tangents (from geometry) are $\pm\dfrac{4}{3}$	B1	3.1a
	So $-\dfrac{c}{d} = \pm\dfrac{4}{3}$ and $\dfrac{d}{c} = \mp\dfrac{3}{4}$	M1	3.1a
	So $\tan\theta = \pm\dfrac{3}{4}$	A1	1.1b
		(5)	
			(9 marks)

Notes:	
(a)	
M1:	Obtains an equation in terms of x and y using the given information
A1*:	Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle
(b)	
M1:	Draws a circle with centre at (10, 0)
A1:	(Radius is 8) so circle does not cross the y axis
(c)	
M1:	Attempts to convert line equation into a cartesian form
A1:	Obtains a simplified line equation
B1:	Uses geometry to deduce the gradients of the tangents
M1:	Understands the connection between arg a and the gradient of the tangents and uses this connection
A1:	Correct answers

Q10.

Question	Scheme	Marks	AOs
(a)	Let $z = x + \mathrm{i}$	M1	2.1
	$w = (x+\mathrm{i})^2 = (x^2 - 1) + 2x\mathrm{i}$	A1	1.1b
	Let $w = u + \mathrm{i}v$, then $u = (x^2 - 1)$ and $v = 2x$	M1	2.1
	$\Rightarrow v^2 = 4(u+1)$, which therefore represents a parabola	A1ft	2.2a
		(4)	
(b)	 M1: Sketches a parabola with symmetry about the real axis A1: Accurate sketch	M1 A1	1.1b 1.1b
		(2)	
			(6 marks)

Notes:

(a)

M1: Translates the information that $\mathrm{Im}(z) = 1$ into a cartesian form; e.g. $z = x + \mathrm{i}$

A1: Obtains a correct expression for w

M1: Separates the real and imaginary parts and equates to u and v respectively

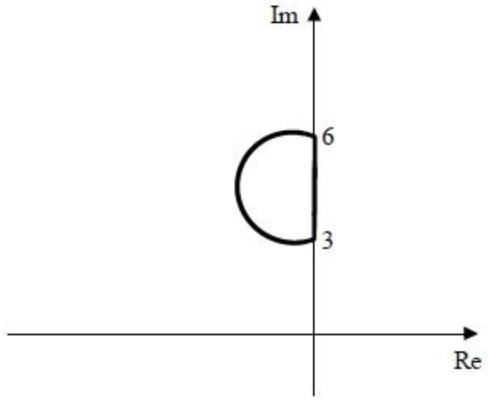
A1ft: Obtains a quadratic equation **and** states that their quadratic equation represents a parabola

(b)

M1: Sketches a parabola with symmetry about the real axis

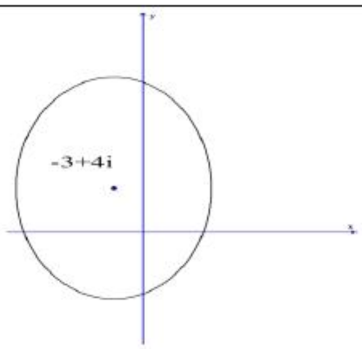
A1: Accurate sketch

Q11.

Question	Scheme	Marks	AOs
(a)		M1 A1 A1	3.1a 1.1b 1.1b
		(3)	
(b)	y-coordinate of centre of circle is 4.5	B1	1.1b
	x-coordinate of centre of circle is $-\dfrac{1.5}{\tan\frac{\pi}{3}}\left(=-\dfrac{\sqrt{3}}{2}\right)$	M1	3.1a
	Radius of circle is $\dfrac{1.5}{\sin\frac{\pi}{3}}$ or $\sqrt{1.5^2+\left(\dfrac{1.5}{\tan\frac{\pi}{3}}\right)^2}$	M1	1.1b
	$d=\sqrt{4.5^2+0.75}+\sqrt{3}$	M1	3.1a
	$d=\sqrt{21}+\sqrt{3}$	A1	1.1b
		(5)	
			(8 marks)

Notes
(a) M1: Interprets the locus correctly as a circle or as an arc of a circle A1: A circle or an arc of a circle passing through or touching at 3 and 6 on the positive imaginary axis. A1: Correct diagram – a major arc that is wholly to the left of the imaginary axis and wholly above the real axis with 3 and 6 marked on the imaginary axis (b) B1: Correct y-coordinate of the centre M1: Correct strategy for finding the x-coordinate of the centre M1: Correct strategy for finding the radius of the circle M1: Fully correct method for the maximum using their values A1: Correct value

Q12.

Question	Scheme	Marks	AOs
(a)	$(x-9)^2+(y+12)^2=4[x^2+y^2]$	M1	2.1
	$3x^2+3y^2+18x-24y-225=0$ which is the equation of a circle	A1*	2.2a
	As $x^2+y^2+6x-8y-75=0$ so $(x+3)^2+(y-4)^2=10^2$	M1	1.1b
	Giving centre at (-3, 4) and radius = 10	A1ft	1.1b
		(4)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Values range from **their** $-3-10$ to their $-3+10$	M1	3.1a
	So $-13 \le \text{Re}(w) \le 7$	A1ft	1.1b
		(2)	
			(8 marks)

Notes:
(a) **M1:** Obtains an equation in terms of x and y using the given information **A1:** Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle **M1:** Completes the square for their equation to find centre and radius **A1ft:**
(b) **M1:** Draws a circle with centre and radius as given from **their** equation **A1:** Correct circle drawn, as above, with centre at $-3+4i$ and passing through all four quadrants
(c) **M1:** Attempts to find where a line parallel to the real axis, passing through the centre of the circle, meets the circle so using " their $-3-10$" to "their $-3+10$" **A1ft:** Correctly obtains the correct answer for their centre and radius