Discrete Random Variables

Questions

Q1.

The discrete random variable X has probability distribution

x	-3	-1	1	2	4
$P(X = x)$	q	$\frac{7}{30}$	$\frac{7}{30}$	q	r

where q and r are probabilities.

(a) Write down, in terms of q, $P(X \leq 0)$

(1)

(b) Show that $E(X^2) = \frac{7}{15} + 13q + 16r$

(2)

Given that $E(X^3) = E(X^2) + E(6X)$

(c) find the value of q and the value of r

(7)

(d) Hence find $P(X^3 > X^2 + 6X)$

(4)

(Total for question = 14 marks)

Q2.

The probability distribution of the discrete random variable X is

$$P(X = x) = \begin{cases} \frac{k}{x} & \text{for } x = 1, 2 \text{ and } 3 \\ \frac{m}{2x} & \text{for } x = 6 \text{ and } 9 \\ 0 & \text{otherwise} \end{cases}$$

where k and m are positive constants.

Given that E(X) = 3.8, find Var(X)

(7)

(Total for question = 7 marks)

Q3.

The discrete random variable X has the following probability distribution.

x	–5	–2	3	4
$P(X = x)$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$

(a) Find Var(X)

(3)

The discrete random variable Y is defined in terms of the discrete random variable X

When X is negative, $Y = X^2$
When X is positive, $Y = 3X – 2$

(b) Find P($Y < 9$)

(3)

(c) Find E(XY)

(2)

(Total for question = 8 marks)

Q4.

The discrete random variable X has probability distribution

x	-3	-2	-1	0	2	5
$P(X=x)$	0.3	0.15	0.1	0.15	0.1	0.2

(a) Find E(X)

(1)

Given that Var(X) = 8.79

(b) find E(X^2)

(2)

The discrete random variable Y has probability distribution

y	-2	-1	0	1	2
$P(Y=y)$	$3a$	a	b	a	c

where a, b and c are constants.

For the random variable Y

$$P(Y \leqslant 0) = 0.75 \quad \text{and} \quad E(Y^2 + 3) = 5$$

(c) Find the value of a, the value of b and the value of c

(5)

The random variable $W = Y - X$ where Y and X are independent.

The random variable $T = 3W - 8$

(d) Calculate $P(W > T)$

(4)

(Total for question = 12 marks)

Q5.

The discrete random variable X has the following probability distribution

x	0	2	3	6
$P(X = x)$	p	0.25	q	0.4

(a) Find in terms of q

(i) E(X)
(ii) E(X^2)

(2)

Given that Var(X) = 3.66

(b) show that $q = 0.3$

(3)

In a game, the score is given by the discrete random variable X

Given that games are independent,

(c) calculate the probability that after the 4th game has been played, the total score is exactly 20

(3)

A round consists of 4 games plus 2 bonus games. The bonus games are only played if after the 4th game has been played the total score is exactly 20

A prize of £10 is awarded if 6 games are played in a round **and** the total score for the round is at least 27

Bobby plays 3 rounds.

(d) Find the probability that Bobby wins at least £10

(6)

(Total for question = 14 marks)

Q6.

The discrete random variable X has probability distribution

x	-5	-1	0	5	b
$P(X = x)$	0.3	0.25	0.1	0.15	0.2

where b is a constant and $b > 5$

(a) Find E(X) in terms of b

(1)

Given that Var(X) = 34.26

(b) find the value of b

(4)

(c) Find P(X 2 < 2 – 3X)

(4)

(Total for question = 9 marks)

Q7.

Members of a photographic group may enter a maximum of 5 photographs into a members only competition.
Past experience has shown that the number of photographs, N, entered by a member follows the probability distribution shown below.

n	0	1	2	3	4	5
$P(N=n)$	a	0.2	0.05	0.25	b	c

Given that $E(4N+2) = 14.8$ and $P(N=5 \mid N>2) = \frac{1}{2}$

(a) show that Var(N) = 2.76

(6)

The group decided to charge a 50p entry fee for the first photograph entered and then 20p for each extra photograph entered into the competition up to a maximum of £1 per person. Thus a member who enters 3 photographs pays 90p and a member who enters 4 or 5 photographs just pays £1

Assuming that the probability distribution for the number of photographs entered by a member is unchanged,

(b) calculate the expected entry fee per member.

