GCE

Further Mathematics B (MEI)

Y435/01: Extra pure

Advanced GCE

Mark Scheme for Autumn 2021

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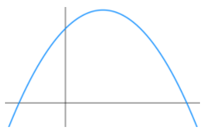
This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Question			Answer	Marks	AO	Guidance	
1	(a)		DR $z = \mathrm{f}(2, y) = 8 + 4y - 2y^2$	M1	1.1	Deriving correct equation of graph of section.	
			$= 10 - 2(y-1)^2 \Rightarrow$ max at (1, 10) or (2, 1, 10)	A1	1.1	Finding TP by completing the square, use of "$-b/2a$", differentiation or mid-point between roots.	Working must be shown.
				B1	1.1	∩-shaped parabola which <u>crosses</u> horizontal axis twice.	Condone incorrect variable names on axes (eg x-y for y-z).
			Crossing z-axis at 8, y-axis at $1 \pm \sqrt{5}$ and showing (1,10) as a max	A1	1.1	Coordinates of intercepts and max must be shown on graph or apparent in working. Allow decimal values (awrt –1.2 and 3.2) for the y-intercepts.	z intercept must be shown as positive and max in 1st quadrant. However, scale is unimportant except that the negative y-intercept must be closer to O than the positive one.
				[4]			

	(b)		$\frac{\partial z}{\partial x} = 3x^2 + 2xy$	B1	1.1		
			$\frac{\partial z}{\partial y} = x^2 - 4y$	B1	1.1		
			$\frac{\partial z}{\partial x} = 0$ $\Rightarrow 3x^2 + 2xy = 0$ $\Rightarrow$ either $x = 0$ or $x = -\frac{2}{3}y$ or $y = -\frac{3}{2}x$	M1	1.1	Setting a partial derivative to 0 and deriving condition(s) on *x* and/or *y*.	Or $\frac{\partial z}{\partial y} = 0$ $\Rightarrow y = \frac{1}{4}x^2$ or $x^2 = 4y$ or $x = \pm 2\sqrt{y}$
			$\frac{\partial z}{\partial y} = 0,\ x = -\frac{2}{3}y \Rightarrow 4y = \frac{4}{9}y^2 \Rightarrow y = 9$	M1	1.1	Substituting condition into other partial derivative equation to derive a non-zero value for *x* or *y*. $\frac{\partial z}{\partial y} = 0,\ y = -\frac{3}{2}x$ $\Rightarrow x^2 + 6x = 0 \Rightarrow x = -6...$	$\frac{\partial z}{\partial x} = 0,\ y = \frac{1}{4}x^2$ $3x^2 + \frac{1}{2}x^3 = 0 \Rightarrow x = -6$ or $\frac{\partial z}{\partial x} = 0,\ x = \pm 2\sqrt{y}$ $12y \pm 4y^{\frac{3}{2}} = 0 \Rightarrow y = 9$
			$x = -6$	A1	1.1		$y = 9$
			$z = -54$ so $(-6, 9, -54)$	A1	1.1	From correct working only.	
			... or $x = 0 \Rightarrow y = 0$ so $(0, 0, 0)$	A1	1.1	Derived from both $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ (could be by observation).	If an extra SP is presented then **A1** can be awarded for either SP correct and then **A0**.
				[7]			

