Poisson Distributions

Questions

Q1.

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of a caller, chosen at random, being connected to the wrong agent is p.

The probability of at least 1 call in 5 consecutive calls being connected to the wrong agent is 0.049

The call centre receives 1000 calls each day.

(a) Find the mean and variance of the number of wrongly connected calls a day.

(7)

(b) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent.

(2)

(c) Explain why the approximation used in part (b) is valid.

(2)

The probability that more than 6 calls each day are connected to the wrong agent using the binomial distribution is 0.8711 to 4 decimal places.

(d) Comment on the accuracy of your answer in part (b)

(1)

(Total for question = 12 marks)

Q2.

A chocolate manufacturer places special tokens in 2% of the bars it produces so that each bar contains at most one token. Anyone who collects 3 of these tokens can claim a prize.

Andreia buys a box of 40 bars of the chocolate.

(a) Find the probability that Andreia can claim a prize.

(2)

Barney intends to buy bars of the chocolate, one at a time, until he can claim a prize.

(b) Find the probability that Barney can claim a prize when he buys his 40th bar of chocolate.

(3)

(c) Find the expected number of bars that Barney must buy to claim a prize.

(1)

(Total for question = 6 marks)

Q3.

A plumbing company receives call-outs during the working day at an average rate of 2.4 per hour.

(a) Find the probability that the company receives exactly 7 call-outs in a randomly selected 3-hour period of a working day

(2)

The company has enough staff to respond to 28 call-outs in an 8-hour working day.

(b) Show that the probability that the company receives more than 28 call-outs in a randomly selected 8-hour working day is 0.022 to 3 decimal places.

(2)

In a random sample of 100 working days each of 8 hours,

(c) (i) find the expected number of days that the company receives more than 28 call-outs,

(1)

(ii) find the standard deviation of the number of days that the company receives more than 28 call-outs,

(2)

(iii) use a Poisson approximation to estimate the probability that the company receives more than 28 call-outs on at least 6 of these days.

(3)

(Total for question = 10 marks)

Q4.

The discrete random variables *W*, *X* and *Y* are distributed as follows

$$W \sim \mathrm{B}(10, 0.4) \qquad X \sim \mathrm{Po}(4) \qquad Y \sim \mathrm{Po}(3)$$

(a) Explain whether or not Po(4) would be a good approximation to B(10, 0.4)

(1)

(b) State the assumption required for $X + Y$ to be distributed as Po(7)

(1)

Given the assumption in part (b) holds,

(c) find $\mathrm{P}(X + Y < \mathrm{Var}(W))$

(2)

(Total for question = 4 marks)

Q5.

Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

(Total for question = 10 marks)

Mark Scheme – Poisson Distributions

Q1.

Question	Scheme	Marks	AOs
(a)	$P(X \geqslant 1) = 1 - P(X = 0)$ $1 - P(X = 0) = 0.049$	B1	3.1b
	$P(X = 0) = 0.951$	B1	1.1b
	$x^5 = 0.951$ $x = 0.99$	M1	3.1b
	$p = 0.01$	A1	1.1b
	$X \sim B(1000, 0.01)$	M1	3.3
	Mean $= np = 10$	A1ft	1.1b
	Variance $= np(1-p) = 9.9$	A1ft	1.1b
		(7)	
(b)	$X \sim Po(\text{“}10\text{”})$ then require: $P(X > 6) = 1 - P(X \leqslant 6)$	M1	3.4
	$= 1 - 0.1301$		
	$= 0.870$	A1	1.1b
		(2)	
(c)	The approximation is valid as : the number of calls is large	B1	2.4
	the probability of connecting to the wrong agent is small	B1	2.4
		(2)	
(d)	The answer is accurate to 2 decimal place	B1	3.2b
		(1)	
			(12 marks)

	Notes
(a)	**B1**: Realising that the P(at least 1 call) $= 1 - \mathrm{P}(X = 0)$ **B1**: Calculating $\mathrm{P}(X = 0) = 0.951$ **M1**: Forming the equation $x^5 =$ "their 0.951" may be implied by $p = 0.01$ **A1**: 0.01 only **M1**: Realising the need to use the model B(1000, 0.01) This may be stated or used **A1**: mean =10 or ft their p but only if $0 < p < 1$ **A1**: Var = 9.9 or ft their p but only if $0 < p < 1$
(b)	**M1**: Using the model Po("their 10") (this may be written or used) and $1 - \mathrm{P}(X \leqslant 6)$ **A1**: awrt 0.870 Award M1 A1 for awrt 0.870 with no incorrect working
(c)	**B1**:Explaining why approximation is valid - need the context of number and calls
	B1: need the context connecting, wrong agent
(d)	**B1**: Evaluating the accuracy of their answer in (b). Allow 2 significant figures

Q2.