(3)

Bai suggests that, as the mean and variance are close, a Poisson distribution could be used to model the number of photographs entered by a member next year.

(c) State a limitation of the Poisson distribution in this case.

(1)

(Total for question = 10 marks)

Mark Scheme – Discrete Random Variables

Q1.

	Scheme	Marks	AO
(a)	$\underline{q+\frac{7}{30}}$	B1 (1)	1.1b
(b)	$E(X^2)=(-3)^2\times q + (-1)^2\times\frac{7}{30} + 1^2\times\frac{7}{30} + 2^2\times q + 4^2\times r$	M1	1.1b
	$=\underline{\frac{7}{15}+13q+16r}$ (*)	A1*cso (2)	1.1b
(c)	$E(X)=-3q + -\frac{7}{30} + \frac{7}{30} + 2q + 4r \quad \{=4r-q\}$	M1	3.1a
	$E(X^2+6X)=\frac{7}{15}+7q+40r$	A1	1.1b
	$E(X^3)=(-3)^3\times q + (-1)^3\times\frac{7}{30} + 1^3\times\frac{7}{30} + 2^3\times q + 4^3\times r$	M1	3.4
	$=64r-19q$	A1	1.1b
	Sum of probabilities = 1 gives: $2q+r=\frac{16}{30}$ (o.e.)	M1	1.1b
	Solve: $24r-26q=\frac{7}{15}$ and $r+2q=\frac{8}{15}$ e.g. $37r=\frac{111}{15}$	dM1	1.1b
	So $\underline{r=\frac{1}{5}}$ and $\underline{q=\frac{1}{6}}$	A1 (7)	1.1b
(d)	$X^3>X^2+6X \Rightarrow X(X-3)(X+2)>0$	M1	2.1
	Use of sketch or table to see: $-2<X<0$ or $X>3$	A1	1.1b
	So $P(X^3>X^2+6X)=P(X=-1 \text{ or } 4)$	M1	2.2a
	$=\frac{7}{30}+"r"=\underline{\frac{13}{30}}$	A1ft (4)	1.1b
ALT	X | -3 | -1 | 1 | 2 | 4 X^3 | -27 | -1 | 1 | 8 | 64 X^2+6X | -9 | -5 | 7 | 16 | 40	**(14 marks)**	

Notes

(b)
M1 for at least 3 correct terms of the expression for $E(X^2)$
A1*cso evidence of M1 scored with no incorrect working seen leading to correct answer (*)
Allow $-3^2\times q + -1^2\times\frac{7}{30}$ etc if followed by $9q$ + …but not if simply followed by given answer

(c)
1st M1 for realising the need to find $E(X)$ – a correct attempt with at least 3 correct terms
1st A1 for the correct expression (needn't be simplified at this stage)
2nd M1 for a correct attempt at $E(X^3)$ with at least 3 correct terms seen
Treat no $\frac{7}{30}$ terms as one correct term
2nd A1 for $64r-19q$ (must be simplified) or for $24r-26q=\frac{7}{15}$
3rd M1 for using sum of probabilities = 1 to form an equation in q and r (needn't be simplified)
Must be correct or clearly state that Σprobs = 1 being attempted with only one slip
4th dM1 for solving their 2 linear equations in q and r (dep on 3rd M1 and 1st or 2nd M1)
Must see correct method to reduce to a linear equation in one variable
3rd A1 for $r=\frac{1}{5}$ and $q=\frac{1}{6}$ or any exact equivalents (dep on 2 correct equations seen)

(d)
1st M1 for 1st stage towards solving the inequality (factorising the cubic)
1st A1 for solving the inequality
2nd M1 for identifying the values of X required i.e. – 1 and 4
2nd A1ft for $\frac{13}{20}$ or exact equivalent e.g. $0.4\dot{3}$ (Allow ft of "their r" + $\frac{7}{30}$)

ALT
Table 1st M1 for at least 4 correct values for X^3 and X^2+6X (must be labelled)
1st A1 for all 10 correct values. [NB Can score M1A0M1A1ft in (d)]

Q2.

