OCR
Oxford Cambridge and RSA

Friday 22 October 2021 – Afternoon

A Level Further Mathematics B (MEI)

Y434/01 Numerical Methods

Time allowed: 1 hour 15 minutes

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You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

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Answer **all** the questions.

1 **(a)** **(i)** Determine the relative error when

- 1.414214 is used to approximate $\sqrt{2}$,
- 1.414214^2 is used to approximate 2. **[3]**

(ii) Write down the relationship between your answers to part **(a)(i)**. **[1]**

(b) **Fig. 1** shows some spreadsheet output.

	A	B	C
1	2	1.414214	2

Fig. 1

The formula in cell B1 is $\boxed{= \text{SQRT(A1)}}$

and the formula in cell C1 is $\boxed{=\text{B1\^{}2}}$.

Ben evaluates 1.414214^2 on his calculator and obtains 2.000 001 238. He states that this shows that the value displayed in cell C1 is wrong.

Explain whether Ben is correct. **[1]**

2 The table shows some values of x and the associated values of $\mathrm{f}(x)$.

x	1	2	3	4	5
$\mathrm{f}(x)$	–0.65	–0.35	1.77	5.71	11.47

(a) Complete the difference table in the Printed Answer Booklet. **[2]**

(b) Explain why the data may be interpolated by a polynomial of degree 2. **[1]**

(c) Use Newton's forward difference interpolation formula to obtain a polynomial of degree 2 for the data. **[3]**

3 The method of False Position is used to find a sequence of approximations to the root of an equation. The spreadsheet output showing these approximations, together with some further analysis, is shown below.

	C	D	E	F	G	H	I	J
4	a	f(a)	b	f(b)	x_{new}	f(x_{new})	difference	ratio
5	1	–1.8248	2	17.2899	1.09547	–1.80507		
6	1.09547	–1.80507	2	17.2899	1.18097	–1.75418	0.08551	
7	1.18097	–1.75418	2	17.2899	1.25641	–1.66246	0.07544	0.88229
8	1.25641	–1.66246	2	17.2899	1.32164	–1.52781	0.06523	0.86458
9	1.32164	–1.52781	2	17.2899	1.37672	–1.35706	0.05508	0.84439
10	1.37672	–1.35706	2	17.2899	1.42208	–1.1642	0.04536	0.8236
11	1.42208	–1.1642	2	17.2899	1.45853	–0.96616	0.03646	0.80376
12	1.45853	–0.96616	2	17.2899	1.48719	–0.77825	0.02866	0.78598
13	1.48719	–0.77825	2	17.2899				
14								

The formula in cell D5 is $\boxed{= \text{SINH}(\text{C5}\text{^}2) - \text{C5}\text{^}3 - 2}$.

(a) Write down the equation which is being solved. **[1]**

The formula in cell C6 is $\boxed{= \text{IF}(\text{H5} < 0, \text{G5}, \text{C5})}$.

(b) Write down a similar formula for cell E6. **[1]**

(c) Calculate the values which would be displayed in cells G13 and G14 to find further approximations to the root. **[3]**

(d) Explain what the values in column J tell you about

- the order of convergence of this sequence of estimates,
- the speed of convergence of this sequence of estimates. **[2]**

4 The table shows some values of x and the associated values of $f(x)$.

x	4	4.0001	4.001	4.01	4.1
$f(x)$	4	4.0002386	4.0023871	4.0239468	4.2472072

(a) Calculate four estimates of the derivative of $f(x)$ at $x = 4$. **[4]**

(b) **Without** doing any further calculation, state the value of $f'(4)$ as accurately as you can, justifying the precision quoted. **[2]**

5 When Nina does the weekly grocery shopping she models the total cost by adding up the cost of each item in her head as she goes along. To simplify matters she rounds the cost of each item to the nearest pound.

One week Nina buys 48 items.

(a) Calculate the maximum possible error in Nina's model in this case. **[2]**

Nina estimated the total cost of her shopping to be £92. The actual cost is £90.23.

(b) Explain whether this is consistent with Nina's model. **[1]**

The next week her husband, Kareem, does the weekly shopping. He models the total cost by chopping the cost of each item to the nearest pound as he goes along.

On this occasion Kareem buys 52 items.

(c) Calculate the **expected** error in Kareem's model in this case. **[2]**

Using his model Kareem estimates the total cost as £76.

The total cost of the shopping is £103.24.

(d) Explain how such a large error could arise. **[1]**

The next week Kareem buys n items.

(e) Write down a formula for the maximum possible error when Kareem uses his model to estimate the total cost of his shopping. **[1]**

(f) Explain how Kareem's model could be adapted so that his formula gives the same expected error as Nina's model when they are both used to estimate the total cost of the shopping. **[2]**

6 The equation $0.5\ln x - x^2 + x + 1 = 0$ has two roots α and β, such that $0 < \alpha < 1$ and $1 < \beta < 2$.

(a) Use the Newton-Raphson method with $x_0 = 1$ to obtain β correct to **6** decimal places. **[4]**

Fig. 6.1 shows part of the graph of $y = 0.5\ln x - x^2 + x + 1$.

