OCR
Oxford Cambridge and RSA

# Monday 4 October 2021 – Afternoon

## A Level Further Mathematics B (MEI)

**Y420/01** Core Pure

**Time allowed: 2 hours 40 minutes**

**You must have:**
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

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**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **144**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

**ADVICE**
- Read each question carefully before you start your answer.

**Section A** (31 marks)

Answer **all** the questions.

**1** **(a)** Express $\dfrac{1}{(2r-1)(2r+1)}$ in partial fractions. **[3]**

**(b)** Hence find $\displaystyle\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}$, expressing the result as a single fraction. **[4]**

**2** **In this question you must show detailed reasoning.**

Find the gradient of the curve $y = 6\arcsin(2x)$ at the point with $x$-coordinate $\frac{1}{4}$. Express the result in the form $m\sqrt{n}$, where $m$ and $n$ are integers. **[4]**

**3** **In this question you must show detailed reasoning.**

The complex numbers $z_1$ and $z_2$ are given by $z_1 = -2+2\text{i}$ and $z_2 = 2\left(\cos\frac{1}{6}\pi + \text{i}\sin\frac{1}{6}\pi\right)$.

**(a)** Find the modulus and argument of $z_1$. **[2]**

**(b)** Hence express $\dfrac{z_1}{z_2}$ in exact modulus-argument form. **[4]**

**4** **In this question you must show detailed reasoning.**

Determine the mean value of $\dfrac{1}{1+4x^2}$ between $x = -1$ and $x = 1$. Give your answer to **3** significant figures. **[4]**

**5** **(a)** Use a Maclaurin series to find a quadratic approximation for $\ln(1+2x)$. **[1]**

**(b)** Find the percentage error in using the approximation in part **(a)** to calculate $\ln(1.2)$. **[3]**

**(c)** Jane uses the Maclaurin series in part **(a)** to try to calculate an approximation for $\ln 3$.

Explain whether her method is valid. **[2]**

**6** Given that $y = mx$ is an invariant line of the transformation with matrix $\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}$, determine the possible values of $m$. **[4]**

**Section B** (113 marks)

Answer **all** the questions.

**7** Prove that $\sum_{r=1}^{n} \frac{r}{2^{r-1}} = 4 - \frac{n+2}{2^{n-1}}$ for all $n \geqslant 1$. **[6]**

**8** The equation $4x^4 - 4x^3 + px^2 + qx - 9 = 0$, where $p$ and $q$ are constants, has roots $\alpha$, $-\alpha$, $\beta$ and $\frac{1}{\beta}$.

**(a)** Determine the exact roots of the equation. **[5]**

**(b)** Determine the values of $p$ and $q$. **[4]**

**9** The transformation T of the plane has associated matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} -1 & 0 \\ -2 & 1 \end{pmatrix}$.

**(a)** On the grid in the Printed Answer Booklet, plot the image OA′B′C′ of the unit square OABC under the transformation T. **[2]**

**(b)** **(i)** Calculate the value of $\det \mathbf{M}$. **[1]**

**(ii)** Explain the significance of the value of $\det \mathbf{M}$ in relation to the image OA′B′C′. **[2]**

**(c)** T is equivalent to a sequence of two transformations of the plane.

**(i)** Specify fully **two** transformations equivalent to T. **[3]**

**(ii)** Use matrices to verify your answer. **[3]**

**10** **(a)** Show on an Argand diagram the points representing the three cube roots of unity. **[2]**

**(b)** **(i)** Find the exact roots of the equation $z^3 - 1 = \sqrt{3}\,\mathrm{i}$, expressing them in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta < \pi$. **[5]**

**(ii)** The points representing the cube roots of unity form a triangle $\Delta_1$. The points representing the roots of the equation $z^3 - 1 = \sqrt{3}\,\mathrm{i}$ form a triangle $\Delta_2$.

State a sequence of two transformations that maps $\Delta_1$ onto $\Delta_2$. **[2]**

**(iii)** The three roots in part **(b)(i)** are $z_1$, $z_2$ and $z_3$.

By simplifying $z_1 + z_2 + z_3$, verify that the sum of these roots is zero. **[2]**

**(iv)** Hence show that $\sin 20° + \sin 140° = \sin 100°$. **[2]**

**11** **(a)** Given that $\mathbf{u} = \lambda\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, find the following, giving your answers in terms of $\lambda$.