Question	Scheme	Marks	AOs
	$\Sigma p = 1 \rightarrow k + \frac{k}{2} + \frac{k}{3} + \frac{m}{12} + \frac{m}{18} = 1$ $\Sigma px = 3.8 \rightarrow k + \frac{k}{2}(2) + \frac{k}{3}(3) + \frac{m}{12}(6) + \frac{m}{18}(9) = 3.8$	M1	3.1a
	$\frac{11k}{6} + \frac{5m}{36} = 1 \ [= 66k + 5m = 36]$	A1	1.1b
	$3k + m = 3.8$	A1	1.1b
	Solving simultaneously to eliminate one variable	dM1	1.1b
	$k = \frac{1}{3}$ and $m = \frac{14}{5}$	A1	1.1b
	$\mathrm{E}(X^2) = 1^2 \times k + 2^2 \times \frac{k}{2} + 3^2 \times \frac{k}{3} + 6^2 \times \frac{m}{12} + 9^2 \times \frac{m}{18} [= 23]$	M1	1.1b
	$\mathrm{Var}(X) = 23 - 3.8^2$		
	$= \underline{8.56}$	A1	1.1b
			(7 marks)

Notes	
	M1: Attempt at both required equations with at least one term in k and one term in m correct **A1:** Correct equation using $\Sigma p = 1$ **A1:** Correct equation using $\Sigma px = 3.8$ **dM1:** (dep on 1st M1) Solving simultaneously (may be implied by one correct value found) **A1:** both values correct (may be implied by correct answer) **M1:** Attempt to find $\mathrm{E}(X^2)$ using their value of k and their value of m with at least 3 correct products or correct ft products Note: $\mathrm{E}(X^2) = 6k + 7.5m$ **A1:** 8.56 cao

Q3.

Question	Scheme	Marks	AOs
(a)	$[\mathrm{E}(X) =](-5)\times\frac{1}{12}+(-2)\times\frac{1}{6}+(3)\times\frac{1}{4}+(4)\times\frac{1}{2}[=2]$	M1	1.1b
	$[\mathrm{E}(X^2) =](-5)^2\times\frac{1}{12}+(-2)^2\times\frac{1}{6}+(3)^2\times\frac{1}{4}+(4)^2\times\frac{1}{2}[=13]$ (oe)	M1	1.1b
	$\mathrm{Var}(X) = \mathrm{E}(X^2)-[\mathrm{E}(X)]^2 = 13-2^2 \quad = \underline{9}$	A1	1.1b
		(3)	
(b)	x: (−5), −2, 3, (4); y: (25), 4, 7, (10); p: $(\frac{1}{12})$, $\frac{1}{6}$, $\frac{1}{4}$, $(\frac{1}{2})$	M1	3.1a
	$\mathrm{P}(Y<9) = \mathrm{P}(X=-2)+\mathrm{P}(X=3)\ [=\frac{1}{6}+\frac{1}{4}]$	M1	1.1b
	$= \underline{\frac{5}{12}}$	A1	1.1b
		(3)	
(c)	$\mathrm{E}(XY) = (-5)(25)\frac{1}{12}+(-2)(4)\times\frac{1}{6}+(3)(7)\times\frac{1}{4}+(4)(10)\times\frac{1}{2}$	M1	3.1a
	$= \underline{13.5}$	A1	1.1b
		(2)	
			(8 marks)

Notes	
(a)	**M1:** Attempt at $\mathrm{E}(X)$ with at least 3 correct products seen **M1:** Attempt at $\mathrm{E}(X^2)$ with at least 3 correct products seen **A1:** 9 cao Alternative **M1:** Attempt at $\mathrm{E}(X)$ with at least 3 correct products seen **M1:** Attempt at expression for $\mathrm{E}((X-\mu)^2) = (-5-2)^2\times\frac{1}{12}+(-2-2)^2\times\frac{1}{6}+(3-2)^2\times\frac{1}{4}+(4-2)^2\times\frac{1}{2}$ with at least 3 correct terms **A1:** 9 cao
(b)	**M1:** Finding distribution of Y **M1:** $\mathrm{P}(X=-2)+\mathrm{P}(X=3)$ or $\mathrm{P}(Y=4)+\mathrm{P}(Y=7)$ **A1:** $\frac{5}{12}$ (condone awrt 0.417)
(c)	**M1:** Attempt at $\mathrm{E}(XY)$ with at least 2 correct terms **A1:** 13.5

Q4.