2	(a)		From Lagrange's Theorem the order of any subgroup of G must be a factor of 8 and 6 is not a factor of 8	B1 [1]	2.4	Or "order of any subgroup of G (or an group of order 8) must be 1, 2 or 4 (or 8)" or "order of any subgroup must be a factor of the order of the group and 6 is not a factor of 8".	If referenced, Lagrange's Theorem does not have to be quoted provided that it is applied. So **B1** for eg "6 is not a factor of 8 so by Lagrange's Theorem there can be no subgroup of G of order 6" but **B0** for eg "By Lagrange's Theorem there can be no subgroup of G of order 6".
2	(b)		g^2 (or g^6) g^6 (or g^2) and no other	B1 B1 [2]	2.2a 2.2a		May see eg gg or $g \circ g$ used here and/or throughout. Allow any multiplicative notation and any symbol for a binary operation.
2	(c)		$e \leftrightarrow 0$ $g \leftrightarrow 1$, $g^2 \leftrightarrow 2$, $g^3 \leftrightarrow 3$, $g^4 \leftrightarrow 4$, $g^5 \leftrightarrow 5$, $g^6 \leftrightarrow 6$, $g^7 \leftrightarrow 7$ $g \leftrightarrow 3$, $g^2 \leftrightarrow 6$, $g^3 \leftrightarrow 1$, $g^5 \leftrightarrow 7$, $g^6 \leftrightarrow 2$, $g^7 \leftrightarrow 5$ $g \leftrightarrow 5$, $g^2 \leftrightarrow 2$, $g^3 \leftrightarrow 7$, $g^5 \leftrightarrow 1$, $g^6 \leftrightarrow 6$, $g^7 \leftrightarrow 3$ and $g \leftrightarrow 7$, $g^2 \leftrightarrow 6$, $g^3 \leftrightarrow 5$, $g^5 \leftrightarrow 3$, $g^6 \leftrightarrow 2$, $g^7 \leftrightarrow 1$	B1 B1 B1 B1	2.2a 2.2a 2.2a 2.2a	Only needs to be seen once. Any one. Any other. Other two. Ignore repeats.	 $g^4 \leftrightarrow 4$ does need not be seen again $g^4 \leftrightarrow 4$ does need not be seen again
			Alternative method: $e \leftrightarrow 0$ Either $g \leftrightarrow 1$ or $g \leftrightarrow 3$ or $g \leftrightarrow 5$ or $g \leftrightarrow 7$ $g \leftrightarrow 1$, $g^2 \leftrightarrow 2$, $g^3 \leftrightarrow 3$, $g^4 \leftrightarrow 4$, $g^5 \leftrightarrow 5$, $g^6 \leftrightarrow 6$, $g^7 \leftrightarrow 7$ $g \leftrightarrow 3$, $g^2 \leftrightarrow 6$, $g^3 \leftrightarrow 1$, $g^5 \leftrightarrow 7$, $g^6 \leftrightarrow 2$, $g^7 \leftrightarrow 5$ and $g \leftrightarrow 5$, $g^2 \leftrightarrow 2$, $g^3 \leftrightarrow 7$, $g^5 \leftrightarrow 1$, $g^6 \leftrightarrow 6$, $g^7 \leftrightarrow 3$ and $g \leftrightarrow 7$, $g^2 \leftrightarrow 6$, $g^3 \leftrightarrow 5$, $g^5 \leftrightarrow 3$, $g^6 \leftrightarrow 2$, $g^7 \leftrightarrow 1$	 B1 M1 A1 A1		 Only needs to be seen once Giving all 4 possible isomorphism options for any generator of G (ie g, g^3, g^5 or g^7) Completing the specification of any one isomorphism Other three. Ignore repeats.	 $g^4 \leftrightarrow 4$ does need not be seen again
				[4]			

3	(a)		$\det(\mathbf{A}-\lambda\mathbf{I}) = \begin{vmatrix} 3-\lambda & 3 & 0 \\ 0 & 2-\lambda & 2 \\ 1 & 3 & 4-\lambda \end{vmatrix}$	M1	1.1a	Formation of appropriate determinant soi.	
			$= (3-\lambda)[(2-\lambda)(4-\lambda)-2\times 3]-3(0-2\times 1)$ oe	M1	1.1	Attempt to expand determinant. Allow one slip.	May see eg expansion by 1st col: $(3-\lambda)[(2-\lambda)(4-\lambda)-6]+1(6-0)$ Or other formulation eg: $((3-\lambda)(2-\lambda)(4-\lambda)+6+0)$ $-(0+6(3-\lambda)+0)$
			$= -\lambda^3 + 9\lambda^2 - 20\lambda + 12 = 0$	A1 [3]	1.1	Must be an equation. ISW.	
3	(b)		1, 2 and 6 substituted into **(a)** equation to verify	B1 [1]	1.1	eg checking trace is insufficient.	
3	(c)		$3a + 3b = a$ or $2a$ or $6a$ and $2b + 2c = b$ or $2b$ or $6b$ and $a + 3b + 4c = c$ or $2c$ or $6c$	M1	1.1	Correctly forming 3 equations in 3 unknowns for one of their eigenvalues. May see explicit choice of eg $c = 1$ to form 3 equations in 2 unknowns.	Or formation of appropriate determinant eg $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 4-\lambda & -4 \\ 0 & -1 & 3-\lambda \end{vmatrix}$.
			$\lambda = 1$: $2a = -3b$, $b = -2c$ or $\lambda = 2$: $c = 0$, $a = -3b$ or $\lambda = 6$: $a = b$, $c = 2b$	M1	1.1	Attempt to solve equations for at least one of their eigenvalues leading to two unknowns in terms of 3rd.	Attempt to expand determinant (might be in terms of λ) eg $\begin{pmatrix} 8-7\lambda+\lambda^2 \\ 6-2\lambda \\ 2 \end{pmatrix}$. Can be inferred by 2 correct coefficients.
			$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	A1	1.1	or any non-zero multiple.	
			$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	A1 [4]	1.1	or any non-zero multiple.	

