

GCE

Further Mathematics B (MEI)

Y434/01: Numerical methods

Advanced GCE

Mark Scheme for Autumn 2021

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2021

Annotations and abbreviations

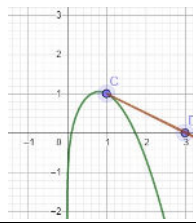
Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

Question		Answer	Marks	AOs	Guidance			
1	(a)	i	$\frac{1.414214-\sqrt{2}}{\sqrt{2}}$ or $\frac{1.414214^2-2}{2}$ oe soi	M1	1.1a	ignore modulus signs		
			0.000000309449 isw	A1	1.1	to 2 sf or more		
			0.000000618898 isw	A1	1.1	to 2 sf or more		
			[3]					
1	(a)	ii	the second relative error is double the first relative error oe	B1	2.2a			
			[1]					
1	(b)		Ben is wrong because the spreadsheet stores 1.414214 to a higher precision than is displayed (and so when the square of this number is calculated, 2 is returned) isw	B1	2.4	or 1.414214 is an approximation to $\sqrt{2}$ so $1.414214^2 \neq 2$ oe		
			[1]					
2	(a)		x	f(x)	Δ	Δ^2	M1 1.1 A1 1.1 all correct	finds 4 Δ values, allow one error
			1	-0.65				
					0.3			
			2	-0.35		1.82		
					2.12			
			3	1.77		1.82		
					3.94			
			4	5.71		1.82		
		5.76						
5	11.47							
			[2]					

Question		Answer	Marks	AOs	Guidance
2	(b)	the second differences are constant oe	B1	1.1	allow the 3 rd differences are zero
			[1]		
2	(c)	$-0.65 + 0.3(x-1) + 1.82 \times \frac{(x-1)(x-2)}{2!}$ [P ₂ (x) =] 0.91x ² - 2.43x + 0.87	M1	1.1	must be correct form; allow 1 substitution error
			A1	1.1	two of three terms correct
			A1	1.1	all correct
			[3]		
3	(a)	$\sinh x^2 - x^3 - 2 = 0$	B1	1.1	must see = 0
			[1]		
3	(b)	=IF(H5>0,G5,E5)	B1	1.1	or =IF(H5<0,E5,G5)
			[1]		must see =
3	(c)	$\frac{1.48719 \times 17.2899 - 2 \times -0.77825}{17.2899 - -0.77825} \text{ oe}$ awrt 1.50928 awrt 1.52603	M1	3.1a	may be implied by 1.509...
			A1	1.1	NB f(1.50928) = -0.6111 to 4 sf
			A1	1.1	
			[3]		

Question		Answer	Marks	AOs	Guidance		
3	(d)	the ratios are decreasing which suggests the convergence is (slightly) faster than 1 st order	B1	2.2b	allow between 1 st and 2 nd order	do not allow eg not first order	
		the ratios are close to 1 which suggests the convergence is slow	B1	2.2b			
			[2]				
4	(a)	$\frac{4.2472072-4}{0.1}$ or $\frac{4.0239468-4}{0.01}$ or	M1	3.1a	use of forward difference method	may be implied by one correct answer	
		$\frac{4.0023871-4}{0.001}$ or $\frac{4.0002386-4}{0.0001}$					
		2.472072 (with $h = 0.1$)	A1	1.1			any two correct
		2.39468 (with $h = 0.01$)	A1	1.1			any three correct
		2.3871 (with $h = 0.001$)	A1	1.1			all four correct
		2.386 (with $h = 0.0001$)					
			[4]				
4	(b)	comparison of last two estimates	M1	1.1	if M0 allow SC1 for 2.39 is secure or 2.386 is possible regardless of justification		
		2.39 is secure or 2.386 is possible	A1	2.2b			
			[2]				
5	(a)	48×0.5 soi	M1	3.3			
		£24	A1	3.4			
			[2]				
5	(b)	consistent because $1.77 < 24$	B1	2.4	allow consistent because error < mpe		
			[1]				

Question		Answer	Marks	AOs	Guidance
5	(c)	52×0.495	M1	3.3	
		£25.74	A1	3.4	
			[2]		
5	(d)	this could happen if a large number of items eg cost less than £1 eg cost £1.99 or £2.99 etc eg more than 50p over the pound eg the mean error per item was 52.38p	B1	3.5a	
			[1]		
5	(e)	mpe = £0.99 <i>n</i>	B1	3.4	condone omission of units, allow 99 <i>n</i> pence
			[1]		
5	(f)	expected error for Nina’s model is £0 since you would expect to round half the prices up and half down oe or expected error in Kareem’s model is –£0.495 <i>n</i> since you would expect the average “chop” to be 49.5p oe so new model should be “estimated cost” + £0.495 <i>n</i>	B1	2.4	U6
			B1	3.5c	