Question	Scheme	Marks	AOs
(a)	$E(X) = -0.1$ oe	B1	1.1b
		(1)	
(b)	$\text{Var}(X) = E(X^2) - ("-0.1")^2$	M1	1.2
	$E(X^2) = 8.8$	A1	1.1b
		(2)	
(c)	$(-2)^2 \times 3a + (-1)^2 \times a\left[+0^2 \times b\right] + 1^2 \times a + 2^2 \times c = ["2"]$	M1	1.1b
	$7a + 2c = 1$ oe	A1	1.1b
	One of $a + c = 0.25$ **or** $4a + b = 0.75$ **or** $5a + b + c = 1$	M1	3.1a
	Two of $a + c = 0.25$ **or** $4a + b = 0.75$ **or** $5a + b + c = 1$	A1	1.1b
	$a = 0.1$ and $b = 0.35$ and $c = 0.15$	A1	1.1b
		(5)	
(d)	$P(W > T) = P(W > 3W - 8) = P(W < 4)$	M1	3.1a
	$P(W < 4) = 1 - \left[P(X = -3) \times P(Y = 1) + P(X = -3) \times P(Y = 2) + P(X = -2) \times P(Y = 2)\right]$ **or** $= P(X \geqslant -1) + P(X = -2) \times P(Y \neq 2) + P(X = -3) \times P(Y \leqslant 0)$	M1dep	1.1b
	$= 1 - \left[0.3 \times "0.1" + 0.3 \times "0.15" + 0.15 \times "0.15"\right]$ **or** $0.55 + 0.15 \times \left[1 - "0.15"\right] + 0.3 \times \left["0.3" + "0.1" + "0.35"\right]$	M1dep	1.1b
	$= \underline{\mathbf{0.9025}}$	A1	1.1b
		(4)	
			(12 marks)

Notes:		
(a)	**B1:**	-0.1 oe
(b)	**M1:**	For recalling and using a correct formula
	A1:	8.8
(c)	**M1:**	For use of $\sum y^2 P(Y = y)\left[= 2\right]$ or $\sum \left(y^2 + 3\right) P(Y = y)\left[= 5\right]$ 3 correct products seen
	A1:	For correct equation with a's collected
	M1:	For use of $\sum P(Y = y) = 1$ or $P(Y \leqslant 0) = 0.75$ or $1 - P(Y \leqslant 0) = 0.25$
	A1:	For 2 correct equations
	A1:	a, b and c correct. Award full marks if all 3 correct
(d)	**M1:**	For using the information given to work out the values of W. Allow $Y - X$ instead of W
	dM1:	For using the information given to work out which are the relevant combinations of X and Y. The irrelevant ones must not be used.
	M1:	Previous method must be awarded. All required cases identified and their probabilities of a, b and c used. Allow in terms of a, b and c
	A1:	0.9025 (accept awrt 0.903 or exact fraction $\frac{361}{400}$)

Q5.