3	(d)		$\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$	M1	3.1a	Forming matrix of their eigenvectors, **E**.	
			$\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}^{-1} = \frac{1}{10}\begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix}$ oe	A1FT	3.1a	BC. Finding inverse of their matrix of eigenvectors.	May be in decimal form: $\begin{pmatrix} -0.2 & -0.6 & 0.4 \\ -0.5 & -0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 \end{pmatrix}$
			$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix}$	B1	3.1a	Matrix of eigenvalues must be consistent with matrix of eigenvectors. Allow 1^n.	
			$\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix}\frac{1}{10}\begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix}$	M1	3.1a	Forming $\mathbf{E\Lambda}^n\mathbf{E}^{-1}$. Can be awarded if $\mathbf{\Lambda}^n$ incorrect or uncalculated but eigenvectors must be in same order as eigenvalues.	
			$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix}\begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} =$ $\begin{pmatrix} -2 & -6 & 4 \\ -5\times 2^n & -5\times 2^n & 5\times 2^n \\ 6^n & 3\times 6^n & 3\times 6^n \end{pmatrix}$	M1	1.1	Proper attempt to multiply either the first two or the last two (of 3) in the correct order (with or without $\frac{1}{10}$).	$\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 6^n \end{pmatrix} =$ or $\begin{pmatrix} 3 & -3\times 2^n & 6^n \\ -2 & 2^n & 6^n \\ 1 & 0 & 2\times 6^n \end{pmatrix}$
			$\frac{1}{10}\begin{pmatrix} 3 & -3 & 1 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}\begin{pmatrix} -2 & -6 & 4 \\ -5\times 2^n & -5\times 2^n & 5\times 2^n \\ 6^n & 3\times 6^n & 3\times 6^n \end{pmatrix} =$ $\frac{1}{10}\begin{pmatrix} -6+15\times 2^n+6^n & -18+15\times 2^n+3\times 6^n & 12-15\times 2^n+3\times 6^n \\ 4-5\times 2^n+6^n & 12-5\times 2^n+3\times 6^n & -8+5\times 2^n+3\times 6^n \\ -2+2\times 6^n & -6+6^{n+1} & 4+6^{n+1} \end{pmatrix}$	A1 [6]	1.1	or $\frac{1}{10}\begin{pmatrix} 3 & -3\times 2^n & 6^n \\ -2 & 2^n & 6^n \\ 1 & 0 & 2\times 6^n \end{pmatrix}\begin{pmatrix} -2 & -6 & 4 \\ -5 & -5 & 5 \\ 1 & 3 & 3 \end{pmatrix} =$ etc. Condone 6×6^n unsimplified.	