**(i)** $\mathbf{u}.\mathbf{v}$ **[1]**

**(ii)** $\mathbf{u}\times\mathbf{v}$ **[2]**

**(b)** Hence determine

**(i)** the acute angle between the planes $2x+y-3z=10$ and $x+2y-2z=10$, **[3]**

**(ii)** the shortest distance between the lines $\frac{x-3}{3}=\frac{y}{1}=\frac{z-2}{-3}$ and $\frac{x}{1}=\frac{y-4}{2}=\frac{z+2}{-2}$, giving your answer as a multiple of $\sqrt{2}$. **[3]**

**12** **Fig. 12** shows a rhombus OACB in an Argand diagram. The points A and B represent the complex numbers $z$ and $w$ respectively.

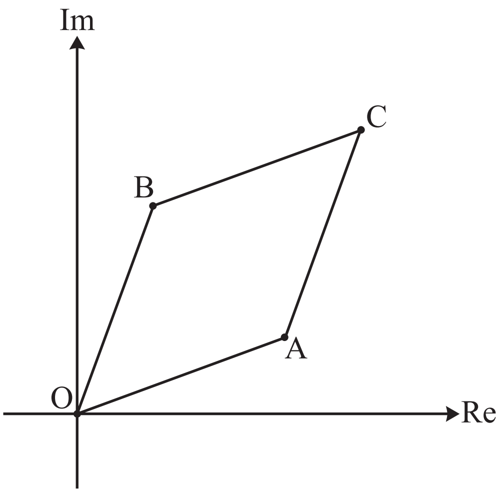


**Fig. 12**

Prove that $\arg(z+w)=\frac{1}{2}(\arg z+\arg w)$.

[A copy of **Fig. 12** is provided in the Printed Answer Booklet.] **[4]**

**13** Find the general solution of the differential equation $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}-3y=2\mathrm{e}^x$. **[7]**

**14** A curve has polar equation $r = a(\cos\theta + 2\sin\theta)$, where $a$ is a positive constant and $0 \leqslant \theta \leqslant \pi$.

**(a)** Determine the polar coordinates of the point on the curve which is furthest from the pole. **[7]**

**(b)** **(i)** Show that the curve is a circle whose radius should be specified. **[6]**

**(ii)** Write down the polar coordinates of the centre of the circle. **[1]**

**15** The equations of three planes are

$$-4x + ky + 7z = 4,$$
$$x - 2y + 5z = l,$$
$$2x + 3y + z = 2.$$

Given that the planes form a sheaf, determine the values of $k$ and $l$. **[6]**

**16** **(a)** Show using exponentials that $\cosh 2u = 1 + 2\sinh^2 u$. **[4]**

**(b)** Show that $\int_0^2 \frac{x^2}{\sqrt{4+x^2}}\,dx = 2\sqrt{2} - 2\ln(1+\sqrt{2})$. **[10]**

**17** In a chemical process, a vessel contains 1 litre of pure water. A liquid chemical is then passed into the top of the vessel at a constant rate of $a$ litres per minute and thoroughly mixed with the water. At the same time, the resulting mixture is drawn from the bottom of the vessel at a constant rate of $b$ litres per minute. You may assume that the chemical mixes instantly and uniformly with the water. After $t$ minutes, the mixture in the vessel contains $x$ litres of the chemical.

**(a)** **(i)** Show that the proportion of chemical present in the vessel after $t$ minutes is $\frac{x}{1+(a-b)t}$. **[2]**

**(ii)** Hence show that $\frac{dx}{dt} + \frac{bx}{1+(a-b)t} = a$. **[2]**

**(b)** First, consider the case where $b = a$.

**(i)** Solve the differential equation to find $x$ in terms of $a$ and $t$. **[4]**

**(ii)** Given that after 1 minute the vessel contains equal amounts of water and chemical, find the rate of inflow of chemical. **[2]**

**(c)** Now consider the case where $b = 2a$.

**(i)** Explain why the differential equation in part **(a)(ii)** is now invalid for $t \geqslant \frac{1}{a}$. **[1]**

**(ii)** Find the maximum amount of chemical in the vessel. **[9]**

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