Question	Scheme	Marks	AOs
(a)(i)	$E(X)=[0\times p]+(2\times 0.25)+3q+(6\times 0.4)\,[=2.9+3q]$	B1	1.1b
(ii)	$E(X^2)=[0\times p]+(2^2\times 0.25)+3^2q+(6^2\times 0.4)\,[=15.4+9q]$	B1	1.1b
		(2)	
(b)	$("15.4+9q")-("2.9+3q")^2=3.66$	M1	1.1b
	$9q^2+8.4q-3.33=0 \Rightarrow q=0.3$ and $-37/30$	M1	1.1b
	$q=0.3$* since q cannot be negative	A1cso*	2.4
	SC $("15.4+9\times 0.3")-("2.9+3\times 0.3")^2$ can get M1M0A0		
		(3)	
(c)	$P(x_1+x_2+x_3+x_4=20)$ = P(6,6,6,2 or 6,6,2,6 or 6,2,6,6 or 2,6,6,6)	M1	1.1b
	$=4\times 0.4^3\times 0.25$	M1	1.1b
	$=0.064$ oe	A1	1.1b
		(3)	
(d)	$P(x_5+x_6\geqslant 7)$ = P(6,6 or 6,3 or 6,2)	M1	3.1a
	$=(0.4^2)+2\times(0.4\times 0.3)+2\times 0.4\times 0.25$ [= 0.6]	M1	1.1b
	$P(\text{score}\geqslant 27)$ = "0.064" × "0.6" [= 24/625 = 0.0384]	M1	1.1b
	$Y\sim$ B(3, "0.0384")	dM1	3.3
	$P(Y\geqslant 1)=1-P(Y=0)$	M1	1.1b
	$=0.1108\ldots$	A1cso	1.1b
		(6)	
Notes			**(14 marks)**

(a)(i)	B1:	Correct expression for E(X) need not be simplified
(ii)	B1:	Correct expression for E(X^2) need not be simplified
(b)	M1:	Using "their E(X^2)" − "their (E(X))^2" = 3.66
	M1:	Rearranging to get a correct 3 term quadratic (condone missing = 0) leading to 0.3 and − 37/30(awrt −1.23) or $(10q-3)(30q+37)$
	A1cso:*	cso with a comment why −37/30 is eliminated. Minimum required is $q>0$ or they say it is impossible.
(c)	M1:	Realising that combination is 6662. Any order. Implied by $0.4^3\times 0.25$
	M1:	Correct calculation
	A1:	0.064 oe only eg 8/125
(d)	M1:	Realising all the different combinations 7 or more can be scored from 2 games. (no need for arrangements) Implied by (0.4^2) and (0.4×0.3) and (0.4×0.25)
	M1:	Fully correct method.
	M1:	For multiplying "their (c)" with "their $P(x_5+x_6\geqslant 7)$" providing at least 2 combinations are used to find $P(x_5+x_6\geqslant 7)$"
	dM1:	Dependent on 3rd M1 being awarded for using or writing B(3, "their $P(x_1+x_2+x_3+x_4+x_5+x_6\geqslant 27)$") $(1-"0.0384")^3$ or
	M1:	For writing or using $1-P(Y=0)$ eg $1-(1-"0.0384")^3$
	A1cso:	awrt 0.111 from correct working
NB (b)	1st 3 marks	
Fully correct method	"0.064"$\times(0.4^2)+0.064\times 2\times(0.4\times 0.3)+0.064\times 2\times(0.4\times 0.25)$ is M1M1M1	
All 3 but no arrangements ie	"0.064"$\times(0.4^2)+0.064\times(0.4\times 0.3)+0.064\times(0.4\times 0.25)$ M1M0M1	
At least 2 combinations used for	>7 eg $0.064\times(0.4\times 0.3)+0.064\times(0.4^2)$ or $2\times(0.4\times 0.3)$ M0M0M1	

Q6.

Question	Scheme	Marks	AOs
(a)	$[\mathrm{E}(X)=]0.2b-1$	B1 (1)	1.1b
(b)	$\mathrm{E}(X^2)=25\times0.3+1\times0.25[+0\times0.1]+25\times0.15+0.2b^2\left[=11.5+0.2b^2\right]$	M1	1.1b
	$"11.5+0.2b^2"-("0.2b-1")^2[=34.26]$	M1	3.1a
	$0.16b^2+0.4b-23.76[=0]$ or $\frac{4}{25}b^2+\frac{2}{5}b-\frac{594}{25}[=0]$	M1	1.1b
	$b=\underline{11}$ [since $b>5$]	A1 (4)	2.2a
(c)	X | -5 | -1 | 0 | 5 | "11" X^2 | 25 | 1 | 0 | 25 | "121" $2-3X$ | 17 | 5 | 2 | -13 | "-31" X^2-2 | 23 | -1 | -2 | 23 | "119" $-3X$ | 15 | 3 | 0 | 15 | "-33" X^2+3X | 10 | -2 | 0 | 40 | "154" X^2+3X-2 | 8 | -4 | -2 | 38 | "152"	M1 A1ft	2.1 1.1b
	$\mathrm{P}(X^2<2-3X)=\mathrm{P}(X=-1)+\mathrm{P}(X=0)$	M1	2.2a
	$=\underline{0.35}$	A1 (4)	1.1b
			Total 9