4	(a)		CF: $u_{n+2} - 3u_{n+1} - 10u_n = 0$ and $u_n = \alpha r^n$ $\Rightarrow r^2 - 3r - 10 = 0$	M1	1.1a	Deriving the auxiliary equation (allow one sign error).	
			$\Rightarrow r = 5$ or $r = -2$ CF is $\alpha 5^n + \beta(-2)^n$	A1FT	1.1	FT correct roots of their AE to form CF (do not ISW).	Condone missing brackets around –2 unless misused.
			Trial function: $u_n = an + b$	B1	1.1a	Correct form.	Other forms eg $an^2 + bn + c$ are allowable provided $a = 0$ derived.
			$a(n+2) + b - 3[a(n+1) + b] - 10(an+b)$ $= 24n - 10$	M1	1.1	Substituting their form correctly into recurrence relation.	
			$\Rightarrow (a - 3a - 10a) = 24$ and $2a + b - 3a - 3b - 10b = -10$	M1	1.1	Deriving two equations in a and b using a correct method (eg comparing coefficients)	
			$a = -2$ and $b = 1$ so GS is $u_n = 1 - 2n + \alpha 5^n + \beta(-2)^n$	A1 **[6]**	1.1	Full form of GS, including $u_n =$, must be seen.	cao
4	(b)		Either: $n = 0 \Rightarrow 1 + \alpha + \beta = 6$ or: $n = 1 \Rightarrow 1 - 2 + 5\alpha - 2\beta = 10$	M1	1.1	Substituting $n = 0$ or $n = 1$ in their GS to derive an equation in α & β.	This mark can be awarded if one of their equations is wrong.
			$\alpha + \beta = 5$ and $5\alpha - 2\beta = 11$ $\Rightarrow 2\alpha + 2\beta = 10 \Rightarrow 7\alpha = 21$	M1	1.1	Deriving 2 equations from substituting $n = 0$ & 1, at least one correct for their GS, and attempting to solve .	Attempt to solve can be implied by correct answer or valid algebra but incorrect answer with no working **M0**
			$\alpha = 3$ and $\beta = 2$ so $u_n = 1 - 2n + 3\times5^n + 2\times(-2)^n$	A1FT **[3]**	1.1	FT from their GS. Allow non-embedded values if GS seen in **(a)**. Do not ISW.	
4	(c)		From recurrence relation: $u_2 = 3u_1 + 10u_0 + 24\times0 - 10$ $= 3\times10 + 10\times6 - 10 = 80$ From particular solution: $u_2 = 1 - 2\times2 + 3\times5^2 + 2\times(-2)^2$ $= 1 - 4 + 75 + 8 = 80$	B1 **[1]**	2.5	Both expressions properly seen (ie it must be clear that candidates are correctly using two different methods to find u_2).	

4	(d)		$v_n = \frac{1-2n}{p^n} + 3\left(\frac{5}{p}\right)^n + 2\left(\frac{-2}{p}\right)^n$	M1	3.1a	Writing v_n in a form which enables the limit to be deduced.	
			If $\lvert p \rvert < 5$ then $v_n \to \infty$ while if $\lvert p \rvert > 5$ then $v_n \to 0$ as $n \to \infty$	B1	2.1	Convincing argument. FT for GS of the form: $c - dn + \alpha s^n + \beta t^n$ (where $\lvert s \rvert > \lvert t \rvert$).	At most one of c and d is 0. s and t are not equal and both not 0. Both α and β are not 0. Either $\lvert s \rvert > 1$ or $\lvert t \rvert > 1$ (or both).
			$p = 5$	A1	2.2a	FT. $p = s$ (must be a number).	**A0** If $s = -t$.
			$q = 3$	A1	2.2a	FT. $q = \alpha$ (must be a number).	**A0** If $s = -t$. If **M0** then **SC2** for $p = 5$, $q = 3$.
				[4]			

Question			Answer	Marks	AO	Guidance	
5			$\frac{\partial g}{\partial x} = 2x$ or $\frac{\partial g}{\partial y} = 2y$ or $\frac{\partial g}{\partial z} = 4z$	M1	3.1a	g(x, y, z) = $x^2 + y^2 + 2z^2$ and surface is g = 126. Finding one correct partial derivative.	May be rewritten as $z = \mathrm{f}(x, y) = \sqrt{63 - \frac{1}{2}y^2 - \frac{1}{2}x^2}$ but condone ±.
			$\nabla g = \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix}$ is the normal to the tangent plane at each point.	A1	3.1a	Finding the normal vector.	$\nabla g = \begin{pmatrix} -\frac{x}{2z} \\ -\frac{y}{2z} \\ -1 \end{pmatrix}$ oe
			$\nabla g.\mathbf{n} = \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix} . \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 4z$	M1	3.1a	Dotting normal with normal to x-y plane.	
			$= \left\lvert \begin{pmatrix} 2x \\ 2y \\ 4z \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rvert \cos\frac{\pi}{3}$	M1	2.2a	Expressing dot product in other form using correct value of angle.	
			$= \sqrt{(2x)^2 + (2y)^2 + (4z)^2} \times 1 \times \frac{1}{2}$ $= 2\sqrt{x^2 + y^2 + 4z^2} \times \frac{1}{2} = \sqrt{126 - 2z^2 + 4z^2}$ $= \sqrt{126 + 2z^2}$	M1	1.1	Using $\cos\frac{\pi}{3} = \frac{1}{2}$, forming magnitude of both normals and reducing to form $\sqrt{a + bz^2}$ oe (could be done after squaring).	or $\sqrt{a + bx^2 + by^2}$ (could see eg $x^2 + y^2 = 108$ oe after equating to $4z$ and eliminating z).
			$\sqrt{126 + 2z^2} = 4z \Rightarrow 126 + 2z^2 = 16z^2$ $\Rightarrow 14z^2 = 126 \Rightarrow z^2 = 9 \Rightarrow z = \pm 3$ $z \geq 0 \Rightarrow z = 3$ which is the equation of Π.	A1 [6]	3.2a	Not ± in final answer.	

6	(a)		$\frac{1}{(q+1)}+\frac{1}{(q+1)(q+2)}+\frac{1}{(q+1)(q+2)(q+3)}+\ldots$ $<\frac{1}{(q+1)}+\frac{1}{(q+1)^2}+\frac{1}{(q+1)^3}+\ldots$	M1	2.1	Correct statement that given series is less than an infinite GP (could be eg $\frac{1}{q}+\frac{1}{q^2}+\ldots$ or $\frac{1}{3}+\frac{1}{3^2}+\ldots$).	
			$=\frac{\frac{1}{q+1}}{1-\frac{1}{q+1}}$	A1	2.1	FT on their $\frac{a}{1-r}$.	
			$=\frac{1}{q+1-1}=\frac{1}{q}$	A1 [3]	2.1	AG. Intermediate step must be seen.	
6	(b)		$q\geq 1\Rightarrow \frac{1}{q}\leq 1$	M1	2.2a		
			But $S<\frac{1}{q}\Rightarrow S<1$; clearly $S>0$ so $0<S<1$ so $S\notin$.	A1 [2]	2.2a	AG. $S>0$ must be stated but need not be justified.	Since $0<\frac{1}{q}\leq 1$ and $S<\frac{1}{q}$ then $0<S<1$ and $\therefore S\notin$.
6	(c)		$\mathrm{e}=\sum_{r=0}^{\infty}\frac{1}{r!}=\frac{p}{q}\Rightarrow \mathrm{e}q!=\sum_{r=0}^{\infty}\frac{q!}{r!}=p(q-1)!$	M1	3.1a	Multiplying both sides by $q!$ No need to mention $q\geq 1$ in this part.	
			$\therefore p(q-1)!=\sum_{r=0}^{\infty}\frac{q!}{r!}=\sum_{r=0}^{q}\frac{q!}{r!}+\sum_{r=q+1}^{\infty}\frac{q!}{r!}$ $=q!+q!+\frac{q!}{2!}+\ldots+\frac{q!}{q!}+S$ $=2q!+q(q-1)\ldots\times 3+q(q-1)\ldots\times 4+\ldots+1+S$	M1	2.1	Rewriting to a form in which it is clear that every term on both sides, except S, is an integer.	
			$p(q-1)!$ and $q!+q!+\frac{q!}{2!}+\ldots+1$ are all integers but S is not which is a contradiction.	A1	3.2a	AG	

			Alternative Method: $S = \sum_{r=q+1}^{\infty} \frac{q!}{r!}$	M1		Expressing S as an infinite sum in terms of factorials.	
			$S = q!\sum_{r=0}^{\infty}\frac{1}{r!} - \sum_{r=0}^{q}\frac{q!}{r!} = q!\mathrm{e} - q! - \sum_{r=1}^{q}\frac{q!}{r!}$ $= p(q-1)! - q! - \sum_{r=1}^{q} q(q-1)\ldots(q-r+1)$ since $1 \le r \le q$	M1		Rewriting to a form in which it is clear that every term on both sides, except S, is an integer.	
			$p(q-1)!$, $q!$ and $q(q-1)\ldots(q-r+1)$ are all integers but S is not which is a contradiction.	A1		AG	
				[3]			

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