Oxford Cambridge and RSA

Wednesday 20 October 2021 – Afternoon

A Level Further Mathematics B (MEI)

Y433/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The list below shows the sizes of 13 items.

5 16 12 15 21 10 17 5 3 6 13 24 5

(a) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 45. **[2]**

(b) Show the result of applying a full bin strategy to pack items with the sizes listed above into bins that have a capacity of 45. **[2]**

2 The table below shows the activities involved in a project, together with their immediate predecessors. The table also gives the duration (in hours) of each activity, apart from G and H.

Activity	Immediate predecessors	Duration (hours)
A	–	6
B	–	4
C	–	8
D	C	3
E	A, B, D	6
F	B, D	10
G	C	
H	C	
I	E, F, G	10
J	F, G	7
K	F, G, H	9

(a) Draw an activity network, using activity on arc, to represent the project. **[3]**

It is given that the only critical activities of the project are C, D, F, and I.

(b) Carry out a forward pass and a backward pass through the activity network, showing the early event time and late event time at each vertex of your network. **[3]**

(c) State the minimum completion time for the project. **[1]**

(d) Calculate the interfering float at H, given that the independent float is not 0. **[1]**

It is given that the total float of activity G is at most double the total float of activity E. Let x be the duration, in hours, of activity G.

(e) Determine the range of possible values of x. **[3]**

3 A vertex of a graph for which the order is an odd number is called an odd vertex.

(a) (i) By considering the number of arc endings for a general graph, explain why, for every graph, the sum of the vertex orders must be an even number. **[1]**

(ii) Hence explain why no graph can have an odd number of odd vertices. **[1]**

Fig. 3 shows a weighted graph. The weights represent arc lengths.

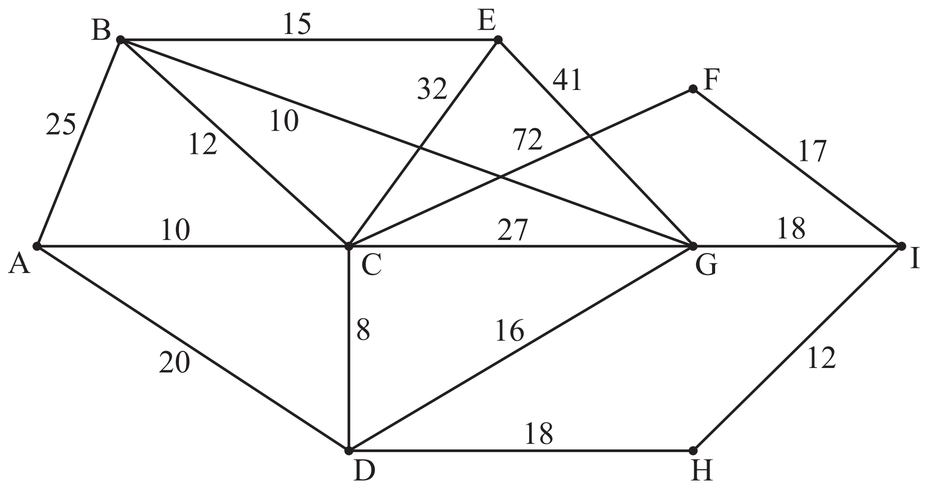

Fig. 3

(b) Apply Dijkstra's algorithm to the copy of the network in the Printed Answer Booklet to find the shortest path from A to F. **[5]**

The following algorithm can be used to find the length of the shortest route in a network that uses every arc at least once, and ends at the vertex where it started.

START with a list of the odd vertices.
STEP 1 For each pair of odd vertices find the shortest path that connects them and the length of this path.
STEP 2 Find the set of paths connecting odd vertices that includes every odd vertex and has the shortest total length.
STEP 3 Add this shortest total to the total weight of the network.
STOP

(c) Complete STEP 1 of the algorithm by completing the table in the Printed Answer Booklet. **[4]**

You are given that the total weight of the network in **Fig. 3** is 353.

(d) Complete STEP 2 and STEP 3 of the algorithm to determine the length of the shortest route that uses every arc of **Fig. 3** at least once, starting and ending at vertex A. **[2]**

4 **Fig. 4.1** represents a system of pipes through which fluid can flow from a source, S, to a sink, T. It also shows two cuts, α and β. The arc weights show capacities in litres per minute.