Qu	Scheme	Marks	AO
(a)	[Let X = no. of prizes Andreia wins] $X \sim \mathrm{B}(40, 0.02)$	M1	3.3
	[Require $\mathrm{P}(X \geqslant 3) = 1 - \mathrm{P}(X \leqslant 2)$] $= 0.04567\ldots$ awrt **<u>0.0457</u>**	A1	1.1b
		(2)	
(b)	[Let Y = no. of the bar when Barney wins] $Y \sim \mathrm{NegBin}(3, 0.02)$	M1	3.3
	[$\mathrm{P}(Y = 40) =$] $\binom{39}{2} \times 0.02^2 \times 0.98^{37} \times 0.02$	M1	3.4
	$= 0.0028071\ldots$ awrt **<u>0.00281</u>**	A1	1.1b
		(3)	
(c)	$\mathrm{E}(Y) = \dfrac{3}{0.02}$ $=$ **<u>150</u>**	B1	1.1b
		(1)	
		(6 marks)	
	Notes		
(a)	M1 for selecting a suitable model i.e. $\mathrm{B}(40, p)$ where p is any probability Written or used, may be implied by a correct ans or $0.037429\ldots$ from $\mathrm{P}(X = 3)$ A1 for awrt 0.0457 (correct answer only 2/2)		
(b)	1st M1 for selecting a suitable model (NB(3, 0.02)) May be implied by a correct expression 2nd M1 for use of model to form a correct expression		
SC	$p \neq 0.02$ Allow prob of the form $\binom{39}{2} p^3(1-p)^{37}$ where $0 < p < 1$ scores M0M1		
	A1 for awrt 0.00281 (accept awrt 2.81×10^{-3}) [correct answer with no working scores 3/3]		
(c)	B1 for 150		

Q3.

Question	Scheme	Marks	AOs
(a)	$X \sim \text{Po}(7.2)$	M1	3.4
	$\text{P}(X = 7) = 0.14858\ldots$ awrt **0.149**	A1	1.1b
		(2)	
(b)	$Y \sim \text{Po}(19.2)$	M1	3.3
	$[\text{P}(Y > 28) =]\ 1 - \text{P}(Y \leq 28) = 1 - 0.9780\ldots = 0.02199\ldots$ **0.022***	A1*	1.1b
		(2)	
(c)(i)	$[100 \times 0.022]$ awrt **2.2**	B1	1.1b
		(1)	
(ii)	$\sqrt{100(0.022)(1-0.022)}$	M1	1.1b
	$= 1.466\ldots$ awrt **1.47**	A1	1.1b
		(2)	
(iii)	$\text{B}(100, 0.022) \rightarrow \text{Po}(2.2)$	M1	3.4
	$\text{P}(W \geq 6) = 1 - \text{P}(W \leq 5)\ [= 1 - 0.9750\ldots]$	M1	1.1b
	$= 0.02490\ldots$ awrt **0.0249**	A1	1.1b
		(3)	
			(10 marks)

Notes	
(a)	**M1:** Writing or using Po(7.2) **A1:** awrt 0.149
(b)	**M1:** Writing or using Po(19.2) **A1*:** cso given answer with correct probability statement (e.g. $1 - \text{P}(Y \leq 28)$) and no incorrect working seen
(c)(i)	**B1:** awrt 2.2 (isw once awrt 2.2 is seen)
(c)(ii)	**M1:** Correct expression including square root **A1:** awrt 1.47 Watch out for $\sqrt{2.2} = 1.483\ldots$ which is M0A0
(iii)	**M1:** Approximating binomial (100, 0.022) with Po(2.2) [may be seen in (i) or (ii)] **M1:** Using $1 - \text{P}(W \leq 5)$ from Poisson distribution **A1:** awrt 0.0249 Note: Using Binomial distribution $1 - 0.9765588\ldots = 0.02344\ldots$ scores M0M0A0

Q4.

Question	Scheme	Marks	AOs
(a)	requires large n/small p so not a good approximation	B1	3.5b
		(1)	
(b)	X and Y must be independent	B1	2.4
		(1)	
(c)	$\text{P}(X + Y < 2.4)$ from Po(7) $[\text{P}(X + Y \leq 2)]$	M1	3.4
	$= 0.029636\ldots$ awrt **0.0296**	A1	1.1b
		(2)	
			(4 marks)
	Notes		
(a)	**B1:** Correct reason why the model would not be appropriate and correct conclusion. Condone e.g. 'p is close to 0.5' for p is not small. Mean is not equal to variance on its own in B0.		
(b)	**B1:** Correct explanation mentioning independence (oe). Ignore extraneous comments.		
(c)	**M1:** Using Po(7) with 2.4 **A1:** awrt 0.0296		

Q5.

Question	Scheme	Marks	AOs
(a)	$X \sim \text{Po}(2.6)$ $\quad Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **0.0544**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow \quad W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **0.205**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **0.648**	A1	1.1b
		(2)	
(d)(i)	Mean $= np =$ **2.4**	B1	1.1b
(d)(ii)	Variance $= np(1-p) = 2.3904$ awrt **2.39**	B1	1.1b
		(2)	
(e)(i)	$[D \sim \text{Po}(2.4) \quad \text{P}(D \leq 4)]$		
	$= 0.9041\ldots$ awrt **0.904**	B1	1.1b
(e)(ii)	Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes	
(a)	M1 for $\text{P}(X=2)\times\text{P}(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer) A1 awrt 0.0544
(b)	M1 for combining Poisson distributions and use of Po('3.8') (may be implied by correct answer) A1 awrt 0.205
(c)	M1 for setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer) A1 awrt 0.648
(d)(i) (d)(ii)	B1 for 2.4 B1 for awrt 2.39
(e)(i) (e)(ii)	B1 for 0.904 B1 for a correct explanation to support use of Poisson approximation in this case