(a)	**B1**	Correct expression for $\mathrm{E}(X)$
(b)	**1st M1**	Correct attempt at $\mathrm{E}(X^2)$ using $\sum x^2\mathrm{P}(X=x)$ at least 3 correct non-zero products Allow $(-5)^2$ etc
	2nd M1	Realising that Var(X) = $\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$ needs to be **used**
	3rd M1	Reducing their equation to a 3 term quadratic. At least 2 terms correct. Allow e.g. $0.16b^2+0.4b=23.76$ Condone missing "=0"
	A1	For 11 only (from the correct equation) so −13.5 must be eliminated Correct answer with no incorrect working seen scores 4/4
(c)	**1st M1**	At least 4 values correct for (X^2 **and** $2-3X$) or for (X^2-2 **and** $-3X$) or X^2+3X or X^2+3X-2 (o.e.) Allow for solving equation with one sign error
	1st A1ft	All correct or correct ft with their b but must have $b>5$ (accurate to 1 sf) Allow solving equation to get awrt −3.6 **and** awrt 0.56 or $\frac{-3\pm\sqrt{17}}{2}$ (ft their $b>5$) If there are omissions **but no errors** in the lists of values then if 2nd M1 and 2nd A1 are scored then the 1st M1 and 1st A1 can be given by implication.
	2nd M1	For identifying the correct values of X required i.e. $X=-1$ and $X=0$
	2nd A1	0.35 **NB** It is possible to score M0A0M1A1 here if their table of values is incorrect Correct answer with no incorrect working seen scores 4/4 (Allow correct use of their $b>5$)

Q7.

Question	Scheme	Marks	AOs
(a)	$4E(N) + 2 = 14.8$ or $E(N) = 3.2$	M1	3.1a
	$0.2+0.1+0.75+4b+5c=3.2$	M1	1.1b
	$\frac{c}{0.25+b+c}=0.5$ or $0.25=c-b$	M1	3.1a
	$b=0.1$ and $c=0.35$		
	$E(N^2)=1\times0.2+4\times0.05+9\times0.25+16\times"0.1"+25\times"0.35"[=13]$	M1	1.1b
	$Var(N)="13"-"3.2"^2$	dM1	1.1b
	$=2.76$ *	A1*	2.1
		(6)	
(b)	fee: 0, 50, 70, 90, 100, 100 / $P(N=n)$: a, 0.2, 0.05, 0.25, b, c	M1	3.3
	$50\times0.2+70\times0.05+90\times0.25+100\times"0.1"+100\times"0.35"$	M1	1.1b
	= 81p	A1	1.1b
		(3)	
(c)	Poisson distribution will assign substantial probability to $N > 5$	B1	3.5b
		(1)	
			(10 marks)

Notes		
(a)	M1:	For using the given information to find E(N)
		ALT $a+b+c=0.5$ oe
	M1:	For use of $\sum nP(N=n)="3.2"$ At least 3 terms correct
		ALT $\sum(4n+2)P(N=n)=14.8 \Rightarrow 2a+1.2+0.5+3.5+18b+22c=14.8$ At least 3 terms correct
	M1:	Forming an equation in b and c using conditional probability
	M1:	For using $\sum n^2P(N=n)$ Allow with the letters b and c
	dM1:	Dependent on previous method mark. Correct method to find Var(N)
	A1*:	All previous marks must be awarded and 2.76 stated
(b)	M1:	Setting up a new model with the correct fees. At least 3 terms correct. Allow 0.5, 0.7, 0.9, 1
	M1:	Correct method for calculating E(fee) Allow with the letters b and c
	A1:	81[p] No units needed. Allow 0.81 if fees are in pounds
(c)	B1:	A correct limitation.