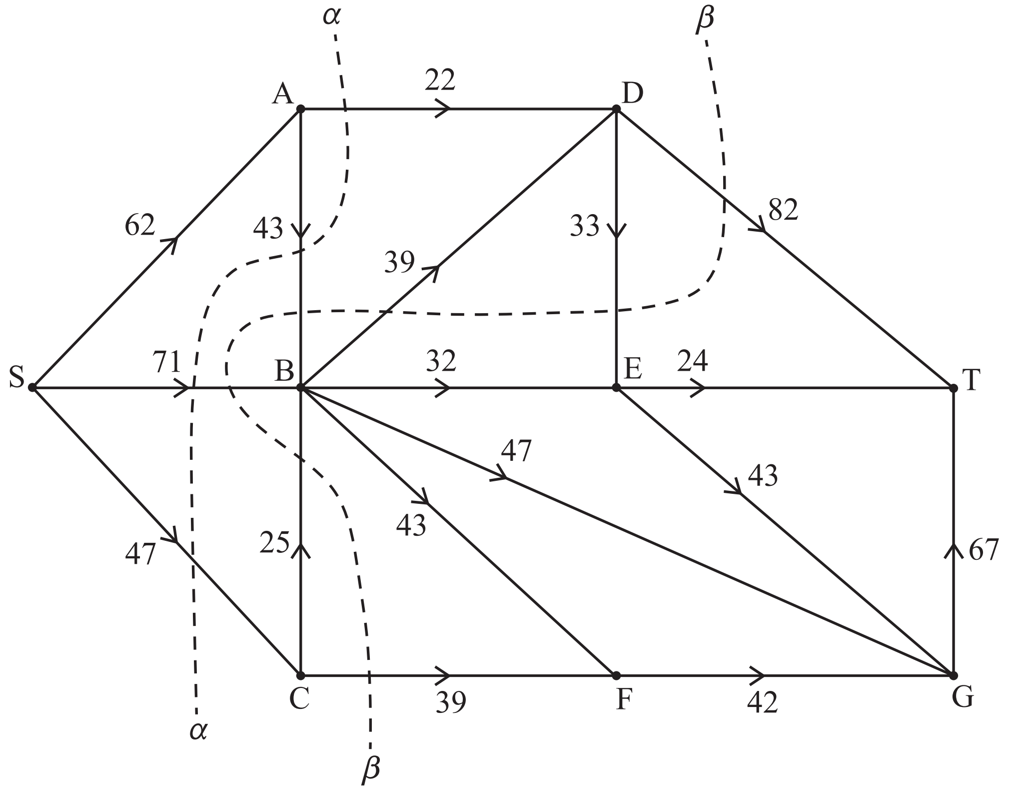

Fig. 4.1

(a) **(i)** Calculate the capacity of the cut α. **[1]**

(ii) Calculate the capacity of the cut β. **[1]**

(b) Using **only** the answers to part **(a)**, state what can be deduced about the maximum possible flow through the system. **[1]**

An LP formulation is set up to find the maximum flow through the network.

Some of the constraints of the LP formulation are shown below.

SA − AB − AD = 0

SC − CB − CF = 0

AD + BD − DE − DT = 0

BF + CF − FG = 0

BG + EG + FG − GT = 0

SA ⩽ 62, SB ⩽ 71, SC ⩽ 47, AB ⩽ 43, AD ⩽ 22, BD ⩽ 39, BE ⩽ 32, BF ⩽ 43, BG ⩽ 47, CB ⩽ 25, CF ⩽ 39, DE ⩽ 33, EG ⩽ 43, FG ⩽ 42

(c) Explain the purpose of the line SC − CB − CF = 0 in the LP formulation. **[1]**

(d) Complete the LP formulation that includes the sink node in the objective function. **[3]**

The complete LP was run in an LP solver and some of the output is shown in **Fig. 4.2**.

VARIABLE	VALUE
SA	34.00000
SC	47.00000
AD	22.00000
CB	8.00000
BD	39.00000
DT	61.00000
EG	0.00000
ET	24.00000
FG	42.00000
GT	67.00000

Fig. 4.2

(e) Use the diagram in the Printed Answer Booklet to show how a flow of 152 litres per minute can be achieved. **[2]**

(f) Prove that this is the maximum possible flow through the system. **[2]**

The capacity of pipe AD is increased to x litres per minute.

(g) Determine the maximum possible flow through the system, stating the corresponding value of x. **[2]**

Turn over for question 5

5 A mathematics competition consists of a written paper that involves questions on the topics of algebra, trigonometry, and calculus only.

From experience of the paper, Leo knows that in the three and a half hours available to complete the paper

- he will answer exactly 50 questions,
- at most half of these will be on the topic of algebra,
- he will answer at least three times as many trigonometry questions as calculus questions.

The table below shows further information about the paper and defines the variables x, y and z.

Type of question	Average time (minutes) to solve each question	Points for each correct answer	Number of questions answered by Leo
Algebra	1	2	x
Trigonometry	4	5	y
Calculus	12	20	z

Leo wants to maximise the number of points that he scores.

(a) The two-stage simplex method could be used to solve Leo's problem.

- Show how the constraints for the problem can be made into equations using slack variables, exactly one surplus variable and exactly one artificial variable.
- Show how the rows for the two objective functions can be formed.
- Complete the initial tableau in the Printed Answer Booklet.

You do **not** need to carry out any iterations of the simplex method. **[9]**

(b) Represent the feasible region for x and y graphically. **[5]**

(c) Explain why the optimal values of x and y can be found by minimising the function $6x + 5y$. **[2]**

(d) **(i)** Hence find the number of each type of question Leo should answer. **[1]**

(ii) State the corresponding number of points Leo will score. **[1]**

(e) Give a reason why Leo may not score the number of points stated in part **(d)(ii)**. **[1]**

END OF QUESTION PAPER

BLANK PAGE