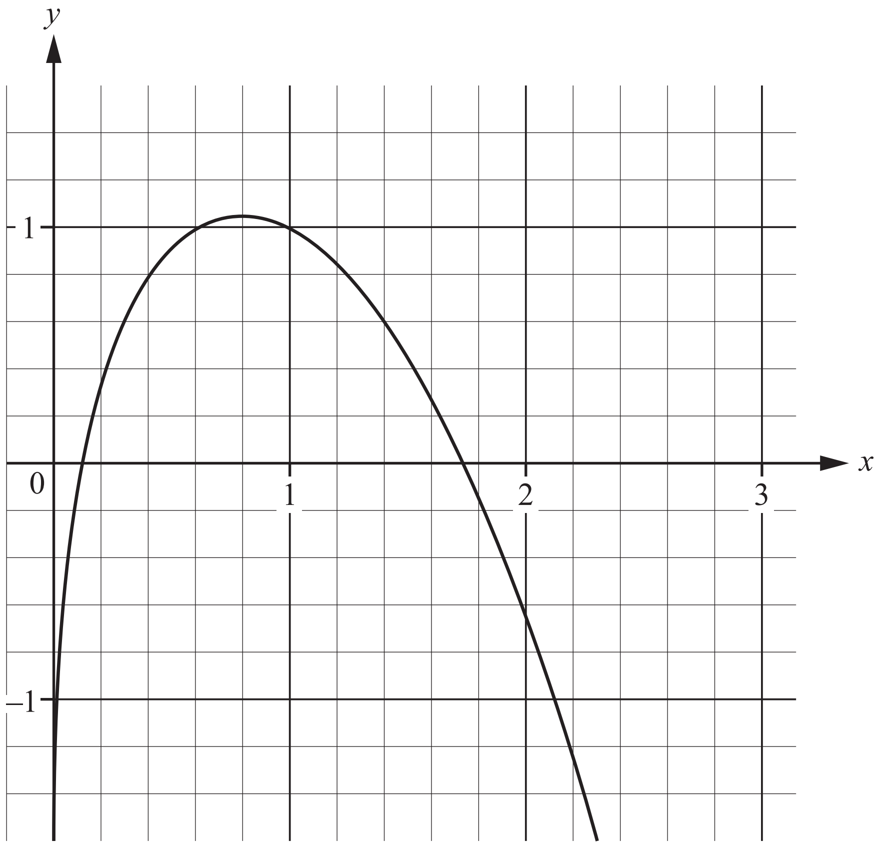

Fig. 6.1

(b) On the copy of **Fig. 6.1** in the Printed Answer Booklet, illustrate the Newton-Raphson method working to obtain x_1 from $x_0 = 1$. **[2]**

Beth is trying to find α correct to 6 decimal places.

(c) Suggest a reason why she might choose the Newton-Raphson method instead of fixed point iteration. **[1]**

Beth tries to find α using the Newton-Raphson method with a starting value of $x_0 = 0.5$. Her spreadsheet output is shown in **Fig. 6.2**.

r	x_r
0	0.5
1	–0.40343
2	#NUM!

Fig. 6.2

(d) Explain how the display #NUM! has arisen in the cell for x_2. **[1]**

Beth decides to use the iterative formula

$$x_{n+1} = \mathrm{g}(x_n) = \sqrt{0.5\ln(x_n) + x_n + 1}.$$

(e) Determine the outcome when Beth uses this formula with $x_0 = 0.5$. **[2]**

(f) Use the relaxed iteration $x_{n+1} = (1-\lambda)x_n + \lambda \mathrm{g}(x_n)$ with $\lambda = -0.041$ and $x_0 = 0.5$ to obtain α correct to **6** decimal places. **[2]**

7 Sarah uses the trapezium rule to find a sequence of approximations to $\int_0^1 \sqrt{\tanh(x)}\,dx$.

Her spreadsheet output is shown in **Fig. 7.1**.

n	T_n	difference	ratio
1	0.43634681		
2	0.5580694	0.121723	
4	0.60199843	0.043929	0.36089
8	0.61787073	0.015872	0.36132
16	0.62357601	0.005705	0.35945
32	0.62561716	0.002041	0.35777

Fig. 7.1

(a) Write down the value of h used to find the approximation 0.62357601. **[1]**

(b) **Without** doing any further calculation, state the value of $\int_0^1 \sqrt{\tanh(x)}\,dx$ as accurately as you can, justifying the precision quoted. **[1]**

(c) Explain what the values in the ratio column tell you about the order of convergence of this sequence of approximations. **[2]**

Sarah carries out further work using the midpoint rule and Simpson's rule. Her results are shown in **Fig. 7.2**.

	M	N	O	P	Q	R
1	n	T_n	M_n	S_{2n}	difference	ratio
2	1	0.43634681	0.679792	0.5986436		
3	2	0.5580694	0.64592745	0.61664144	0.018	
4	4	0.60199843	0.63374304	0.6231615	0.00652	0.362269
5	8	0.61787073	0.62928129	0.62547777	0.00232	0.355253
6	16	0.62357601	0.62765831	0.62629755	0.00082	0.35392
7	32	0.62561716	0.62707259			

Fig. 7.2

(d) Write down an efficient spreadsheet formula for calculating S_{16}. **[2]**

(e) Determine the missing values in row 7. **[3]**

(f) Use extrapolation to determine the value of $\int_0^1 \sqrt{\tanh(x)}\,dx$ as accurately as you can, justifying the precision quoted. **[6]**

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