Question		Answer	Marks	AOs	Guidance							
			[2]									
6	(a)	$\frac{1}{2x} - 2x + 1$ seen $x_{n+1} = x_n - \frac{0.5 \ln(x_n) - x_n^2 + x_n + 1}{\frac{1}{2x_n} - 2x_n + 1}$ oe soi	M1 M1	2.1 1.1	may be implied by correct iterates condone omission of subscripts							
		<table border="1" style="display: inline-table; vertical-align: top;"> <tr><td>1</td></tr> <tr><td>3</td></tr> <tr><td>2.0791668</td></tr> <tr><td>1.7783346</td></tr> <tr><td>1.7360141</td></tr> <tr><td>1.7351281</td></tr> <tr><td>1.7351277</td></tr> </table> 1.735128	1	3	2.0791668	1.7783346	1.7360141	1.7351281	1.7351277	M1 A1	1.1 1.1	at least three further correct iterates derived from starting at 1 if M0 allow SC1 for 1.735128 from N-R method used with different x_0 and at least 3 correct iterates shown
1												
3												
2.0791668												
1.7783346												
1.7360141												
1.7351281												
1.7351277												
			[4]									
6	(b)		M1 A1	2.4 1.1	tangent at (1,1) (1,1) to (3,0)							
			[2]									

Question		Answer	Marks	AOs	Guidance							
6	(c)	N-R generally has 2 nd order convergence whereas fixed point iteration generally has 1 st order convergence	B1	2.4	allow eg N-R converges faster allow eg fixed point iteration more likely to fail oe							
			[1]									
6	(d)	ln(-0.403) is undefined (so the spreadsheet cannot compute a value)	B1	2.2a								
			[1]									
6	(e)	<table border="1" style="display: inline-table; vertical-align: top;"> <tr><td>0.5</td></tr> <tr><td>1.0739769</td></tr> <tr><td>1.4524673</td></tr> <tr><td>1.6245304</td></tr> <tr><td>1.6932631</td></tr> <tr><td>1.7194743</td></tr> <tr><td>1.7293015</td></tr> </table> converges to β	0.5	1.0739769	1.4524673	1.6245304	1.6932631	1.7194743	1.7293015	M1 A1	2.1 2.2a	need to see at least 3 iterates correct to at least 5 sf
0.5												
1.0739769												
1.4524673												
1.6245304												
1.6932631												
1.7194743												
1.7293015												
			[2]									

Question		Answer	Marks	AOs	Guidance
6	(f)	0.5	M1	1.1	at least 3 correct iterates derived from starting at 0.5
		0.4764669			
		0.4528879			if M0 allow SC1 for 0.111082 from relaxation method used with different x_0 and at least 3 correct iterates shown
		0.4293074			
		0.4057756			
		0.3823498			
		...			
		0.1116318	A1	2.2a	
		0.1111278			
		0.1110835			
		0.1110821			
		0.1110821			
		0.111082			
			[2]		
7	(a)	$\frac{1}{16}$ isw or 0.0625 isw	B1	2.2a	
			[1]		
7	(b)	by comparison of T_{16} and T_{32} 0.6 is certain or 0.63 is probable	B1	2.2b	
			[1]		

Question		Answer	Marks	AOs	Guidance
7	(c)	r appears to be between 0.25 and 0.5	B1	2.2b	
		so order of convergence is between 1 st and 2 nd order	B1	2.2b	
		<i>Alternative</i> $r > 0.25$ so convergence slower than 2 nd order $r < 0.5$ so convergence faster than 1 st order	B1 B1		
			[2]		
7	(d)	$\frac{2M_n + T_n}{3}$ or $\frac{4T_{2n} - T_n}{3}$ soi	M1	1.1	
		$= (2 \cdot 0.5 + N_5)/3$ or $= (4 \cdot N_6 - N_5)/3$	A1	1.1	must see =
			[2]		
7	(e)	awrt 0.62658745	B1	1.1	
		awrt 0.00029	B1	1.1	
		awrt 0.354	B1	1.1	
			[3]		

Question		Answer	Marks	AOs	Guidance	
7	(f)	<p>S_{2n} and difference from table used in extrapolation</p> <p>awrt 0.62658745 and awrt 0.00029 used</p> $0.62658745 + 0.00029 \times \frac{r}{1-r}$ <p>awrt 0.62674355 to awrt 0.62675058</p> <p>comparison with their S_{64}</p> <p>0.6267 is secure</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>3.1a</p> <p>1.1</p> <p>1.1</p> <p>1.1</p> <p>3.2a</p> <p>2.2b</p>	<p>eg their 0.62658745 and their 0.00029</p> <p>may see more dp for difference</p> <p>$0.35 \leq r \leq 0.36$</p> <p>or 0.62675 is possible; allow 0.626746</p> <p>the last two A marks are only available if answers obtained from extrapolation to infinity from S_{64}</p>	<p>If M0 allow SC2 for awrt 0.626607 obtained from</p> $\frac{16 \times 0.62658745 - 0.62629755}{15}$ <p>then SC1 for 0.627 obtained from comparison with S_{64}</p>
			[6]			

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored