GCE

Further Mathematics B (MEI)

Y436/01: Further pure with technology

Advanced GCE

Mark Scheme for Autumn 2021

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

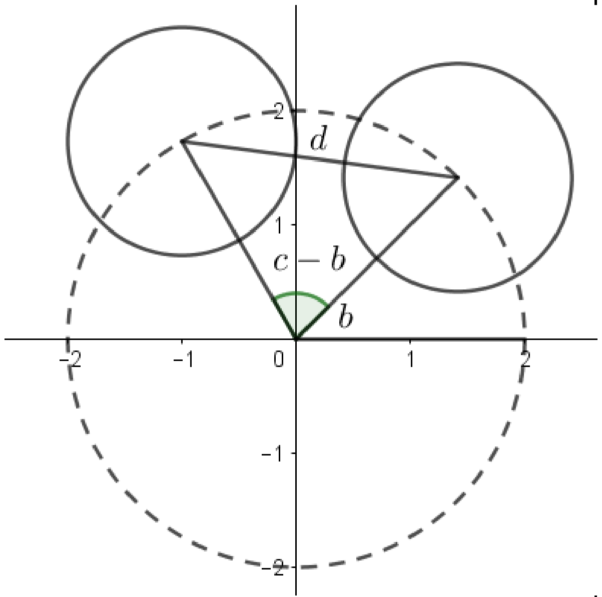
This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

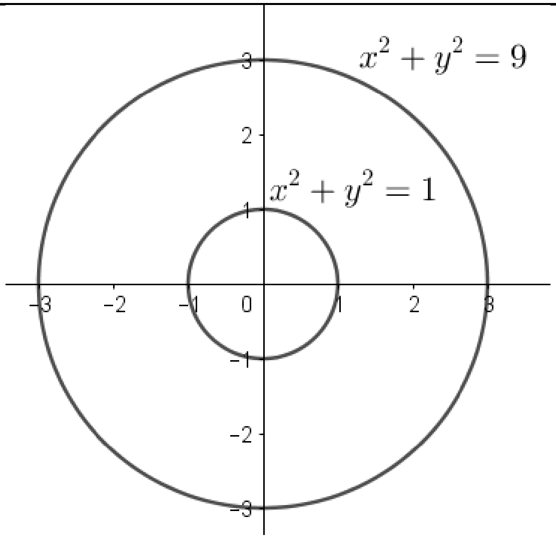
All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

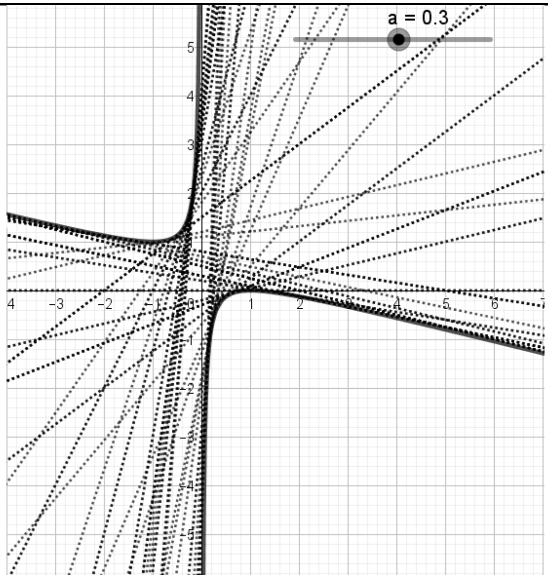
Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Question			Answer	Marks	AOs		Guidance
1	(a)		$x^2 + y^2 = 4$	B1 [1]	1.2		
	(b)		Let d be the distance concerned. Then d is as in the diagram below.  			Diagram not necessary.	
			By reference to a diagram using the cosine rule $d^2 = 2^2 + 2^2 - 2\times 2\times 2\cos(c-b)$ $= 8(1-\cos(c-b))$. Therefore $d = 2\sqrt{2}\sqrt{1-\cos(c-b)}$.	M1 A1 [2]	2.1 1.1	Alternatives here include $d^2 = (2\cos(c) - 2\cos(b))^2 + (2\sin(c) - 2\sin(b))^2$	

Question			Answer	Marks	AOs		Guidance
	(c)		For the circles to touch, need $d = 2$ in the above. $2 = 2\sqrt{2}\sqrt{1-\cos(c-b)}$ $\Rightarrow 1 = \sqrt{2}\sqrt{1-\cos(c-b)}$ $\Rightarrow \frac{1}{\sqrt{2}} = \sqrt{1-\cos(c-b)}$ Then $\Rightarrow \frac{1}{2} = 1-\cos(c-b)$ $\Rightarrow \cos(c-b) = \frac{1}{2}$ $\Rightarrow c-b = \frac{\pi}{3}$, since $0 \le b < c < \pi$	M1 A1 [2]	2.1 1.1	Or just state that it's an equilateral triangle in this case.	
	(d)			B1 B1 [2]	1.1 1.1	Equations are not required in this part.	

Question			Answer	Marks	AOs		Guidance
	(e)		$x^2 + y^2 = 1$ $x^2 + y^2 = 9$	B1 B1 [2]	1.2 1.2		
2	(a)		Gradient of the line through $(0, a)$ and $(1, a^2)$ is $\frac{a^2 - a}{1 - 0} = a(1 - a)$. The line crosses the $y-$axis at $(0, a)$ so the equation of the line is $y = a(a-1)x + a$	M1 A1 [2]	1.1a 1.1b		
	(b)		The two straight lines are $y = b(b-1)x + b$ and $y = c(c-1)x + c$. These are parallel if $b(b-1) = c(c-1)$ $\Rightarrow 0 = c^2 - b^2 + b - c$ $\Rightarrow 0 = (c-b)(c+b) - (c-b)$ $\Rightarrow 0 = (c-b)(c+b-1)$ $\Rightarrow 0 = c + b - 1$ (since $c \neq b$) $\Rightarrow c + b = 1$	M1 M1 A1 [3]	3.1a 2.4 2.1	Note that equation can be solved using CAS which is an acceptable method.	

Question			Answer	Marks	AOs		Guidance
	(c)			B1 B1 [2]	1.1b 1.1b	One mark for each branch of the curve, check for shape, asymptotes, max/min and quadrants and and points on the axes. Note that question requires 'sketch'.	

Question			Answer	Marks	AOs		Guidance
	(d)		Let $\mathrm{f}(x,y,a)=a(a-1)x-y+a$. The envelope is the set of points for which, simultaneously $\mathrm{f}(x,y,a)=0$ and $\frac{\partial f}{\partial a}(x,y,a)=0$, for some value of a.	M1	3.1a		
			$\mathrm{f}(x,y,a)=a^2x-ax-y+a$ $\Rightarrow \frac{\partial f}{\partial a}=2ax-x+1$	M1	1.1		
			$\frac{\partial f}{\partial a}(x,y,a)=0$ implies that $a=\frac{x-1}{2x}$ (note x not zero) ,	M1	1.1		
			If, in addition, f $(x,y,a)=0$ then $0=\left(\frac{x-1}{2x}\right)\left(\frac{x-1}{2x}-1\right)x-y+\frac{x-1}{2x}$	M1	2.2a		
			$\Rightarrow y=\frac{x-1}{2x}\left(\frac{-1-x}{2}+1\right)$ oe isw	A1 [5]	2.1	This is awarded for any correct expression with y in terms of x explicitly. Might also see $y=-\frac{(x-1)^2}{4x}$ or $y=\left(\frac{x-1}{2x}\right)\left(\frac{1-x}{2}\right)$ for example.	

3	(a)	(i)	Appropriate structure program
			
			Loop with correct range dependent on m, n. Check for common divisors with if statement and tracking greatest one found.
			
			Fully correct program.
				M1
				
				M1
				
				A1
				[3]
					3.3
					
					2.1
					
					2.5
						Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors
						
						Example code for Python
						def hcf(m,n):
						    k = min(m,n)
						    hcf=1
						    for i in range(1,k+1):
						        if m%i==0 and n%i==0:
						            hcf=i
						    return hcf
						print(hcf(m,n))	

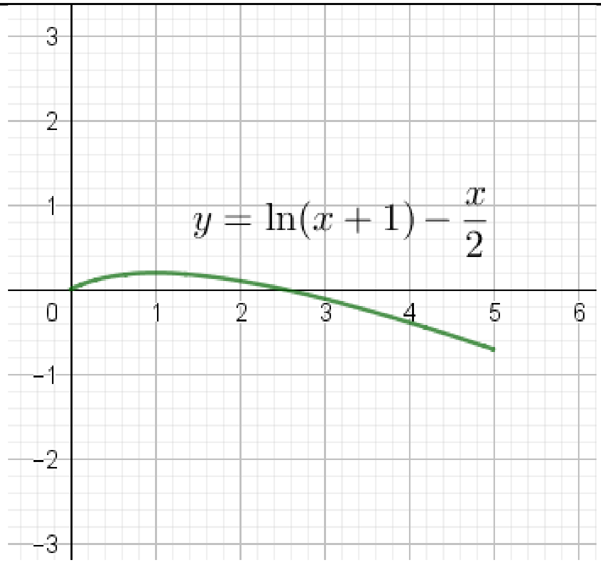
		(ii)	(74333, 89817) = 49	B1 [1]	1.1	Set m = 74333 and n =89817 in the above.	

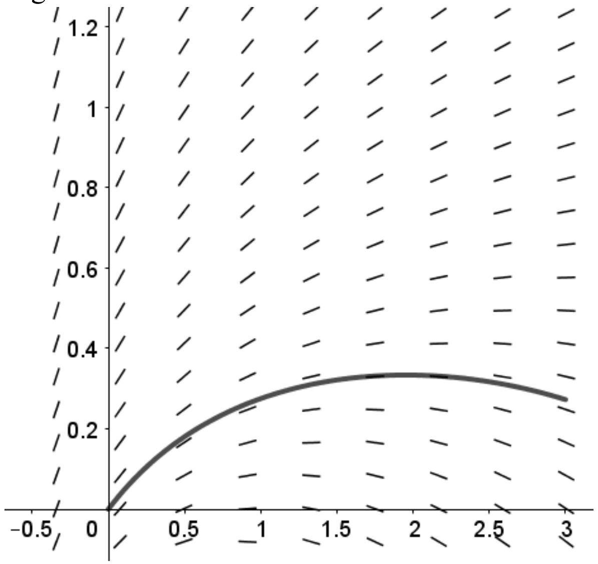
	(b)	(i)	Appropriate structure program
			
			Loop with correct range and counts number of values coprime to k.
			
			Fully correct programme
				M1
				
				M1
				
				A1
				
				[3]
					3.3
					
					2.1
					
					2.5
						Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors
						
						Example code for Python with hcf function as in 2(i) above.
						def phi(k):
						    count = 0
						    for i in range(1,k):
						        if hcf(i,k)==1:
						            count = count + 1
						    return count
						print(phi(k))	

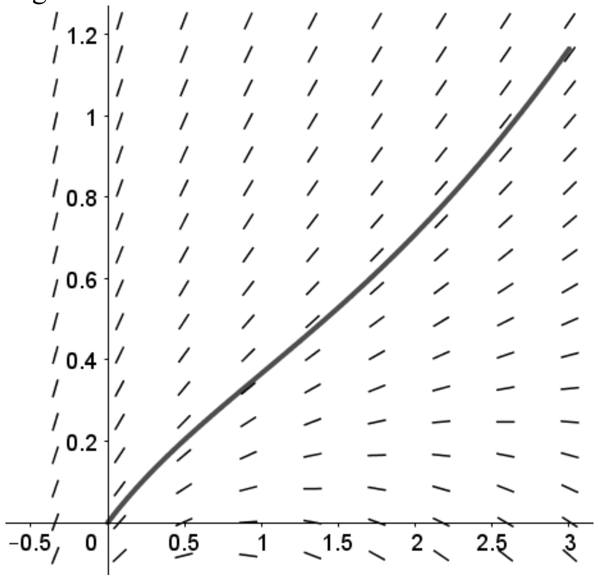
		(ii)	$\varphi(128) = 64$ and $\varphi(1000) = 400$	B1 B1 [2]	1.1 1.1		
		(iii)	$\varphi(2^n) = 2^{n-1}$. This is because all the odd numbers less than 2^n are coprime to 2^n and all the even numbers less than 2^n are not. The are 2^{n-1} such odd numbers.	M1 A1 [2]	2.1 3.2a	Spotting odd/even property. Correct value in terms of *n*.	
		(iv)	$\varphi(10^n) = 4 \times 10^{n-1}$. All numbers less than 10^n with final digit 1, 3, 7 and 9 are coprime to 10^n, any other number is not. There are four such numbers in 1, 2,…,10, four in 11, 12,…,20, four in 21,22,…,30, and so on. There are 10^{n-1} such groups before reaching 10^n. So there are $4 \times 10^{n-1}$ number less than 10^n which are coprime to 10^n.	M1 M1 A1 [3]	2.1 2.2a 3.2a	Spotting end digit property. Applying it across all numbers less than 10^n. Correct value in terms of *n*.	

	(c)	(i)	F(5) = 9, the corresponding fractions are $\frac{1}{5},\frac{1}{4},\frac{1}{3},\frac{2}{5},\frac{1}{2},\frac{3}{5},\frac{2}{3},\frac{3}{4},\frac{4}{5}$. F(6) = 11, the corresponding fractions are $\frac{1}{6},\frac{1}{5},\frac{1}{4},\frac{1}{3},\frac{2}{5},\frac{1}{2},\frac{3}{5},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6}$	**B1** **B1** **[2]**	**1.1** **1.1**		
		(ii)	Adding to the distinct fractions between 0 and 1 with denominator *k*, the only 'new' fractions with denominator $k+1$ have numerators which are coprime to $k+1$. Therefore there are $\varphi(k+1)$ of these.	**B1** **B1** **[2]**	**3.1a** **2.4**		

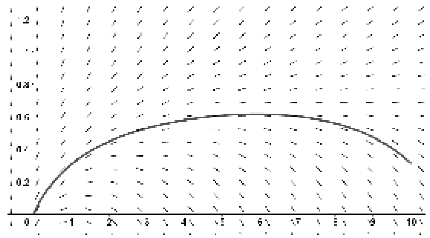
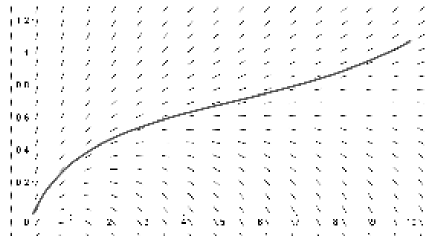
		(iii)	By (c)(ii) required value is $\sum_{k=1}^{100}\varphi(k)$. By adapting previous program this is 3043.	**M1**
				
				A1
				[2]
					3.1a
					
					1.1
						By adding code such as
						def fracs(k):
						    count = 0
						    for i in range(1,k+1):
						        count = count + phi(i)
						    return count
						print(fracs(100))	

4	(a)	(i)	Solution is $y=\ln(x+1)-\frac{x}{2}$	**B1** **[1]**	**1.1a**		
		(ii)		**B1** **[1]**	**1.1**		
		(iii)	With $y=\ln(x+1)-\frac{x}{2}$, $\frac{dy}{dx}=\frac{1}{x+1}-\frac{1}{2}$. Solving $\frac{dy}{dx}=0$ gives $x=1$. When $x=1$, $y=\ln(2)-\frac{1}{2}$	**M1** **A1** **[2]**	**1.1a** **1.1**		
	(b)	(i)	Fig 4.1, a is around 0.5	**B1** **[1]**	**1.1**		

		(ii)	Fig 4.2. a is around 1	**B1** **[1]**	**1.1**		
		(iii)	Fig 4.1 	**B1**	**1.1**	Sufficient to see slight downturn as x increases and only stationary pt is the maximum	

		(iv)	Fig 3.2  	B1 [2]	1.1	Sufficient to be increasing.	
		(v)	One is increasing for the values of x shown. The other has a stationary point (local maximum).	B1 [1]	1.2	Either comment will do. Allow 'one intersects the x-axis (eventually), the other doesn't.'	

	(c)	(i)	A1 contains 0 B1 contains 0 I1 contains 0.1 (the value of h) K1 contains a (the value of a) C1 =$I$1*((1-A1)/(2*(A1+1))+$K$1*ATAN(B1)) D1 =$I$1*((1-(A1+$I$1))/(2*(A1+$I$1+1))+$K$1*ATAN(B1+C1)) A2 =A1+$I$1 B2 = =B1+0.5*(C1+D1) copy down	B1 B1 B1 B1 [4]	3.1a 3.1a 3.1a 2.5	Give reasonable BOD on possible transcription errors and consider correct answers to 4(c)(ii), 4(c)(iii), 4(c)(iv) as evidence of correct formulae in the spreadsheet. Allows for a and h to be varied. Cols for x and y Cols for k_1 and k_2 Formulae for x_{n+1} and y_{n+1}	
		(ii)	Approximation to y when $x = 5.0$ with $a = 0.5$, using $h = 0.1$ is -0.249889 (to 6 d.p.)	B1 [1]	1.1	Correct answer to at least 3 s.f. Must for correct for the number of significant figures given.	
		(iii)	Approximation to y when $x = 5.0$ with $a = 1$, using $h = 0.1$ is 3.160809 (to 6 d.p.)	B1 [1]	1.1	Correct answer to at least 3 s.f. Must for correct for the number of significant figures given.	

		(iv)	Using $a = 0.645$ (with $h = 0.1$) produces an approximate solution which increases initially but then peaks and descreases. Using $a = 0.655$ (with $h = 0.1$) produces an approximate solution which increases indefinitely. This is evidence that a^* is between 0.645 and 0.655. This suggests that $a^* = 0.65$ to 2 d.p.	M1 M1 M1 A1 [4]	3.1a 3.1a 2.2a 2.2b	Further investigation of other values of h and graphing software might be used to confirm this. This should be awarded credit appropriately. E.g here is solution with $a = 0.645$  And here is solution with $a = 0.655$ 	

Question	A01	A02	A03	E	C	A	
1(i)(A)	1			1			C1, C4
1(i)(B)	1	1		2			C4
1(i)(C)	1	1		1	1		C4
1(i)(D)	2			2			C9
1(i)(E)	2			2			C4, C9
1(ii)(A)	2			2			C4
1(ii)(B)		2	1	1	2		C4
1(ii)(C)	2			1	1		C9
1(ii)(D)	2	2	1			5	C9
2(i)(A)		2	1	2	1		T1, T5
2(i)(B)	1			1			T5
2(ii)(A)		2	1	2	1		T6
2(ii)(B)	2			1	1		T5, T6
2(ii)(C)		1	1	1	1		T5, T6
2(ii)(D)		2	1	1	1	1	T5, T6
2(iii)(A)	2			2			T5
2(iii)(B)		1	1			2	T5, T6
2(iii)(C)	1		1			2	T5, T6
3(i)(A)	1			1			C1
3(i)(B)	1			1			C1
3(i)(C)	2			2			C5
3(ii)(A)	2			2			C2, C6
3(ii)(B)	2			2			C6
3(ii)(C)	1				1		C6
3(iii)(A)		1	3		3	1	C7
3(iii)(B)	1				1		C7
3(iii)(C)	1				1		C7
3(iii)(D)		2	2			4	C6, C7, C8
Total	30	17	13	30	15	15	0.00

S&C marks: 1(ii)D 5 marks

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk